



Department of Civil Engineering

# Pedagogical handout

Title

## **PRESTRESSED CONCRETE**

Course intended for students of

Master 2: Metal and Mixed Construction

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## **PREFACE**

This document is an educational resource presented in the form of a course with some practical applications, dedicated to the subject of pre-stressed concrete. It is based on previously developed lecture materials and is primarily intended for Master's students in Civil Engineering, specializing in steel and composite construction.

Given the broad scope of pre-stressing, only the essential points are addressed here, with particular emphasis on the post-tensioning method, without going into excessive details or lengthy calculations. To facilitate understanding, a clear and accessible scientific language has been adopted, supported by diagrams and numerous practical applications designed to help students assimilate the material and develop a solid grasp of the concepts presented.

The document also includes figures and illustrations from recent construction projects in Algeria (projects conducted under Séro-Est, the \*Company for the Study and Construction of Civil Engineering Structures\*, in the Wilaya of Annaba, with construction sites established in the Wilayas of El Tarf, Guelma, and Annaba). These additions provide a more concrete and realistic view of pre-stressed concrete in practice.

We hope this work will be a valuable aid to our students and contribute to enriching the available didactic resources in this field, thereby helping to promote a broader dissemination of the concept of pre-stressing in Algeria.

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## **LIST OF SYMBOLS**

- $K$**  Stiffness of the beam
- $E$**  Modulus of elasticity of the material
- $I$**  Moment of inertia of the cross-section
- $h$**  height of the beam
- $L$**  span of the beam
- $L_0$**  Initial dimension in mm.
- $\Delta\theta$**  Difference between maximum and minimum temperatures (in °C).
- $\lambda$**  Coefficient of thermal expansion of materials (°C<sup>-1</sup>)
- $t(j)$**  The age of the concrete.
- $r_m (cm)$**  Mean radius.
- $f_{prg}$**  Guaranteed maximum stress at rupture;
- $f_{peg}$**  Stress at the conventional yield limit;
- $\eta$**  Cracking coefficient
- $\psi$**  Anchorage coefficient
- $\sigma$**  The stress,
- $F$**  The applied force,
- $B$**  The cross-sectional area
- $M$**  Bending moment.
- $V_i$**  Distance from the neutral axis (typically to the outermost fiber).
- $I$**  Moment of inertia about the bending axis.
- $B$**  Cross-sectional area [cm<sup>2</sup>]
- $S_x$  and  $S_y$**  Static moments [cm<sup>3</sup>]
- $i_x$  and  $i_y$**  Axial moments of inertia [cm<sup>4</sup>]
- $i_{xy}$**  Centrifugal (product) moment of inertia [cm<sup>4</sup>]
- $i_p$**  Polar moment of inertia [cm<sup>4</sup>]
- $W_x$  and  $W_y$**  Section modulus [cm<sup>3</sup>]
- $W_p$**  Torsional resistance modulus [cm<sup>3</sup>]
- $I_x$  et  $I_y$**  Radius of gyration [cm]
- $\rho$**  Section efficiency ratio.

## **GENERAL INTRODUCTION**

Concrete is one of the most widely used construction materials for bridges, buildings, and industrial or hydraulic infrastructure. Its success stems from high compressive strength, the near-universal availability of its constituents (water, aggregates, cement), and relatively low cost compared to steel or timber. However, concrete performs poorly in tension and shear, limiting its direct use in tension zones of beams and slabs.

Combining concrete and steel led to reinforced concrete, a composite that benefits from concrete's compressive capacity and steel's tensile resistance. While this has enabled countless structures for over a century, cracks inevitably develop under heavy loads in tension zones, shifting tensile demand to steel. Even fine cracking undermines durability by allowing aggressive agents to reach reinforcement, while creep and long-term deformations exacerbate serviceability and lifespan concerns.

Traditionally, engineers increased section sizes and reinforcement ratios, at the expense of heavier, costlier, and sometimes less buildable structures. Prestressed concrete emerged as a more advanced alternative. Pioneered by Eugène Freyssinet in 1928, it subjects concrete from the outset to permanent compression using highly tensioned tendons, preempting service loads so previously vulnerable regions remain compressed or only slightly in tension, thereby avoiding cracking.

Two main methods are used: pre-tensioning, where strands are stressed before casting and bond to hardened concrete (common in precast), and post-tensioning, where ducts within hardened elements receive tendons that are stressed by jacks and anchored (widely used for bridges and tanks). These methods extend spans, reduce member depth and weight.

Beyond transportation, prestressed concrete serves large-capacity tanks, power plants, long-span roofs for industrial and commercial buildings, stadiums, and innovative architecture. In Algeria, use was long limited but accelerated through major projects such as the East–West Highway, which fostered technology transfer, workforce training, acquisition of stressing

equipment, and local production of prestressed elements. While bridges remain the main field, applications are gradually broadening as expertise spreads.

The evolution from plain to reinforced to prestressed concrete reflects sustained engineering ingenuity. Reinforced concrete remains indispensable, but prestressed concrete is essential wherever strength, lightness, and durability must converge, representing a genuine shift toward modern, high-performance, long-lasting infrastructure.

This manuscript unfolds over six chapters.

- **Chapter1** introduces prestressing in concrete: reinforced concrete basics, stiffness, cracking and slenderness, origin and terminology, principle, limitations, advantages, applications, comparison between reinforced and prestressed concrete, and specialized materials.
- **Chapter2** details implementation methods: historical background, prestressing force, post-tensioning (steps, advantages, equipment: ducts, anchorages, jacks), pre-tensioning (steps, pros and cons), and a comparison of both approaches.
- **Chapter3** covers materials: concrete placing and physical properties (density, thermal expansion, Poisson's ratio, shrinkage, creep), passive and active reinforcement, required qualities, and geometric/mechanical characteristics.
- **Chapter4** treats isostatic beam design at SLS: section properties, load combinations, normal stress checks, sizing of prestressing force, selection and placement of reinforcement, cable profiles and layouts, passage zones, shear verification, and complementary provisions.
- **Chapter5** addresses ULS resistance: combinations and assumptions, internal forces, neutral axis depth, moment resistance, shear checks, and local verifications at ends and anchorages. Finally,
- **Chapter6** quantifies prestress losses: initial tension, instantaneous losses (friction, anchorage slip, instantaneous concrete strain), delayed losses (shrinkage, creep, steel relaxation), and total loss assessments.

# CHAPTER I: INTRODUCTION TO PRESTRESSING IN CONCRETE

## 1. INTRODUCTION

Concrete is a heterogeneous material that possesses excellent compressive strength but very low tensile strength. In bending, this weakness leads to a sudden and brittle failure because concrete cannot withstand tensile forces, which represents a significant risk to the integrity of unreinforced structures.

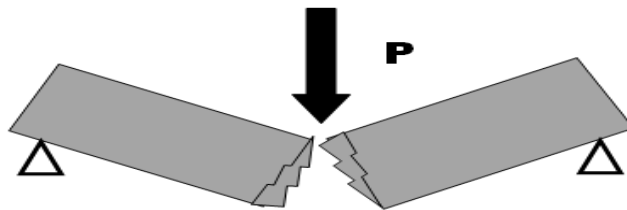


Figure 01. Absence of reinforcement: sudden failure.

Thus, a beam resting on two supports, subjected to the effects of its self-weight ( $G$ ) and a live load ( $Q$ ), experiences bending stresses. These stresses result in a compressed zone in the upper part of the beam and a tensioned zone in the lower part.

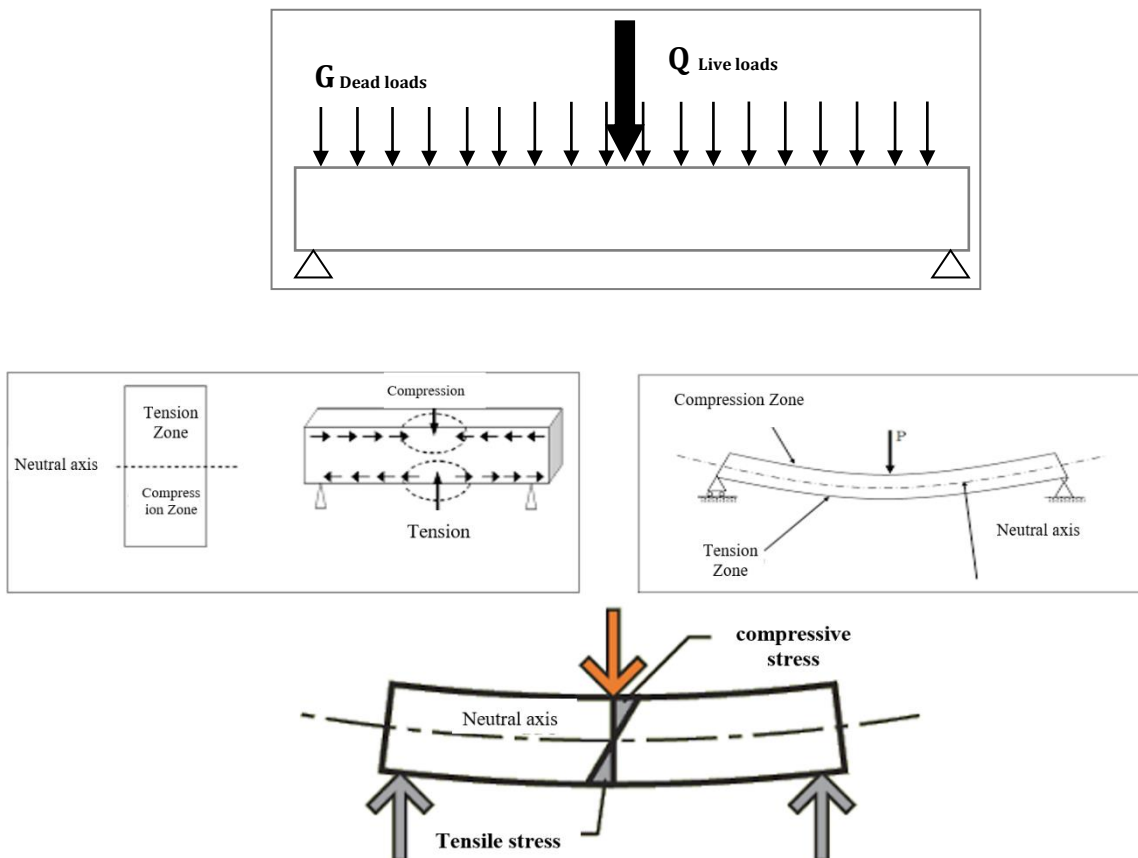


Figure 02. Behavior of a beam (tension zone – compression zone)

## ✚ QUESTION 01

In the case of Figure I.01,

**What solution can be considered to resist tensile forces in concrete?**

## ✚ ANSWER 01

The solution is to add a sufficient amount of reinforcement in the tension zone in order to eliminate the risk of failure, according to the principle of reinforced concrete.

### ***1. REINFORCED CONCRETE***

Reinforced concrete is a composite material that combines the high compressive strength of concrete with the high tensile strength of steel. Since its introduction in the 19th century, it has become a key element in modern construction, used in infrastructures ranging from buildings and bridges to marine structures.

#### ***1.1. Characteristics of Reinforced Concrete***

- **Compressive strength:** Reinforced concrete is known for its ability to withstand heavy compressive loads, making it a preferred choice for weight-bearing structures (walls, columns).
- **Low tensile strength:** One of the main disadvantages of concrete is its low tensile strength. This weakness is compensated by inserting steel reinforcement in the tension zones of the concrete, thus ensuring better performance.
- **Cracking and corrosion:** When concrete is subjected to tension, it cracks, and these cracks can lead to the corrosion of the reinforcement. This affects the long-term durability of structures, especially in aggressive environments (moisture, salts).

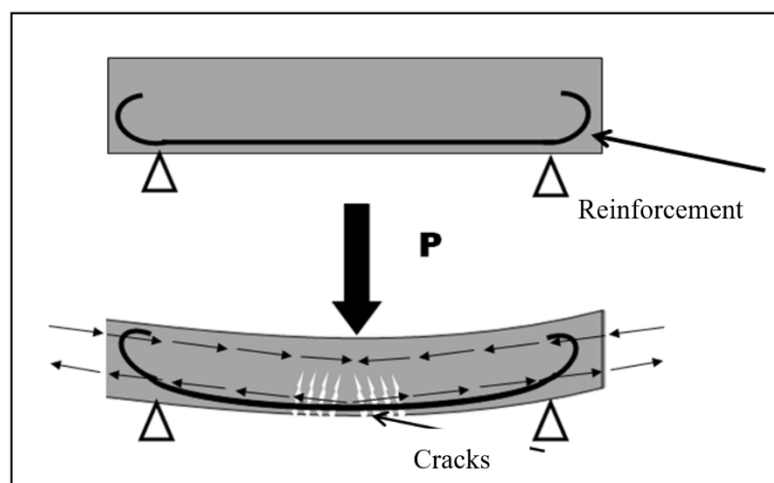


Figure 03. Placement of reinforcement in the tension zone of concrete.

Distributed cracking under tensile stress in concrete.

## ***1.2. Role of Reinforcement***

Reinforcement plays a crucial role by supporting the tensile forces that concrete cannot handle on its own. In areas where the concrete is under tension (zones of bending or tension), the reinforcing steel takes up all the tensile force. However, cracking of the concrete remains an issue that must be controlled to prevent the deterioration of structural elements.

## ***1.3. Solutions for Cracking***

**Crack control:** To limit cracking, it is essential to control the deformations in the tensioned reinforcement by keeping the stresses in the steel below defined thresholds. This helps to limit crack openings and prevent long-term corrosion problems.

**Limiting deformations:** To avoid excessive deflection (deformation) in reinforced concrete structures, it is common to design elements with a certain height-to-span ratio (generally between  $\frac{1}{10}$  et  $\frac{1}{16}$  for beams). However, when the span becomes too large, reinforced concrete can become less competitive compared to steel.

## ***2. STIFFNESS OF MATERIALS: FROM STABILITY TO CRACKING***

### ***2.1. Definition of stiffness***

La rigidité est définie comme la capacité d'un matériau à résister à la déformation lorsqu'il est soumis à une force. Elle est mesurée par le module de rigidité, également connu sous le nom de module de Young, qui quantifie la relation entre la contrainte, exprimée en force par unité de surface, et la déformation, qui représente le changement de longueur du matériau. Cette propriété mécanique est essentielle pour évaluer la performance et la durabilité des matériaux dans diverses applications d'ingénierie.

### ***2.2. Uncracked and Rigid Materials:***

Uncracked materials, such as intact reinforced concrete beams, are considered rigid. The stiffness of a beam can be calculated using the formula  $K = E \cdot I$ ,

where:

- $K$  = stiffness of the beam
- $E$  = modulus of elasticity of the material
- $I$  = moment of inertia of the cross-section

High stiffness ensures that beams retain their original shape under load, thus minimizing excessive deformations.

### ***2.3. Consequences of Cracking***

Cracking in a material leads to a reduction in its inertia, which decreases its overall stiffness. This reduction in stiffness results in increased deflection, compromising structural integrity. Cracks can develop in both the compressed and tensioned zones of beams, affecting their load-bearing capacity.

### ***2.4. Issues Associated with Reinforced Concrete***

Reinforced concrete is designed to resist compressive forces, but cracks can weaken this capacity. Defects related to cracking can reduce the durability and safety of reinforced concrete structures. It is essential to anticipate these issues during design and construction.

### ***2.5. Slenderness and Deflection***

The slenderness of a beam or structural element corresponds to the ratio between its length (L) and its height (H). This parameter plays a key role in structural design, especially when choosing between reinforced concrete and prestressed concrete.

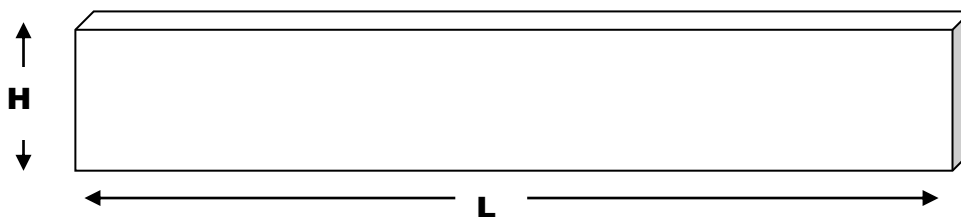
#### **Optimal Slenderness Ratio for Concrete**

In the case of reinforced concrete structures, to ensure sufficient resistance to deformation and to avoid excessive deflection, it is often necessary to design elements with moderate slenderness, generally between 1/10 and 1/16. However, for larger spans, this ratio can lead to massive sections and heavy structures.

$$\frac{1}{16} < \frac{h}{L} < \frac{1}{10}, \text{ where:}$$

- **h**: height of the beam
- **L**: span of the beam

For example, for a beam 20 meters long, the height h should be at least 2 meters.



### Limitations of Reinforced Concrete Beams

- Reinforced concrete beams generally have a maximum span of 10 meters due to limitations in stiffness and durability.
- Beyond this span, it may be more economical to switch to stiffer materials, such as steel, to ensure the safety and performance of the structure.

#### QUESTION 02

In the case of Figure 03,

**How can cracks be eliminated in reinforced concrete?**

#### ANSWER 02

To eliminate cracks in reinforced concrete, it is necessary to apply an axial compressive force that counteracts the tensile stresses caused by loading, following the principle of "prestressed concrete."

### 3. *PRESTRESSING*

#### *3.1. Origin and Terminology of Prestressing in Reinforced Concrete*

The term "**prestressing**" in the context of concrete comes from the combination of the prefix "**pre-**", meaning "**before**", and the word "stressing," which refers to a force or tension applied to a material. The idea **behind prestressing** is to introduce compression into the concrete before it is subjected to external loads.

Therefore, **prestressing** literally means "applying a stress beforehand," in this case, before the application of external loads, in order to eliminate the tensile strength of the concrete.

#### *3.2. Principle of Prestressing*

Prestressing is an innovative technique that eliminates the resistance of concrete to tensile stresses. It involves applying compressive forces in the tension zones of the concrete even before the structure is subjected to external loads.

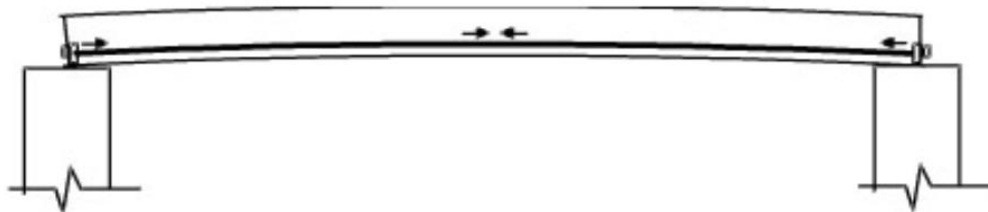


Figure 04. Beam under the effect of an axial load in the tension zone (Prestressing).

### 3.3. Definition of Prestressing

Prestressing is defined as a process that involves subjecting an element to compressive forces before the application of loads. These forces counteract the tensile stresses induced by external loads. The objective is to completely or partially eliminate tension in the concrete in order to prevent any cracking.

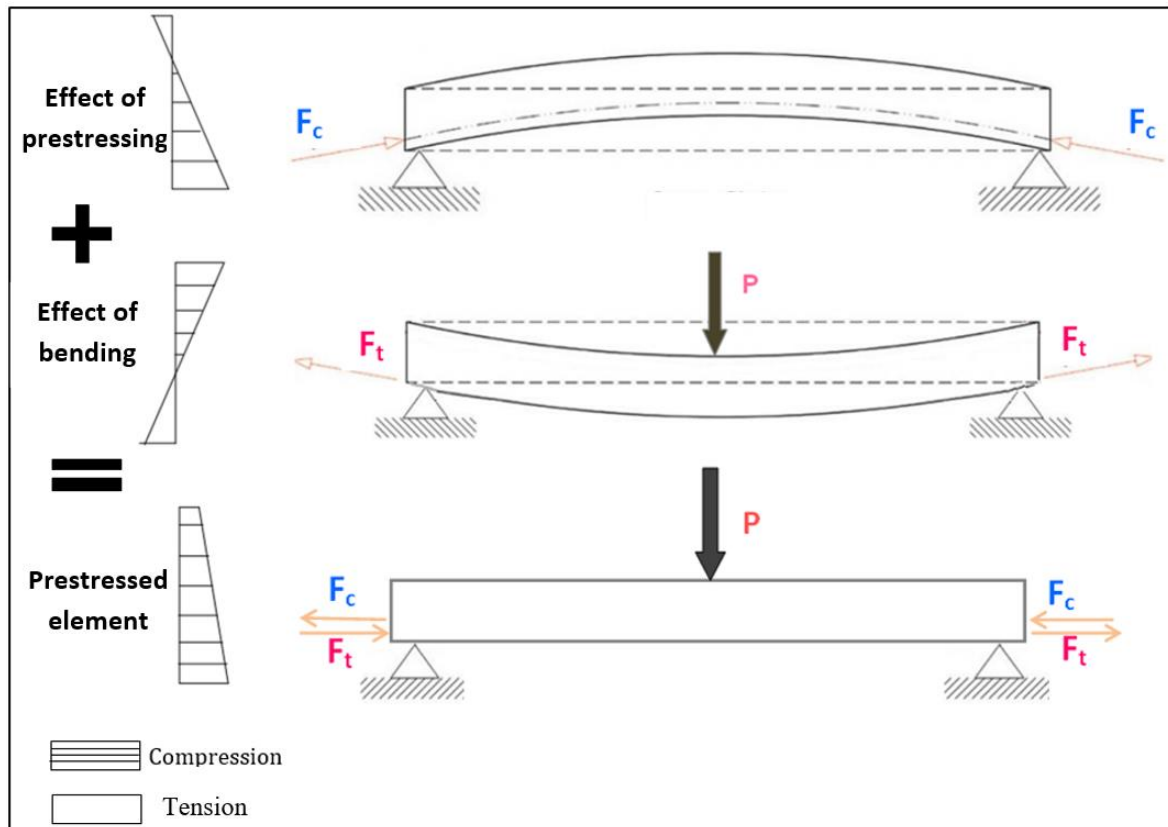


Figure 05. Interaction between the effect of prestressing and bending in a prestressed beam.

Diagram illustrating the effect of prestressing and bending on a beam.

### 3.4. Limitations and Constraints

Although prestressing offers many advantages, it is not without limitations. It requires the use of high-quality materials, such as high-strength steel and specially designed concretes capable of withstanding high compressive forces. In addition, the implementation of prestressing demands technical expertise and specialized equipment, which can increase initial costs.

### 3.5. Slenderness and Prestressed Concrete

One of the major advantages of prestressed concrete is the ability to create elements with greater slenderness without compromising their stiffness or resistance to deformation. Indeed, thanks to the initial compression introduced by prestressing, the cross-section

remains better compressed, allowing for a reduced element height while maintaining performance.

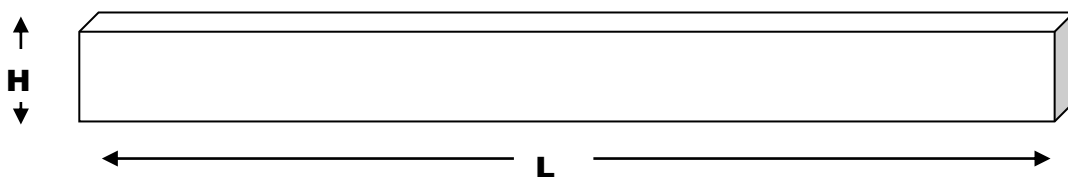
### **Optimal slenderness ratio for prestressed concrete**

Prestressed concrete allows for the design of elements with a higher slenderness ratio, between 1/25 and 1/17 (H/L). This means that the height can be much smaller compared to the length, while still maintaining optimal stiffness and strength.

$$\frac{1}{25} < \frac{h}{L} < \frac{1}{17}, \text{ where:}$$

h: height of the beam, and L: span of the beam.

For example, for a beam 20 meters long, the height h should be at most 0.8 meters.



### ***3.6. Advantages of Prestressing***

- **Elimination of cracking:** By creating a state of compression in the concrete, prestressing prevents the formation of cracks. This improves the durability of the structure by preventing corrosion of the reinforcement.
- **Improved stiffness:** Prestressing helps maintain the stiffness of the section. In the absence of cracks, the concrete remains rigid, which limits deformations under load.
- **Optimization of materials:** Thanks to the absence of cracks and the preservation of stiffness, it is possible to reduce the cross-sections of structural elements, allowing for material savings.

### ***3.7. Applications of Prestressing***

- **Bridges and viaducts:** Large-span structures, such as bridges, greatly benefit from prestressing, as it allows for a reduction in the weight of the structures while increasing their load-bearing capacity.
- **Tunnels and pipelines:** In underground infrastructures, prestressing helps maintain the integrity of concrete walls despite the high pressures exerted by the soil or fluids.

#### 4. COMPARISON BETWEEN REINFORCED CONCRETE AND PRESTRESSED CONCRETE

**Table 1: Comparison between reinforced and prestressed concrete**

Characteristic	Reinforced Concrete	Prestressed Concrete
<b>Cracking</b>	Often present	Eliminated or controlled
<b>Stiffness</b>	Reduced when cracking occurs	Maintained, no cracking
<b>Material usage</b>	Reinforcement limited by cracking	Optimal, no limitation of stress in reinforcement
<b>Material economy</b>	Less economical for large spans	More economical for longer spans

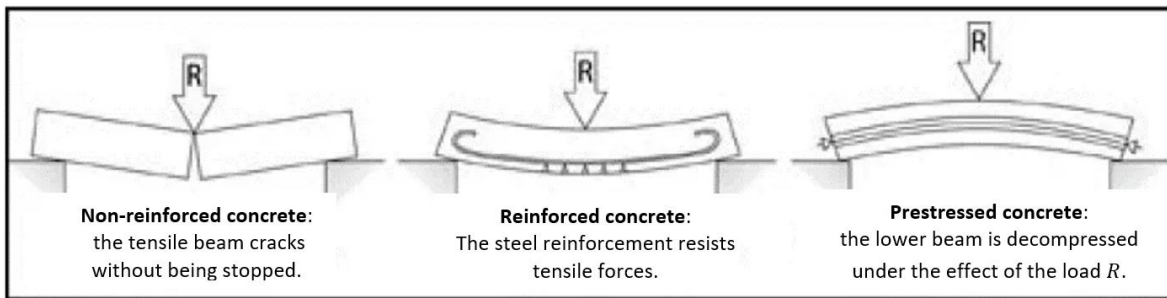


Figure 06: Diagram illustrating the differences between plain concrete, reinforced concrete, and prestressed concrete.

#### 5. SPECIAL MATERIALS FOR PRESTRESSED CONCRETE

Prestressed concrete uses special materials that ensure high mechanical performance and increased durability, thus meeting strict standards to withstand the stresses associated with prestressing. The use of these materials offers several advantages: high mechanical strength allowing for the reduction of cross-sections of load-bearing elements and the creation of lighter structures, improved durability thanks to optimal concrete formulation and protection of steel against corrosion, as well as low long-term maintenance, thereby reducing operating costs.

##### 5.1. Prestressing Steel

Prestressing steel is a special material designed to provide a tensile strength of about 1600 MPa. Used in the form of cables or tendons, this homogeneous steel ensures uniform prestress throughout the structure. Its design allows it to resist both tension and compression, thus ensuring the performance of prestressed structures. Moreover, to prevent

the harmful effects of corrosion, cables and tendons are often protected by sheaths or anti-corrosion coatings, enhancing their durability and effectiveness.

### 5.2. Concrete

In prestressed concrete, the use of high-performance concrete (HPC) with a minimum strength of 40 MPa is essential to withstand prestressing forces. Proper cement content (up to 500 kg/m<sup>3</sup>) and a controlled mix design ensure durability and low porosity, improving resistance to freeze-thaw cycles and aggressive agents. HPC provides better compressive strength and limits cracking, thus ensuring the longevity of structures.

### 5.3. Cement Used for Prestressed Concrete

In prestressed concrete, it is crucial to use high-quality cement, such as CEM I CPA (Ordinary Portland Cement). This cement, composed of 95% clinker and 5% gypsum, guarantees excellent mechanical strength. For prestressed structures, it is recommended to use a type R (Rapid) cement, which allows for rapid strength development, as opposed to type N (Normal), with strength classes often of 52.5 MPa, ensuring optimal solidity for these highly stressed elements.

#### 5.3.1. Types of Cement

Below is a summary table of the different types of cement (CEM I, II, III, IV, and V) as well as their characteristic strength (in MPa) and typical composition:

**Table 2. Types of cement.**

	<b>Cement Type</b>	<b>Clinker %</b>	<b>One of the following constituents (slag, pozzolans, fly ash, limestone, shale, silica fume) %</b>	<b>Secondary constituents %</b>	<b>Strength limit in MPa</b>
CPA	CEM I	95 - 100		0 to 5	32.5 - 42.5 - 52.5
CPJ	CEM II A	80 - 94	• 6 - 20 of one of the constituents, except silica fume (max 10%) and fillers (max 5%)		32.5 - 42.5 - 52.5
	CEM II B	65 - 79	• 21 - 35, with the same limits as above		
CHF	CEM III A	35 - 64	• 36 - 65 % slag.		32.5 - 42.5 - 52.5
	CEM III B	20 - 34	• 66 - 80 % slag.		
	CEM III C	5 - 19	• 81 - 95 % slag.		
CPZ	CEM IV A	65 - 90	• 10 - 35 % pozzolans, fly ash, silica fume	32.5 - 42.5 -	

	CEM IV B	45 – 64	(max 10%) • 36 – 55 % pozzolans, fly ash, silica fume (max 10%)		52.5
CLC	CEM V A	40 – 64	• 18 – 30 % slag and 18 – 30% fly ash or pozzolans.		32.5 - 42.5 -
	CEM V B	20 - 39	• 31 - 50 slag and fly ash or pozzolans.		52.5

**General Use:**

- **32.5 MPa cement:** Used for general works such as foundations, screeds, and light structures.
- **42.5 MPa cement:** Suitable for structural elements such as beams, slabs, and load-bearing walls.
- **52.5 MPa cement:** Used in specialized projects, such as high-strength structures, prestressed elements, and constructions in severe conditions (such as marine environments or areas exposed to freeze/thaw cycles).

**Note:** A higher-strength cement generally requires a finer fineness, a higher CaO content, and a well-balanced composition, thus promoting improved performance.

Fineness, CaO content, and cement composition are key parameters that vary depending on the strength of the cement. These differences influence not only short-term performance (setting and hardening) but also the durability and longevity of concrete structures.

**5.3.2. Other Types of Cement**

**5.3.2.1. Rapid (R) and Normal (N) Cements**

The designations R (Rapid) and N (Normal) in cement types refer to the specific setting and hardening characteristics of the cement. Here is a detailed explanation:

**Tableau 3. Types of cement R & N**

Gnation	Meaning	Characteristics	Use
R Cement	Rapid	Fast setting and hardening	Works requiring rapid hardening
N Cement	Normal	Standard setting and hardening	General use in construction

**5.3.2.2. CRS and HTS Cement**

**a. CRS (Sulfate-Resistant Cement):**

- **Use:** This cement is specifically designed to withstand environments where concrete is exposed to high concentrations of sulfates, such as in soils, groundwater, or wastewater.
- **Properties:** It is formulated with a reduced tricalcium aluminate (C3A) content, making it less reactive to sulfates. Sulfates can cause expansion and cracking in concrete, but CRS minimizes these effects.
- **Applications:** Ideal for structures in contact with aggressive environments such as hydraulic works (wastewater treatment plants), marine structures (coastal works), foundations in sulfate soils, and structures exposed to industrial environments.

**b. HTS (High Silica Cement):**

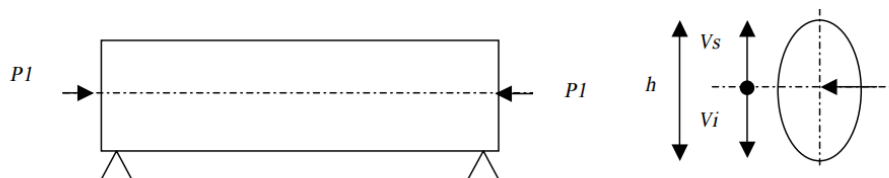
- **Use:** High silica cement is a type of cement that contains a higher proportion of silica ( $\text{SiO}_2$ ). This high silica content improves the long-term strength and durability of concrete, especially in alkaline or acidic environments.
- **Properties:** Thanks to the addition of silica, this cement has increased mechanical strength and greater durability under severe conditions. It is more resistant to chemical attacks and corrosion. The addition of silica can also reduce the porosity of the concrete, thereby increasing its durability.
- **Applications:** HTS cement is often used in environments exposed to aggressive chemical conditions, such as power plants, marine infrastructures, or structures in contact with aggressive waters. It is also preferred for constructions requiring high durability and a long service life.

## 6. APPLICATION EXERCISES

### Exercise 1:

Consider a beam with a rectangular cross-section  $B$  and a moment of inertia  $I$ , subjected to a bending moment  $M$  and a centered prestressing force  $P$ .

- Draw the stress distribution diagram.

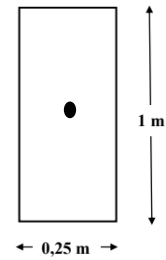


### Exercise 2:

Consider a beam with a rectangular cross-section ( $b \times h = 25 \times 100$ )cm, subjected to an external moment  $M = 0.7\text{MN}\cdot\text{m}$  and a centered prestressing force  $P_1$ .

If the final tensile stress in the lower fibers is assumed to be zero:

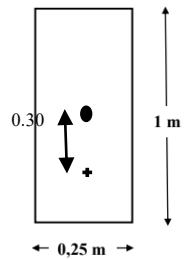
- Draw the stress distribution diagram.
- Deduce the value of  $P_1$ .



**Exercise 3:**

Consider the same beam, subjected to an external moment  $M = 0.7\text{MN}\cdot\text{m}$  and an eccentric prestressing force  $P_2$  with an eccentricity  $e = -30\text{cm}$ .

- Draw the stress distribution diagram.
- Deduce the value of  $P_2$ .
- Provide a conclusion on the two cases.

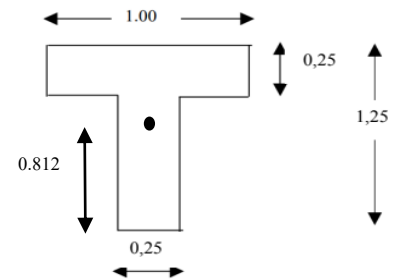


**Exercise 4:**

Consider a T-shaped beam, as shown below, subjected to an external moment  $M = 0.8\text{MN}\cdot\text{m}$  and a centered prestressing force  $P_1$ .

If the final tensile stress in the lower fibers is assumed to be zero:

- Draw the stress distribution diagram.
- Deduce the value of  $P_1$ .

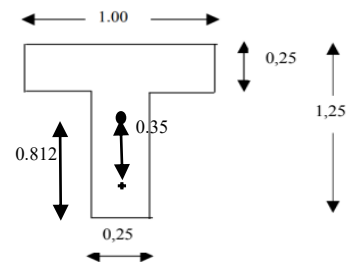


**Exercise 5:**

Consider the same beam, subjected to an external moment  $M = 0.8\text{MN}\cdot\text{m}$  and an eccentric prestressing force  $P_2$  with an eccentricity  $e = -35\text{cm}$ .

If the final tensile stress in the lower fibers is assumed to be zero:

- Draw the stress distribution diagram.
- Deduce the value of  $P_2$ .
- Provide a conclusion on the two cases.



**Exercise 6:**

Consider a beam with a rectangular cross-section ( $50 \times 120$ )cm, subjected to moments  $M_{\min} = 1.2\text{MN}\cdot\text{m}$  and  $M_{\max} = 3.25\text{MN}\cdot\text{m}$ .

The prestressing force and its eccentricity are given as  $P = 5.1\text{MN}$  and  $e_0 = -0.44\text{m}$ .

Determine the stress distribution diagram under maximum and minimum moments.

## ***CHAPTER 02: METHODS OF PRESTRESSING IMPLEMENTATION***

### ***HISTORICAL BACKGROUND***

Prestressing, an essential technique in modern construction, was introduced in 1928 by Eugène Freyssinet, who enabled concrete to work only in compression. According to him, prestressing consists of applying additional forces before service loads appear, in order to keep stresses within the material's allowable limits. This principle prevents cracking and improves the durability of concrete. However, prestressing is not a recent innovation: its roots can be traced back to ancient practices, such as the banding of barrels or wagon wheels, which already foreshadowed the use of this technique.

The idea of prestressing applied to concrete first appeared in the 19th century with the patents of P.H. Jackson (1860) and Dohering (1888). They sought to eliminate concrete cracking by tensioning the reinforcement. However, their efforts were unsuccessful due to phenomena that were poorly understood at the time, such as shrinkage and creep, as well as the use of unsuitable steel quality.

Eugène Freyssinet truly perfected prestressing by understanding the importance of creep and shrinkage from his experiments on the Plougastel Bridge arch in 1926. His method, based on introducing compression to compensate for future tension, laid the foundation for the rise of prestressed concrete after World War II.

This technique was first used for bridges, such as those over the Saale in Germany (1928) and in Aue (1936), and then extended to many other applications, including reservoirs, poles, slabs, and various structures. Today, prestressing can be performed internally, where the cables are embedded in the concrete, or externally, used for large structures such as bridge decks, thus facilitating maintenance. Tensioning, whether internal or external, is a delicate operation that requires precise measurements and rigorous expertise.

#### ***1. THE PRESTRESSING FORCE***

As previously mentioned, prestressing is generally introduced into concrete via tensioned steel reinforcement—wires, strands, or bars—which exert a force on the concrete. High-strength steel is most often tensioned using hydraulic jacks. This tensioning can be performed either

before or after the concrete is poured, resulting in either pre-tensioned or post-tensioned prestressed concrete. This prestressing force can be applied in various ways, with the two most commonly used methods defined according to these modes of prestressing.

**2. METHODS OF PRESTRESSING**

To carry out the prestressing operation, there are two possibilities:

- Post-tensioning prestressing
- Pre-tensioning prestressing

**2.1. Post-tensioning Prestressing**

Post-tensioning is achieved using active reinforcement (cables or strands) that are tensioned after the concrete has been poured and hardened. The concrete must have sufficient compressive strength to withstand the prestressing forces to which it will be subjected. In this process, the cable, housed in its duct and subjected to the action of the jack to which it is attached, elongates and is tensioned to force  $F$ .

The jack, which bears against the concrete of the member to be prestressed, exerts a compressive force equal to  $F$  (principle of action and reaction).  $P$  is thus the prestressing force developed by the cable. Once tensioned, the cable is anchored to the concrete, ensuring the permanence of the compression.

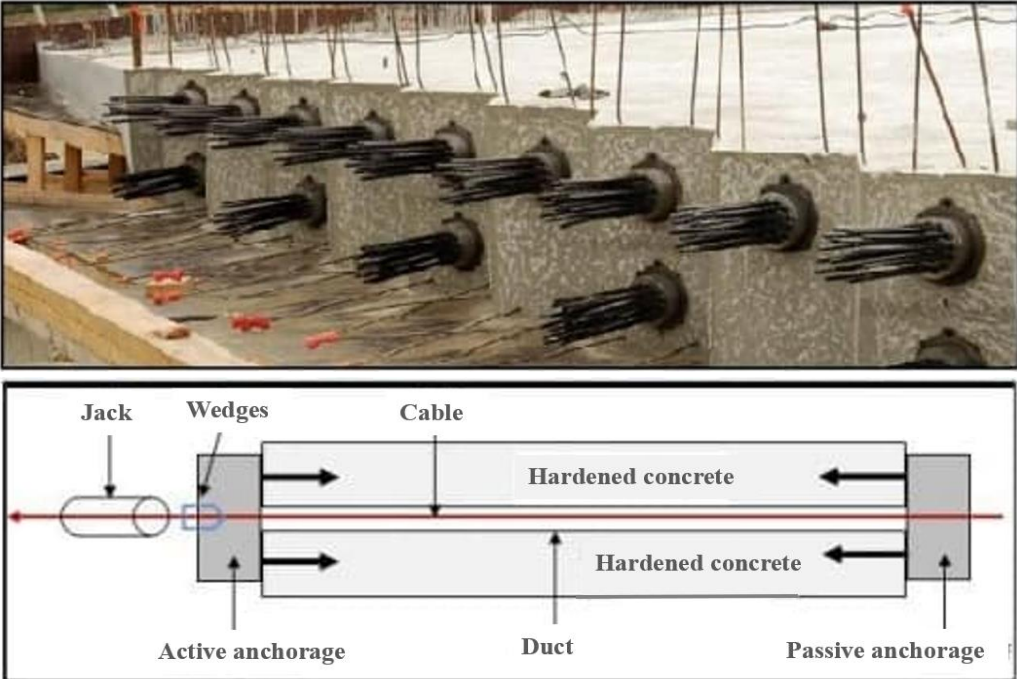


Figure 01. Post-tensioning prestressing process.

### *2.1.1. Steps for Implementing Internal Post-Tensioning Prestressing*



Figure 02. Placement of ducts in the formwork.

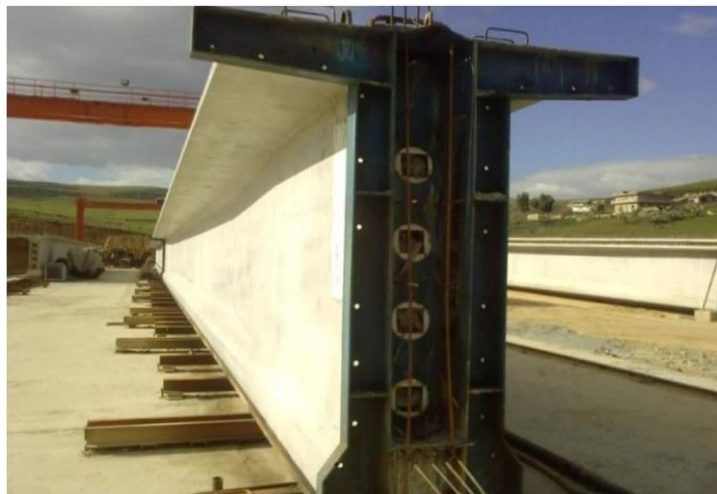


Figure 03. Concrete pouring.



Figure 04. After the concrete has hardened, insertion of the cables into the ducts and their tensioning.



Figure 05. Filling the ducts with injection grout.

**Note:**

In this prestressing process, the cables can be tensioned in two directions by tensioning the cables at both ends (*Active-Active method*) or in one direction only (*Active-Passive method*).

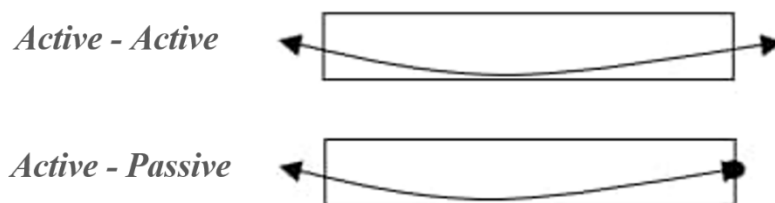


Figure 6. Cable tensioning methods.

### ***2.1.2. Advantages of Post-Tensioning Prestressing:***

1. This method has the advantage of allowing the placement of prestressing reinforcement along curved paths without losing their positions after concrete pouring.
2. Since the prestressing jacks bear directly on the structural element itself, this method does not require any fixed installation.
3. Post-tensioning prestressing allows for various shapes to be chosen for the elements to be prestressed.
4. Before the element is put into service, this method allows for the adjustment of the prestressing force according to the evolution of the loads that the element will have to support.

### ***2.1.3. Equipment Required for Post-Tensioning Prestressing***

**2.1.3.1. Ducts:** These are metal (or plastic) conduits made of strips 0.4 to 0.6 mm thick, wound in a spiral. They are used to hold and protect the prestressing cables after concreting. They must be watertight, flexible, and ensure good bonding with both the concrete and the injection grout.



Figure 7. Metal duct.

### 2.1.3.2. Anchorages:

Anchorages are used to secure the prestressing cables against the hardened concrete of the element. This is a mechanical device necessary to ensure the permanence of the prestressing force on the concrete after the cables have been tensioned. For this reason, this device is extremely important, as the durability and longevity of the structure depend on it. There are two types of anchorages:

#### A. Active Anchorage

This is where the jacks bear during tensioning; it is used to lock the cable at the end where the tensioning is carried out and to ensure the transfer of the prestressing force to the concrete.

It consists of:

- An anchorage plate: This is a part generally made of cast iron, standardized steel, or heat-treated steel, with a strength between 500 and 1000 MPa. Also called the anchorage head, this part is most often cylindrical and provided with holes for the passage of strands and for housing the wedges. It is also equipped with a hole that allows for the injection of grout after the cables have been tensioned.



Figure 8. Anchorage plate (active anchorage).

- The anchorage wedge: This is the second key component for the safety of the anchorage. It is made of steel and operates on the wedge principle, locking the strand inside the anchorage head. Generally, the anchorage wedge takes the form of a cone or a truncated key made of 2 or 3 pieces.

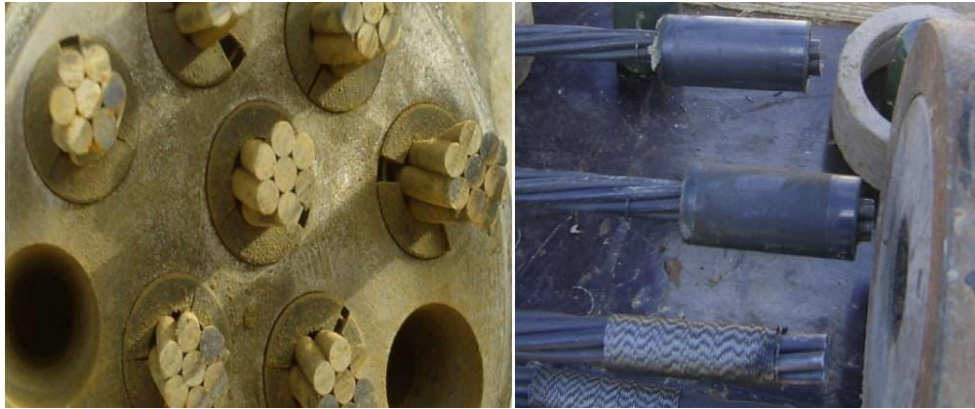


Figure 9. Anchorage wedges.

The bearing plate: This is a metal plate made of steel or cast iron, designed to distribute the prestressing force from the anchorage head to the structure. The bearing (or distribution) plate must ensure both the proper distribution of the prestressing force from the cables to the concrete and the deviation of the strand bundle between the main part of the cable and the area where it passes through the anchorage plate, where it spreads out.

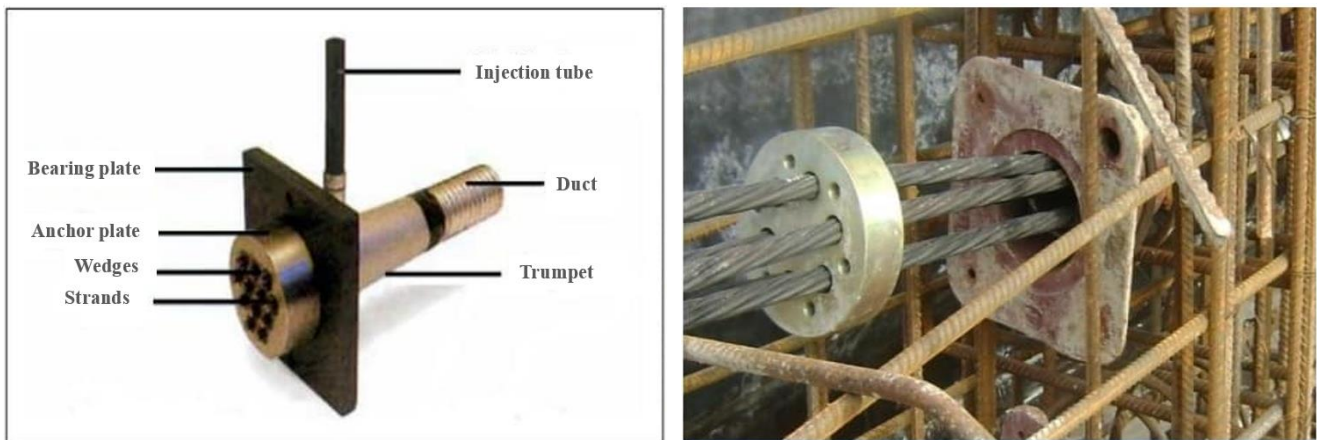


Figure 10. Bearing plate.

***B- Passive (or fixed) anchorage:*** Generally composed of the same components as the active anchorage, this type of anchorage, unlike the previous one, does not allow the tensioning of its cables. It is used in the case of short cables, where the savings from using a passive anchorage can offset the additional steel costs due to friction losses, or when only one end is accessible (for example: anchor rods in rocks). There is the external passive anchorage, which remains accessible after concreting, and the passive anchorage embedded in the concrete, which works either by bearing or by bonding.



Figure 11. Anchorage plate (passive anchorage).

### **2.1.3.3. Jacks:**

Essential for tensioning the cables, they are connected to hydraulic pumps that must be properly calibrated. The jacks used for tensioning the prestressing steel are conventional coaxial jacks with various variants depending on the type of anchorages.



Figure 12. Hydraulic jack.

## **2.2. Pre-tensioning Prestressing:**

Pre-tensioning prestressing is carried out using active reinforcement (cables, bars, or strands) that are tensioned on a tensioning bed before the concrete is poured. In this process, the prestressing steel is first tensioned using jacks bearing against an abutment. When the required elongation is reached, the prestressing steel is locked in place with anchorages. Next, the concrete is poured in direct contact with the prestressing steel within the formwork molds that surround them. Once the concrete has reached sufficient strength (after hardening), the prestress is transferred. The prestressing reinforcement is then released from its anchorages

and cut outside the cast element. Since this reinforcement cannot return to its original length, the force is transferred by bond to the concrete, and the element becomes prestressed.

Pre-tensioning prestressing is mainly used for the mass production of identical prefabricated elements. This prefabrication is done primarily in a factory, but sometimes it can also be done on the construction site itself. Depending on the type of element being prefabricated, the profile of the prestressing steel, which is straight, consists of a single straight line or several straight segments. For pre-tensioning, seven-wire strands are frequently used as prestressing steel, with each strand tensioned individually.

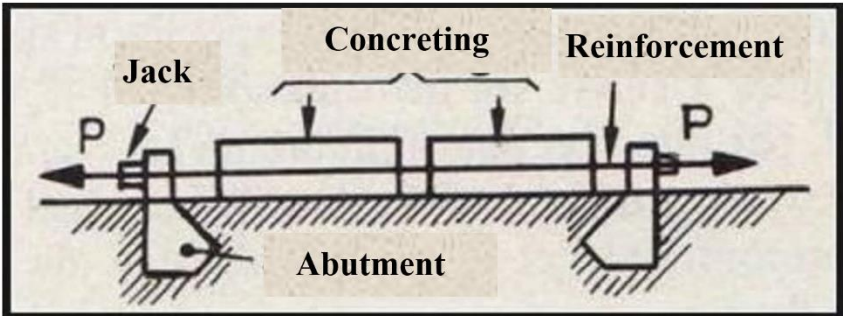


Figure 13. Principle of pre-tensioning prestressing.

**2.2.1. Steps for implementing internal pre-tensioning prestressing:**

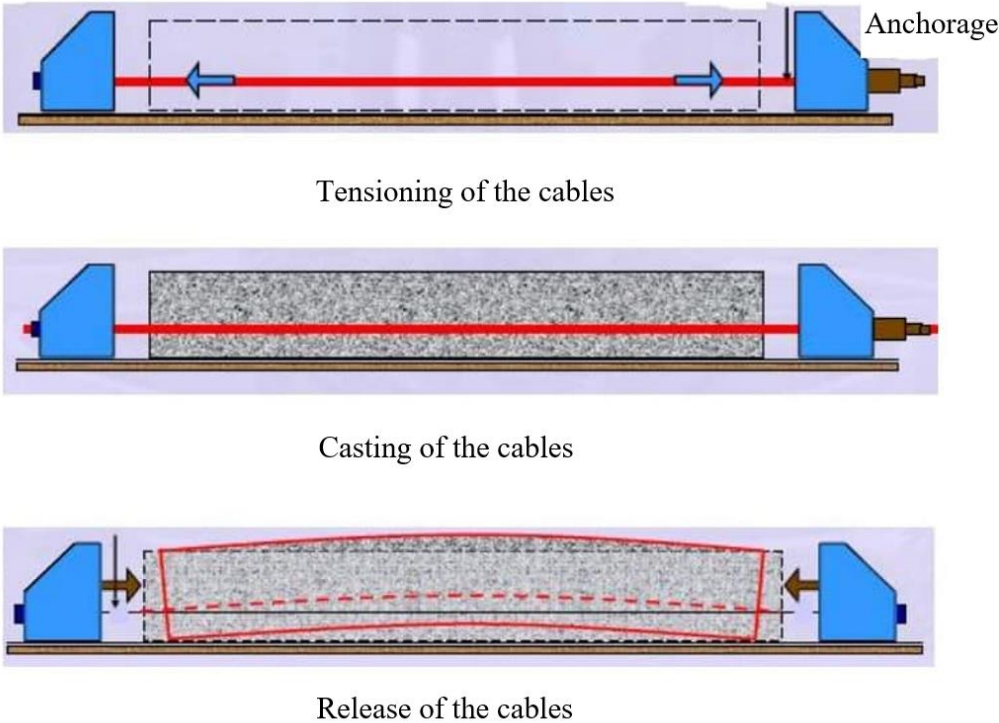


Figure 14. Steps for carrying out pre-tensioning prestressing.

### ***2.2.2. Advantages and Disadvantages of Pre-tensioning Prestressing:***

Compared to post-tensioning, pre-tensioning ensures good bonding between the prestressing steel and the concrete. It also offers savings in terms of ducts, anchorage devices, and the grouting operation. However, this process requires very heavy installations, which limits the choice of shapes for the elements to be prestressed. The major disadvantage of this method lies in the difficulty of creating curved profiles for the prestressing reinforcement and adjusting the intensity of the prestressing force once the concrete has hardened.

### ***3. Comparison of the Two Methods***

A comparison between the two methods (post-tensioning and pre-tensioning) leads to the following observations:

- ***Pre-tensioning***
  1. Savings on ducts, anchorage devices, and the grouting operation.
  2. Requires very heavy installations, which consequently limits the choice of shapes.
  3. Simplicity in the execution of the process.
  4. Good cooperation between the concrete and the reinforcement.
  5. Difficulty in creating curved reinforcement profiles.
  6. Impossible to adjust the force in the reinforcement after tensioning.
- ***Post-tensioning***
  1. Does not require any fixed installation since the prestressing jack bears directly on the element itself.
  2. Allows for a variety of shapes to be chosen.
  3. Possibility to adjust the prestressing force, making it possible to adapt the process to changes in the structure's mass.
  4. Ease of creating curved profiles for the prestressing reinforcement.

## **4. APPLICATIONS**

AN EDUCATIONAL SITE VISIT WITH THE STUDENTS, FOCUSING ON THE USE OF  
PRE-STRESSED CONCRETE.

## ***CHAPTER 03: CHARACTERISTICS OF MATERIALS USED IN PRE-STRESSED CONCRETE***

### ***INTRODUCTION***

Concrete is a heterogeneous material composed of a mixture of binder (cement), aggregates, water, and possibly admixtures. In its hardened state, concrete is characterized by its high compressive strength and low tensile strength. Reinforced concrete combines concrete with high-adhesion steel reinforcements to form elements of various shapes that resist both tension and compression. In prestressed concrete structures, in addition to these two materials (concrete and reinforcements), prestressing steel (cables) is used.

Prestressed concrete requires the use of high-quality materials implemented with care. It allows these materials to be used under high stress, especially during construction (i.e., at the moment of cable tensioning). The steel used for prestressing cables must have high strength to be stretched to a very high initial strain (commonly between 1,200 and 1,500 MPa, except for bars). This initial tension, close to the breaking stress, is not dangerous because it gradually decreases due to prestress losses. Moreover, the final effective tension is higher when the initial tension and thus the steel strength are higher. The concrete must also be of exceptional quality because, as long as it is not prestressed, it risks cracking due to the restraint caused by formworks during shrinkage. To prevent this, concrete must be prestressed very early while still young and having limited strength.

### ***1. THE CONCRETE***

Prestressed concrete requires a deeper knowledge of the concrete material than what is needed for reinforced concrete. Indeed, since prestressing is a permanent and intense stress, it is essential to consider not only the fundamental properties of concrete, such as its characteristic compressive strength, but also certain properties whose effects are not immediate but appear over time. In prestressed concrete, the desired concrete exhibits high strength, sometimes at very early ages, achieved by using CEMI 52.5 MPa cement, and has shrinkage and creep as low as possible. Some requirements, such as workability and high strength, can sometimes conflict. High strengths are obtained with low water-to-cement ratios, but these mixes have reduced workability. However, thanks to advanced technology, it is possible to produce concretes with a water-to-cement ratio as low as 0.25 while maintaining sufficient workability, especially with the use of superplasticizers.

A concrete with the desired behavior is not a matter of chance. To determine the most effective mix design, trial mixes must usually be made, especially if no previous results are available. Cement dosages used for prestressed concrete typically range between 500 and 550 kg/m<sup>3</sup>, and water-to-cement ratios range from 0.30 to 0.45. According to strength classes in the Eurocode, concrete varies between C12-15 (not used in prestressed concrete) and C50-60. Beyond these strengths, concrete is classified as high-performance concrete (HPC). For example, a C40-50 concrete has a characteristic strength of 40 MPa. In prestressed concrete, the characteristic compressive strength at 28 days varies between 30, 35, 40 MPa, up to 45 MPa for prestressed beam bridges.

### ***1.1. Concrete implementation***

The placement of concrete plays a significant role in its expected qualities, such as mechanical strength, compactness, and conversely, porosity and permeability. This placement is facilitated by:

- Good vibration.
- Continuous grading of aggregates.
- The use of admixtures (plasticizers, fluidifiers, etc.).

Conversely, placement is hindered by:

- A high density of reinforcement, especially if the spacing between the bars is small.
- Low temperatures.
- Large casting thicknesses.

During concrete casting, it is recommended not to let the concrete fall from a significant height (more than one meter) without special precautions to avoid segregation; that is, the accumulation of heavy aggregates at the bottom of the formwork and the rise of water and laitance at the top, which would harm its compactness and strength.

### ***1.2. Physical characteristics of concrete***

#### ***1.2.1. Density***

It varies from 2.2 to 2.5 t/m<sup>3</sup> depending on the amount of steel embedded in the concrete. In calculations, it is generally considered equal to 2.5 t/m<sup>3</sup>.

#### ***1.2.2. Thermal expansion***

Thermal expansion of materials is the increase in volume, generally imperceptible, of a body when its temperature rises at constant pressure. This expansion is explained by the increased

thermal agitation of the particles constituting the body. If the body is long, its expansion will be especially noticeable in length; this is called linear expansion. Like any material, concrete reacts to temperature. Its effect is very important, especially in statically indeterminate structures where significant damage can occur.

The coefficient of thermal expansion is given by the following relation:

$$\epsilon = \frac{\Delta L}{L_0} = \lambda \cdot \Delta\theta \quad (\text{III-01}).$$

Where:

$L_0$  : Initial dimension in mm.

$\Delta\theta$  : Difference between maximum and minimum temperatures (in °C).

$\lambda$  : Coefficient of thermal expansion of materials (°C<sup>-1</sup>)

#### SOME EXAMPLES OF THERMAL EXPANSION COEFFICIENT

- Concrete:  $10 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$
- Steel:  $12 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$
- Aluminum:  $23 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$
- PVC:  $80 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$
- Wood: 35 to  $55 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$
- PMMA:  $80 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$
- Glass:  $9 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$

#### 1.2.3. Poisson's ratio:

if  $\epsilon_L$  is the longitudinal relative strain and  $\epsilon_t$  the transverse relative strain, the Poisson's ratio

is given by:  $\nu = \frac{\epsilon_t}{\epsilon_L}$  :

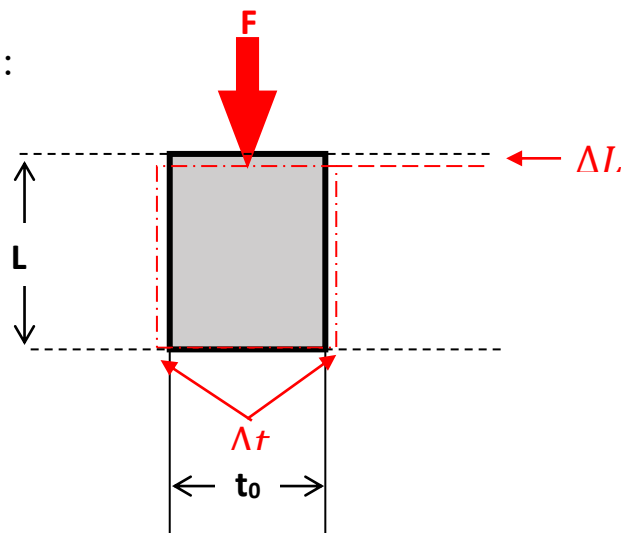


Figure1: Poisson's ration mechanisme.

Knowing that:  $\epsilon_t = \Delta t / t_0$  et  $\epsilon_L = \Delta L / L_0$

$$\nu = \frac{\epsilon_t}{\epsilon_L} = \frac{\Delta t / t}{\Delta L / L_0} \quad (\text{III-02}).$$

Le coefficient The Poisson's ratio of concrete is taken as:

- $\nu = 0$ : For ultimate stress calculations (cracked concrete).
- $\nu = 0.2$ : For deformation calculations (uncracked concrete).

#### 1.2.4. Shrinkage

It is the spontaneous shortening of concrete during its hardening in the absence of any stress.

This shortening can be expressed as:

$$\epsilon_r(t) = \frac{\Delta L}{L_0} = \epsilon_r * r(t) \quad (\text{III-03}).$$

Where:

- $\epsilon_r$ : The final shrinkage.
- $r(t)$ : A function varying from 0 to 1 as time varies from 0 to  $\infty$  infinity starting from the concrete casting.

Shrinkage is due to the progressive evaporation of water not bound in the cement hydration reaction. This phenomenon only begins after curing if the humidity is maintained at 100% during curing. Furthermore, the volume reduction due to shrinkage is not totally reversible. In other words, if after some time the concrete element is placed back in a 100% humid atmosphere, there is some swelling, but it does not regain its original volume. It is easy to understand that the final shrinkage  $\epsilon_r$  depends on:

- The composition of the concrete (increasing function of the water/cement).
- The ambient humidity  $\rho_{ph}$  (decreasing function of  $\rho_{ph}$ ): concrete kept in water even exhibits negative shrinkage.

- The transverse dimensions of the element, represented by its mean radius:  $r_m = B/U$ , B with B the cross-sectional area and U its perimeter in contact with the ambient (decreasing function of  $r_m$ ).
- Time (increasing function of time).

Under Algerian climate conditions, the following standard values are often used for  $\epsilon_r(t)$  (Algeria climate zoning map, CBA 93):

- $-1.5 \times 10^{-4}$  mm/mm for very humid regions.
- $-2.0 \times 10^{-4}$  mm/mm for humid regions.
- $-3.0 \times 10^{-4}$  mm/mm for dry temperate regions.
- $-4.0 \times 10^{-4}$  mm/mm for hot and dry regions.
- $-5.0 \times 10^{-4}$  mm/mm for very dry or desert regions.

Moreover, the BPEL proposes, for the evolution law of shrinkage:

$$r(t) = \frac{t}{t+9r_m} \quad \text{(III-04).}$$

Where:

$t(j)$  is the age of the concrete.

$r_m$  (cm) : mean radius.

### ***1.2.5. Creep***

Creep corresponds to an increasing deformation over time under constant stress. This phenomenon is also related to the migration of water within the concrete. It depends on a number of parameters, such as:

- The average thickness of the element.
- The applied stress.
- The cement dosage.
- The water content.
- The humidity.
- The temperature.
- The age at which tension is applied.

The creep deformation at time  $t$  of a concrete subjected to an age  $j = t_1 - t_0$  under a constant stress  $\sigma_1$  is expressed by the formula:

$$\epsilon_{ic} = \frac{\Delta L}{L_0} \quad (\text{III-05}).$$

From experience:  $\epsilon_{fl}(\infty) = 2 * \epsilon_i$

$$\text{So : } \epsilon_T = \epsilon_i + \epsilon_{fl} = \epsilon_i + 2\epsilon_i = 3 \epsilon_i \quad (\text{III-06}).$$

$\epsilon_{ic} = \frac{\sigma_1}{E_{i28}}$  : Conventional instantaneous deformation under the effect of stress  $\sigma_1$

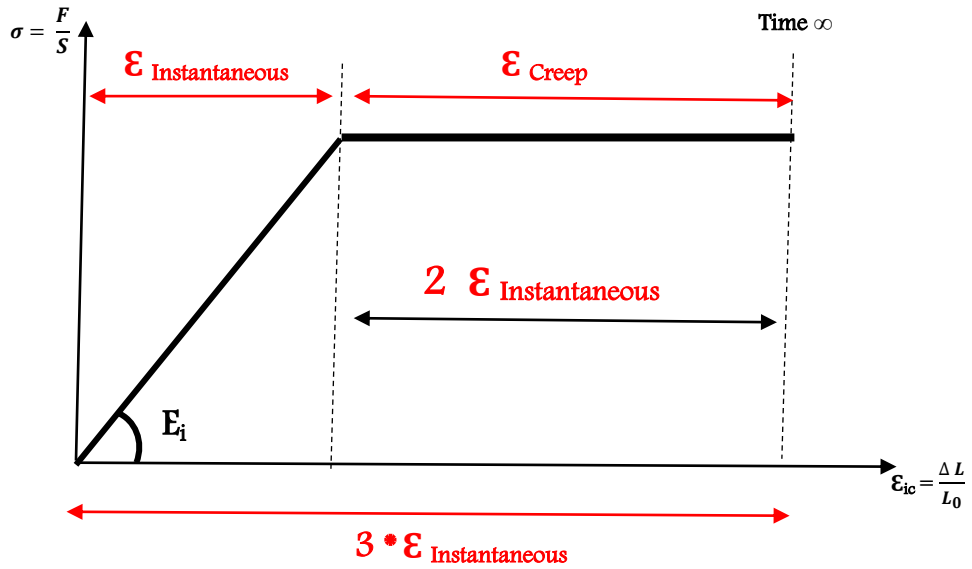
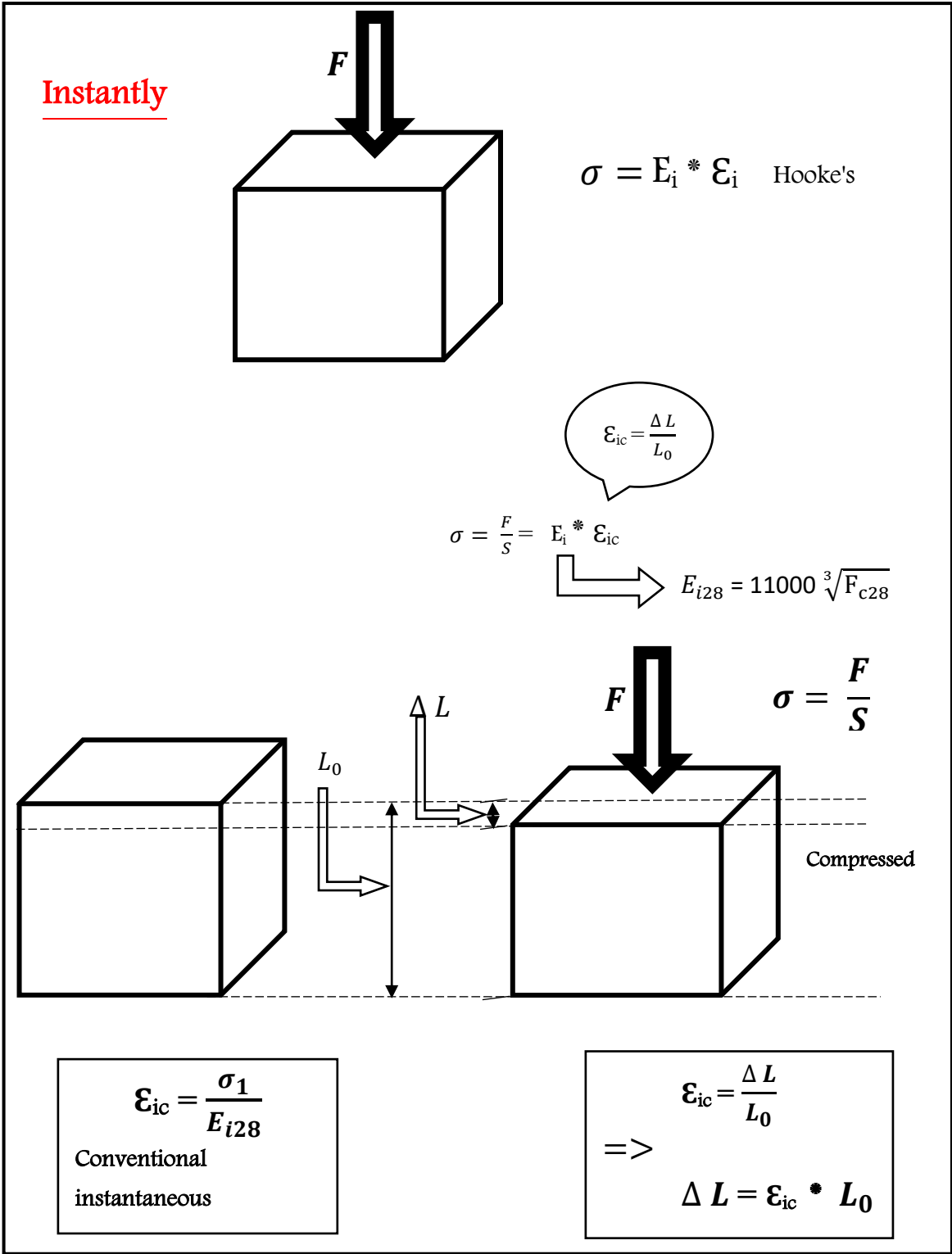
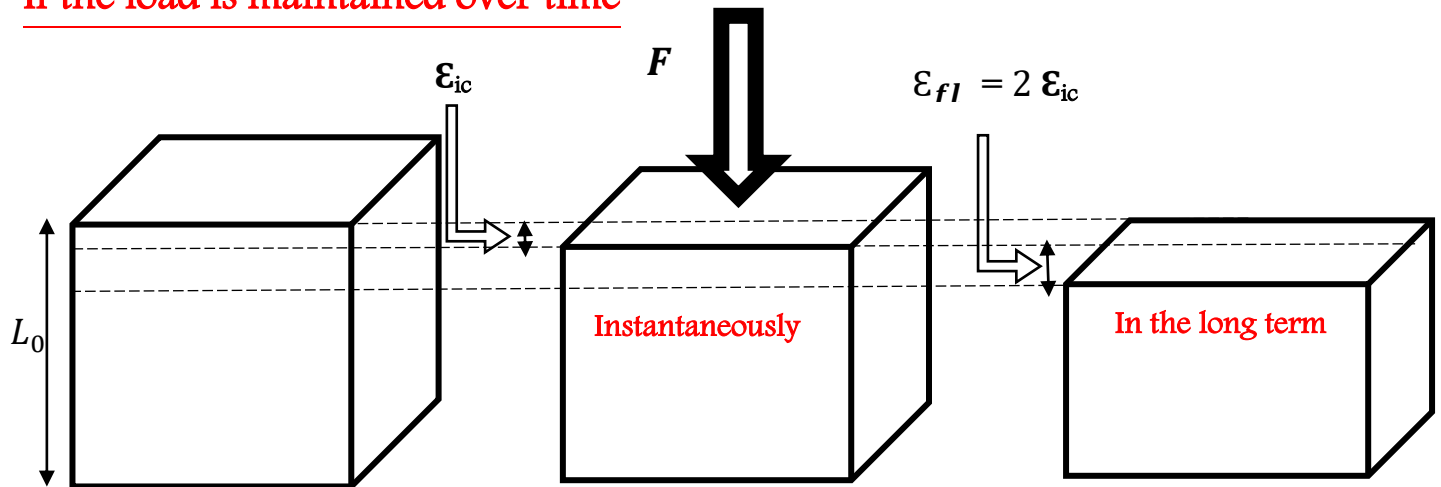


Figure 2: Stress-strain diagram for creep



Concrete is not a perfectly elastic material. Subjected to a constant stress, it deforms over time. Its delayed deformation is approximately twice its instantaneous deformation.

If the load is maintained over time



From experience.  $\epsilon_{fl}(\infty) = 2 \cdot \epsilon_i$

So:  $\epsilon_t = \epsilon_i + \epsilon_{fl} = \epsilon_i + 2\epsilon_i = 3 \epsilon_i$

Knowing that:  $\Delta L = \epsilon_i \cdot L_0$

➤ **Example :**

Consider a beam prestressed by a cable with a force  $P(x)$  (at the end of the prestressing operation). The stress in the concrete, at the cable level,  $P_0 = 1000 \text{ KN}$

**Data:**

Class 30/35  $F_{c28} = 30 \text{ MPa}$

➤ **Solution**

$$\sigma = \frac{F}{S} = \frac{1000 \text{ KN}}{0.3 \cdot 0.5} = \frac{1 \text{ MN}}{0.3 \cdot 0.5} = \frac{1}{0.15} = 6.66 \text{ MPa (MN/m}^2\text{)}$$

$$E_{i28} = 11000 \sqrt[3]{30} = 34180 \text{ MPa}$$

$$\epsilon_{ic} = \frac{\sigma}{E_{i28}} = \frac{6.66}{34180} = 0.195 \text{ ‰}$$

$$\epsilon_{ic} = \frac{\Delta L}{L_0} \Rightarrow \Delta L_f = \epsilon_{ic} \cdot L_0 = 20000 \text{ mm} \cdot 0.195 \text{ ‰} = 3.9 \text{ mm (Concrete is a very rigid$$

material)



the prestressing force due to cable shortening. To minimize this tension loss, the concrete must exhibit very limited deformation.

- Low sensitivity to delayed deformation effects: Over time, delayed deformations reduce the prestressing force. An optimal mix design is critical, especially concerning the water-to-cement ratio.
- Good workability: To ensure proper filling of all mold regions, particularly those with dense reinforcement.
- High compactness: This reduces porosity and permeability, which is crucial because concrete acts as the main protective barrier against corrosion of the cables, especially in pretensioned structures.
- Watertightness.
- Chemical inertness towards the prestressing cables.

## 2. ***MECHANICAL CHARACTERISTICS: REINFORCEMENTS***

The steels used in prestressing are of two different types:

- Active steels that create and maintain the prestress;
- Passive steels necessary for assembly, to resist shear forces, and to limit cracking.

### ***2.1. Passive reinforcements***

These are reinforcements identical to those used in reinforced concrete; they are only tensioned by the deformation of the element.

#### ***2.1.1. Description of different types of steel***

The steels generally used are classified into several categories:

- Smooth round bars.
- Deformed bars with high adhesion.
- Wires (both high adhesion wires and smooth wires).
- Welded meshes.

Generally, for passive reinforcements in prestressed concrete, the distinction is made between:

- Longitudinal passive steels
- Transversal passive steels

### 2.1.2. Characteristics of passive reinforcements

The characteristics of passive reinforcements to be considered in calculations are:

**Table 1. Characteristics of passive reinforcements**

Type	Designation	Yield strength (MPa)
Smooth round bars	FeE215	215
	FeE235	235
High-adherence bars	FeE400	400
	FeE500	500
High-adherence stranded wires and welded meshes	FeTE500 / TSHA	500
Smooth welded meshes	TSL	500

- Nominal cross-section of the reinforcement
- Longitudinal deformation modulus

The longitudinal deformation modulus of steel,  $E_s$  is taken as 200,000 MPa.

#### Yield strength guaranteed

The steel is defined by the guaranteed value of its yield strength, denoted  $f_e$ .

Table II.1 gives the conventional designations, grades, and the current yield strengths available on the market.

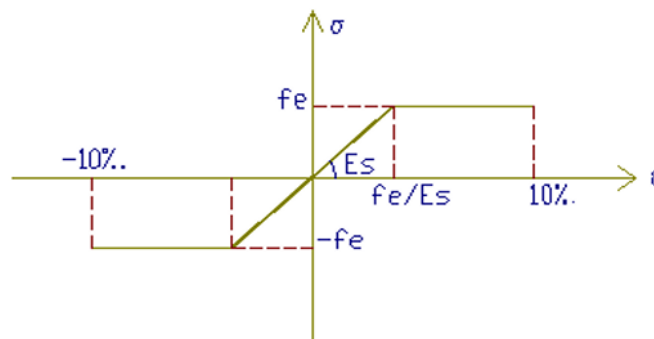


Figure 3: Stress-Strain Diagram

### ***The Bonding Capacity of Reinforcement with Concrete***

This capacity is characterized by the so-called bond coefficients for cracking and anchorage, denoted respectively by  $\eta$  and  $\psi$ .

Cracking coefficients:

- $\eta = 1$  for smooth round bars
- $\eta = 1.6$  for high-adherence bars or wires of diameter  $\geq 6\text{mm}$
- $\eta = 1.3$  for high-adherence bars of diameter  $< 6\text{mm}$

Anchorage coefficients:

- $\psi = 1$  for smooth round bars
- $\psi = 1.5$  for high-adherence bars or wires

### ***2.2. Active Reinforcement***

Active steels are the steels for prestressing; they are put under tension. Unlike passive reinforcements in reinforced concrete, these come from specially made quality steel.

The use of prestressing reinforcement requires steel that meets certain conditions. These have been classified according to:

- Category: wire, bars, strands.
- Strength class.

#### **2.2.1. Required Qualities**

- High mechanical strength.
- Sufficient ductility.
- Good resistance to corrosion.
- Low relaxation.
- As low a cost as possible.

#### **2.2.2. Geometric Characteristics**

### **Wires**

Wires are reinforcements whose largest cross-sectional dimension is less than 12.5 mm; they are supplied in coils.

There is a distinction:

- Wires of round smooth steel with symbol L.
- Other wires than round and smooth, with symbol L.

The wires are defined by their nominal diameter, to which corresponds a nominal conventional section according to Table 2.

**Table 2. The geometric characteristics of Wires.**

Diameter	4	5	6	7	8	10	12.2
Section	12.6	19.6	28.3	38.5	50.3	78.5	117

### **Bars**

Bars are defined as round and smooth reinforcements with a diameter greater than 12.5 mm, or non-round or non-smooth bars that cannot be supplied in coils.

The geometric characteristics are the diameter and the conventional section defined as follows in Table 3.

**Table 3. The geometric characteristics of bars.**

Diameter	20	22	26	32	36
Section	314	380	531	804	1018

### **Strands**

A strand is an assembly of 3 or 7 wires twisted helically and distributed in one layer, sometimes around a central wire.

Strands are characterized by the number of wires, diameter, and cross-sectional area. Table II.4 provides the corresponding values.

**Table 4: Geometrical characteristics of prestressing strands**

Type	3 wires	7 wires	7 wires	7 wires standard	7 wires standard	7 wires super	7 wires super
Diameter	5.2	6.85	9.3	12.5	15.2	12.9	15.7
Section	13.6	28.2	52	93	139	100	150



Figure 4 : strand with 7 wires.

### **2.2.3. Design Characteristics**

The design characteristics of prestressing steels to be considered in calculations are:

- Nominal cross-section of the reinforcement;
- Guaranteed maximum stress at rupture  $f_{prg}$ ;
- Stress at the conventional yield limit  $f_{peg}$ ;
- Relaxation coefficient  $\rho_{1000}$ 
  - $\rho_{1000} = 2.5\%$  for TBR class (Very Low Relaxation)
  - $\rho_{1000} = 8\%$  for RN class (Normal Relaxation)
- Bond to concrete;
- Coefficient of thermal expansion  $10^{-5}$  per degree Celsius.
- Longitudinal deformation modulus:

- $E_p = 200,000$  MPa for wires and bars
- $E_p = 190,000$  MPa for strands

The diagrams to be conventionally used for calculations are as follows:

- Stress-strain diagrams.
- For strand wires and tendons

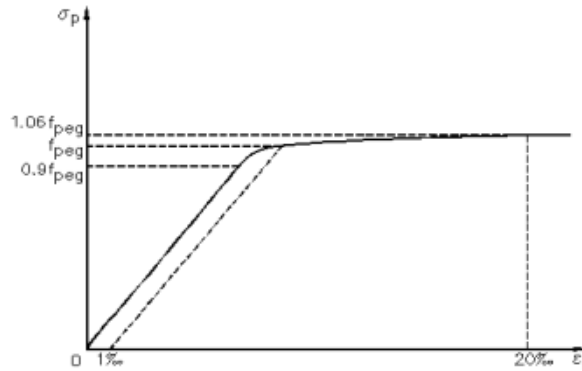


Figure 5. Stress-strain diagrams and For strand wires and tendons

$$\text{For } \sigma_p \leq 0,9 f_{peq} \quad \epsilon_p = \frac{\sigma_p}{E_p} \quad (\text{III-07}).$$

$$\text{For } \sigma_p > 0,9 f_{peq} \quad \epsilon_p = \frac{\sigma_p}{E_p} + 100 * \left( \frac{\sigma_p}{f_{peq}} - 0,9 \right)^5 \quad (\text{III-08}).$$

**For quenched and tempered wires and for bars:**

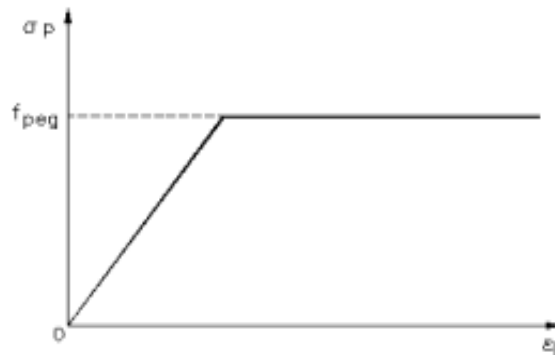


Figure 6. diagram for strand wires and tendons if high precision is not required

$$\text{For: } \epsilon_p \leq \frac{f_{peq}}{E_p} \quad \sigma_p = E_p * \epsilon_p \quad (\text{III-09}).$$

$$\text{Otherwise: } \sigma_p = f_{peq}$$

This last diagram is allowed for strand wires and tendons if high precision is not required.

### 3. APPLICATIONS

#### Application 1

Determine, for a concrete of  $f_{c28} = 30$  MPa, the following characteristics:

- The compressive strength at day  $j = 7$  and 90 days
- The tensile strength at day  $j = 7$  and 90 days
- The instantaneous longitudinal modulus of deformation at day  $j = 7$  and 90 days
- The delayed longitudinal modulus of deformation at day  $j = 7$  and 90 days

#### Solution

- Compressive strength at day  $j = 7$  and 90 days

$j = 7$  days:

$$f_{c7} = \frac{j}{4.76 + 0.83j} \times f_{c28} = 20.04 \text{ MPa}$$

$j = 90$  days:

$$f_{c90} = f_{c28} = 30 \text{ MPa}$$

- Tensile strength at day  $j = 7$  and 90 days

$j = 7$  days:

$$f_{t7} = 0.6 + 0.06f_{c7} = 1.80 \text{ MPa}$$

$j = 90$  days:

$$f_{t90} = f_{t28} = 0.6 + 0.06f_{c28} = 2.4 \text{ MPa}$$

- Instantaneous longitudinal modulus of deformation at day  $j = 7$  and 90 days

$j = 7$  days:

$$E_7 = 11000\sqrt{f_{c7}} = 30056.3 \text{ MPa}$$

$j = 90$  days:

$$E_{i90} = 11000\sqrt{f_{c28}} = 34179.6 \text{ MPa}$$

## Application 2

For a concrete with  $f_{c_{28}} = 35$  MPa:

- Determine the ultimate compressive stress limit under normal and accidental conditions.
- Determine the compressive stress limit in service.
- Plot the stress-strain diagram of concrete.
- In the absence of conclusive experimental evidence, when a more precise evaluation of deformations is needed, plot the stress-strain diagram of concrete.

## Application 3

For a passive steel FeE400:

- Determine the ultimate stress limit under normal and accidental conditions.
- Determine the service stress limit in the case of prejudicial and very prejudicial cracking.
- Plot the stress-strain diagram for the steel.

## Application 4

For a prestressing steel  $f_{peg} = 1583$  MPa

- Plot the stress-strain diagram for the steel.

## Application 5

Determine the shrinkage deformation of a beam with a section (0.30, 0.70) m tensioned at  $j = 7$  days under the following conditions:

- A humid climate, Zone A
- A temperate dry climate, Zone B
- A hot and dry climate, zones B', C, and D1
- A very dry or desert climate, zones D2 and D3

**Note:** See Algerian code CBA93.

## **CHAPTER 04: CALCULATION OF ISOSTATIC BEAMS AT THE SERVICEABILITY LIMIT STATE**

### **1. CALCULATION SECTIONS**

In calculating the geometric properties of a section (position of the centroid, area, moments of inertia, etc.), we account for the dimensions of the section in the considered phase.

#### **Geometric Characteristics of Sections**

Solving problems in Strength of Materials requires understanding the geometric properties of cross-sections.

The fundamental principle involves determining the stresses acting on a section and comparing the maximum stress to the allowable stress:  $\sigma \leq [\sigma]$

**Simple Tension:** 
$$\sigma = \frac{F}{B} \quad (\text{IV-01}).$$

- $\sigma$  is the stress,
- $F$  is the applied force,
- $B$  is the cross-sectional area.

**Simple Bending:** 
$$\sigma = \frac{M.V_i}{I} \quad (\text{IV-02}).$$

- $M$ : Bending moment.
- $V_i$ : Distance from the neutral axis (typically to the outermost fiber).
- $I$ : Moment of inertia about the bending axis.

**Combined Bending:** 
$$\sigma = \frac{F}{B} + \frac{M.V_i}{I} \quad (\text{IV-03}).$$

This represents the superposition of the effects of tension or compression and bending, which is common in elements subjected to combined loads.

The geometric characteristics to be studied are:

#### **Geometric Properties to Analyze**

- Cross-sectional area ( $B$ ) [ $\text{cm}^2$ ]
- Static moments ( $S_x$  and  $S_y$ ) [ $\text{cm}^3$ ]
- Axial moments of inertia ( $I_x$  and  $I_y$ ) [ $\text{cm}^4$ ]
- Centrifugal (product) moment of inertia ( $I_{xy}$ ) [ $\text{cm}^4$ ]

- Polar moment of inertia ( $I_p$ ) [ $\text{cm}^4$ ]
- Section modulus ( $W_x$  and  $W_y$ ) [ $\text{cm}^3$ ]
- Torsional resistance modulus ( $W_p$ ) [ $\text{cm}^3$ ]
- Radius of gyration  $i_x$  et  $i_y$  [ $\text{cm}$ ]
- Section efficiency ratio  $\rho$

### 1.1. Static Moment

The static moments of the area of a section with respect to the X and Y axes are given by the following expressions.

$$S_x = \int_A y \, dA \qquad S_y = \int_A x \, dA \qquad \text{(IV-04.05).}$$

- A: Area of the section
- y: Distance of an area element  $dA$  from the X-axis
- x: Distance of an area element  $dA$  from the Y-axis.

When the X-axis or the Y-axis passes through the centroid of the section, the static moments  $S_x$  and  $S_y$  are equal to zero. This is because the centroid is the point around which the area distribution is symmetrical, thus canceling out the static moment.

**Note:** If the X or Y axis passes through the centroid, the static moments  $S_x$  and  $S_y$  equal zero (due to symmetry).

### 1.2. Moments of Inertia ( $I_x, I_y$ ) [ $\text{cm}^4$ ]

The moments of inertia of the area of a section with respect to the X and Y axes are given by the following formulas:

**Theorem:** The axial moment of inertia of a section with respect to any axis is equal to the moment of inertia of that section with respect to the parallel centroidal axis plus the product of the area of the section by the square of the distance from the centroid of the section to the considered axis (Huygens' theorem).

The moments of inertia of a section with respect to the X and Y axes quantify the section's resistance to deformation around these axes. These moments of inertia play a key role in the

strength of materials, especially for the analysis of bending and the stability of structures. Here are the formulas:

$$I_x = \int_A y^2 dA \quad I_y = \int_A x^2 dA \quad (\text{IV-06.07})$$

where:

- $I_x$  is the moment of inertia with respect to the X-axis,
- $I_y$  is the moment of inertia with respect to the Y-axis,
- $y$  is the distance from an area element  $dA$  to the X-axis,
- $x$  is the distance from an area element  $dA$  to the Y-axis.

There is also a product of inertia,  $I_{xy}$ , which is defined as:

$$I_{xy} = \int x y dA \quad (\text{IV-08}).$$

This product of inertia is useful for evaluating the moments of inertia with respect to oblique axes or the principal axes of inertia of the section.

➤ **Huygens' Theorem (or the Parallel Axis Theorem)**

This theorem allows the calculation of the moment of inertia of a section with respect to any axis parallel to the axis passing through the centroid of the section. The theorem states that:

$$I = I_G + Ad^2 \quad (\text{IV-09}).$$

where:

- $I$  is the moment of inertia with respect to the considered axis,
- $I_G$  is the moment of inertia of the section with respect to an axis passing through the centroid of the section, parallel to the considered axis,
- $A$  is the area of the section,
- $d$  is the distance between the centroid of the section and the considered axis.

This theorem is essential for structural analysis because it allows transferring moments of inertia calculated around central axes (passing through the centroid of a simple section) to parallel axes of composite sections in beams and other structural elements.

### 1.3. Section Modulus

The section modulus is a quantity that expresses the capacity of a section to resist bending moments without yielding. It is directly related to the distribution of material around the axis of the section and is calculated based on the moment of inertia of the section and the distance to the farthest fiber. The section modulus is defined for each axis as follows:

$$W_{x \min} = \frac{I_x}{y_{\max}} \quad , \quad W_{y \min} = \frac{I_y}{x_{\max}} \quad (\text{IV-10.11}).$$

where:

- $W_{y \min}$  and  $W_{x \min}$  are the section moduli with respect to the X and Y axes,
- $I_x$  and  $I_y$  are the moments of inertia with respect to the X and Y axes,
- $y_{\max}$  the distance between the X-axis and the farthest fiber in the section,
- $x_{\max}$  is the distance between the Y-axis and the farthest fiber in the section.

### Usefulness of the Section Modulus

The section modulus is an important characteristic in engineering and materials mechanics because it is directly related to bending strength. The higher the section modulus, the greater the bending moment the section can withstand before reaching its maximum allowable stress. It is commonly used in the design of beams and other structural elements, where it helps determine the distribution of materials to optimize resistance to bending forces.

### 1.4. Radius of Gyration

The radius of gyration of a section is a measure that characterizes the distribution of material around an axis. It is often used in mechanics of materials and stability analysis, especially to assess the tendency of a section to deform under axial compression (such as in cases of buckling).

The radius of gyration is defined by the following formulas for the X and Y axes:

$$i_x = \sqrt{\frac{I_x}{B}} \quad i_y = \sqrt{\frac{I_y}{B}} \quad (\text{IV-12.13}).$$

where:

- $i_x$  and  $i_y$  are the radii of gyration with respect to the X and Y axes,
- $I_x$  and  $I_y$  are the moments of inertia of the section with respect to the X and Y axes,
- $B$  is the area of the section.

### **Usefulness of the Radius of Gyration**

The radius of gyration is an indication of the "spread" of the area around an axis. The larger the radius of gyration, the farther the material is from the axis, which can increase resistance to buckling in elements subjected to compression.

In engineering, the radius of gyration is often used to calculate the buckling factor in columns, as it is directly related to the stability of the section.

#### **1.5. Section Efficiency**

The efficiency of a section is a measure of how effectively the material is distributed within a section, particularly in relation to its ability to resist internal forces such as bending. It is defined by the following formula:

$$\rho = \frac{I}{B \cdot V_i \cdot V_s} \quad (\text{IV-14}).$$

where:

- $\rho$  is the efficiency of the section,
- $I$  is the moment of inertia of the section,
- $B$  is the width of the section,
- $V_i$  is the velocity or the component of the force in the direction of the section,
- $V_s$  is another variable that could be related to stress or axial force (the exact meaning of  $V_s$  depends on the context).

The efficiency (yield) of a section gives an idea of how effectively the material in the section is used to withstand internal forces. A high efficiency indicates that the material is well distributed to maximize the section's strength while minimizing material consumption.

#### **1.6. Types of sections**

##### **1.6.1. Gross section**

This corresponds to the concrete alone, calculated based on the formwork dimensions shown on the plans, without taking into account the reinforcement or the voids corresponding to

ducts intended for prestressing reinforcement or their anchorages (see Fig. 01). The gross section is used for the assessment of:

- The self-weight of a structure.
- The stiffness of the various elements making up the structure (for the purpose of calculating hyperstatic actions).
- The deformations for parts of works that meet the conditions of class II.

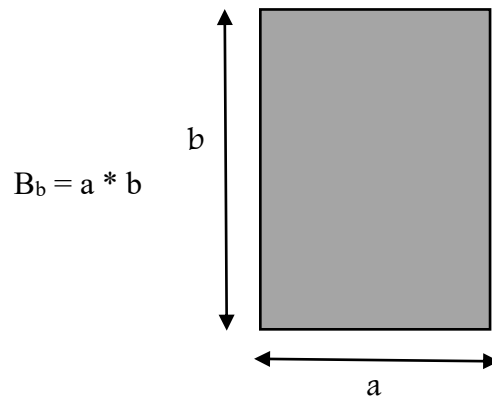


Figure1. Gross section

### 1.6.2. Net section:

This is the total concrete section with the ducts and anchorages deducted (Figure 02). It is used for the calculation of stresses at the serviceability limit state (SLS) in class I and II.

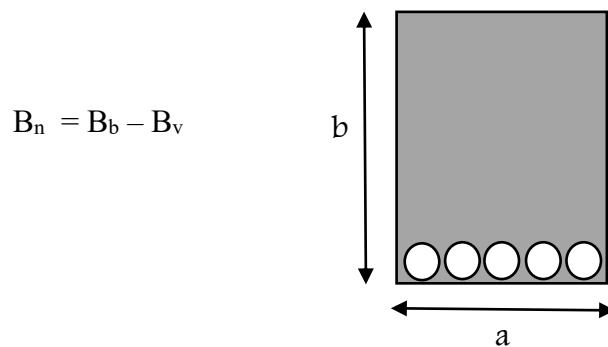


Figure 2. Net section

### 1.6.3. Homogeneous section:

This is the section obtained by adding to the net section the area of the bonded longitudinal reinforcement multiplied by an appropriate equivalence coefficient "n".

$$\text{Homogeneous section} = \text{net section} + (\text{area of bonded longitudinal reinforcement}) \times (\text{equivalence coefficient}).$$

1 cm<sup>2</sup> of steel = 5 cm<sup>2</sup> of concrete

n = 5: for short-term actions.

1 cm<sup>2</sup> of steel = 15 cm<sup>2</sup> of concrete

n = 15: for long-term actions.

That is to say:

short-term  $S_n = B_n + 5 A_p$

long-term  $S_n = B_n + 15 A_p$

#### 1.6.4. *Reduced Homogeneous Section (Cracked Section):*

For the calculation of stresses in class III (Reinforced Concrete), it is necessary to use reduced homogeneous sections, defined as follows:

- Only the compressed part of the concrete section,
- The area of the passive longitudinal reinforcement multiplied by the equivalence coefficient  $n_v = 15$ ,
- The fraction  $\rho$  of the area of the prestressing longitudinal reinforcement multiplied by the equivalence coefficient  $n_v = 15$ .

**Reduced homogeneous section = (area of compressed concrete only) + (area of passive reinforcement)  $\times$  (nv) + (area of prestressing reinforcement)  $\times$  (nv)  $\times$  ( $\rho$ ).**

Where:

$\rho = 1$ : In the case of pre-tensioning

$\rho = 0.5$ : In the case of post-tensioning with grout injection

$\rho = 0$ : In the case of post-tensioning when the tendons are not bonded.

#### 1.6.5. *Cover Section*

This is the area bounded by the outline of the section and two lines parallel to the considered axis of bending, enclosing all the prestressing reinforcement, at a minimum allowed distance "c" (see Figure 03). This section is used for certain verifications in class II.

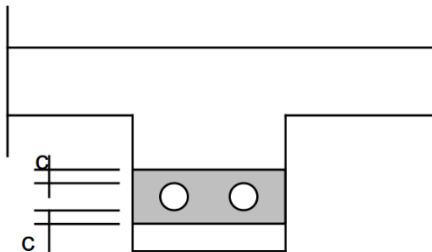


Figure3. Cover Section.

## **2. LOAD COMBINATIONS AT THE SERVICEABILITY LIMIT STATE**

Load combinations are fundamental in the design of concrete structures because they ensure safety, functionality, and durability throughout the life of the structure. At the serviceability limit state (SLS), the goal is to ensure user comfort and the longevity of the structure without exceeding certain deformations or stresses in the materials.

### **2.1. Types of Loads**

The main types of loads are:

- Permanent loads (G): The self-weight of the structure and permanently fixed elements (floors, coverings, load-bearing walls, etc.).
- Live or variable loads (Q): Temporary loads due to occupancy, furniture, snow, wind, seismic actions, etc.
- Indirect actions: Effects such as shrinkage, creep, temperature variations, and released prestress may also play a role.

### **2.2. Main Combinations at SLS**

The formulation of load combinations at SLS follows current regulations (such as Eurocode 2). For simply supported beams, the common distinguished combinations are:

- Quasi-permanent combination: Used for checks related to long-term deformations and comfort (deflections, long-term cracking).
- Frequent combination: To control stresses in frequently occurring service states, aiming to avoid excessive crack opening.
- Characteristic combination: Used when verifying exceptional actions (for example, the transient passage of a heavy vehicle).

The general expression of a load combination at SLS is:

$$S = G_{k,j} + \sum Q_{k,1} + \sum \psi_i Q_{k,i} \quad (\text{IV-15}).$$

where  $G_{k,j}$  are characteristic values of permanent loads,  $Q_{k,i}$  are characteristic values of variable loads, and  $\psi_i$  are combination factors adapted to the type of verification (quasi-permanent, frequent, characteristic).

Each combination aims to reproduce the actual conditions experienced by the beam during operation in order to control cracking, deflections, and the level of residual stress in the concrete or steel.

### **2.3. Verification Class at the Serviceability Limit State**

The term “verification class” refers to the criterion controlled during the dimensioning at SLS, directly related to the use of the beam and requirements for comfort and durability.

### **2.4. Main Classes at SLS**

- Stress verification: Maximum stresses in the concrete (compression) and in the reinforcement (tension) must not exceed the normative limits to avoid progressive damage and ensure reversibility of deformations.
- Cracking control: In zones where cracks can appear, their width must be limited to guarantee durability (protection against steel corrosion) and visual comfort. Norms usually impose a maximum crack width, typically around 0.2 to 0.3 mm in normal environments.
- Deformation limitation: Instantaneous and long-term deflections (related to creep) must be compatible with the intended use. Maximum limits (for example, L/250 to L/500 depending on the span length) are imposed to avoid aesthetic defects and discomfort for users.
- Stability verification: Ensuring that action combinations do not lead to global or local instability of the beam, especially at support points.

### **2.5. Practical Verification Procedure**

1. Calculate, for each load combination, the state of internal forces: moments, shear forces, normal forces.
2. Apply formulas for maximum stress in concrete ( $\sigma_{c,max}$ ) and in steel ( $\sigma_{s,max}$ ).
3. Compare results to limit values prescribed by the regulations (For example,  $\sigma_{c,adm}$ ,  $\sigma_{s,adm}$ ).
4. Check maximum deflections using charts or appropriate formulas, including the time-dependent effect (creep, shrinkage).
5. Verify crack widths using recommended analytical methods or specialized software for reinforced concrete design.

This procedure ensures systematic verifications to guarantee that the prestressed beam will fulfil its function without pathologies or damage during service.

### **3. JUSTIFICATION WITH RESPECT TO NORMAL STRESSES**

#### **3.1. Verification at the Serviceability Limit State**

##### 1. Verification Principle

This verification involves calculating the stresses in the concrete and comparing them with the allowable limit stresses. It must be established for each construction phase and during service.

The stress calculation is done using the following algebraic expression:

$$\sigma(y) = \frac{P}{B} + (P \cdot e_p + M) \cdot \frac{y}{I} \quad (\text{IV-16}).$$

In the general case, the following must be satisfied:

$$\sigma_{\min} \leq \sigma(y) \leq \sigma_{\max}$$

The limit stresses are not the same for different load combinations, for verifications during construction phases, and for verifications during service phases.

#### **3.2. Calculation Assumptions**

Calculations in the regular section are carried out based on two fundamental assumptions:

- Cross sections remain plane.
- Material stresses are proportional to their strains.

Depending on the type of verification, additional assumptions apply:

##### **a) Calculation in uncracked section**

- The concrete in tension resists tension.
- No relative slip occurs between materials.

This latter hypothesis implies that normal stresses caused by all actions other than permanent actions can be calculated over the entire homogeneous section.

##### **b) Calculation in cracked section**

- The concrete in tension is neglected.
- No relative slip occurs between materials.
- When the concrete strain cancels at a reinforcement bar, the stress in this bar is:
  - ✓ 0 if it is passive reinforcement.
  - ✓  $\sigma_{pd} + n_i \sigma_{bpd}$  (with  $n_i=5$ ) if it is prestressing reinforcement.

- The stress in passive reinforcement as well as the variation of over-stress in the prestressing steel, which occurs after decompression of the concrete, are evaluated using the equivalence coefficient  $n_v=15$ .

### ***3.3. Dimensioning of Prestressing Force***

The prestressing force  $F_p$  must be calculated to partially or totally compensate for the tensile forces generated by live and permanent loads, in order to limit cracking and deformations at the serviceability limit state (SLS). This force is generally determined based on the allowable stresses in concrete and steel, as well as the internal forces applied (bending moments, axial forces).

The initial prestressing force is chosen to reduce tensile stresses in the concrete to an acceptable value, or even to annul them in certain critical zones.

It is imperative to take into account losses of prestressing force due to relaxation of the prestressing steel, creep and shrinkage of the concrete, as well as mechanical losses caused by anchorage friction.

The final effective force after losses must ensure the desired long-term performance and must not exceed the maximum admissible stresses in the materials.

### ***3.4. Selection of Active Reinforcement***

Active reinforcements include high-strength steel cables, tendons, or wires capable of sustaining high tension forces.

The type of reinforcement is selected based on the precision of placement, the required curvature in the beam, and the anchorage conditions.

The number of reinforcements is determined according to the calculated prestressing force by dividing this force by the unit tensile capacity of the steel elements.

### ***3.5. Arrangement of Reinforcements***

The geometric arrangement of prestressing cables or wires is designed to follow the stress flow lines in the beam, often taking the form of parabolas or polygonal segments.

The positioning of active reinforcements within the section is crucial to optimize the efficiency of prestressing: they must be placed near the tensile fiber to maximize the compressive effect on the concrete.

Precautions must be taken regarding the spacing of the cables to ensure effective anchorage, to avoid concrete damage, and to facilitate manufacturing and installation.

The arrangement must also consider shear stresses (tangential stresses), often complemented by passive reinforcement.

#### **4. CABLE LAYOUTS**

After defining the loads, section characteristics, and prestressing (force  $P$  and eccentricity  $e_0$ ), it is necessary to determine the cable layout throughout the sections along the beam. Two cases are considered:

- **Simply supported beam:** The prestressing force  $P$  and eccentricity  $e_0$  are applied in the most stressed central section. The cables are raised near the supports to resist shear forces (because there are no bending moments at the supports). Note the exception: the cables are straight when using bonded wires.
- **Continuous (statically indeterminate) beam:** It is necessary to calculate the prestressing force  $P_i$  and eccentricity  $e_{0i}$  at every section. If the prestressing consists of continuous cables, the unknown is the eccentricity  $e_{0i}$ , defined by the cable profile. However, this leads to an increase in prestressing force; for economical reasons, cables are stopped in some spans. The cables dimensioned in certain sections and stopped in others must satisfy:
  - Longitudinal flexural resistance: both during construction and in service (SLS and ULS).
  - Shear resistance (raised cables near the supports).
  - Numerous practical considerations (concrete cover, assemblies, void pressure, anchorage, etc.).

In practice, the process proceeds as follows:

- Determination of the external forces and deduction of the moment  $M$ .
- Determination of the necessary prestressing force ( $P = \Delta M / (c + c')$ ).
- Selection of the most suitable prestressing units.
- Deduction of the number of cables in the main sections ( $n = P / \text{force of one cable}$ ).
- Determination of cable eccentricity in the main sections.
- Cable layout drawing based on these results and respecting practical constraints.
- Verification of bending by calculation of the extreme resulting stresses.
- The same process is adopted for multi-span beams taking into account the hyperstatic moments.

##### **4.1. Cable Profile Raising**

Raising the cables near the beam ends presents the following benefits:

- Reduction of the simply supported prestressing moment.
- Reduction of the shear force due to prestressing ( $P \sin\alpha$ ).
- Ease of accommodation of cables at the beam ends.

In practice, the cable raisings overlap and get closer as they approach the beam end. Furthermore, angular deviations must be limited to reduce friction losses in curves. Cables raised exiting on the top table of beams can be tensioned in a second phase, whereas those exiting at the end generally can only be tensioned before beam placement.

#### 4.2 Cable Anchorage Blockouts

In cable anchorage, three families of steel are considered:

- $A_1$ : bursting steels determined by  $A_1 = P(0.5 - \sin\alpha) / \sigma_{adm}$
- $A_2$ : steels for resisting tensile thrust, such as  $A_2 = P\alpha / \sigma_{adm}$
- $A_3$ : stitching steels:  $A_3 = (P \cos\alpha / \sigma_{adm}) - (A_1 + A_2)$ .

#### 4.3 Practical Cable Layout Arrangements

##### 4.3.1 Transversal Arrangements

These include:

- Grouping cables in bundles: maximum of 2 for large units.
- Placement of cables in the beam flanges (Fig4. a).
- Placement of cables at the bottom of boxes (Fig4. b).
- Concreting and vibration chimneys (Fig4. c).

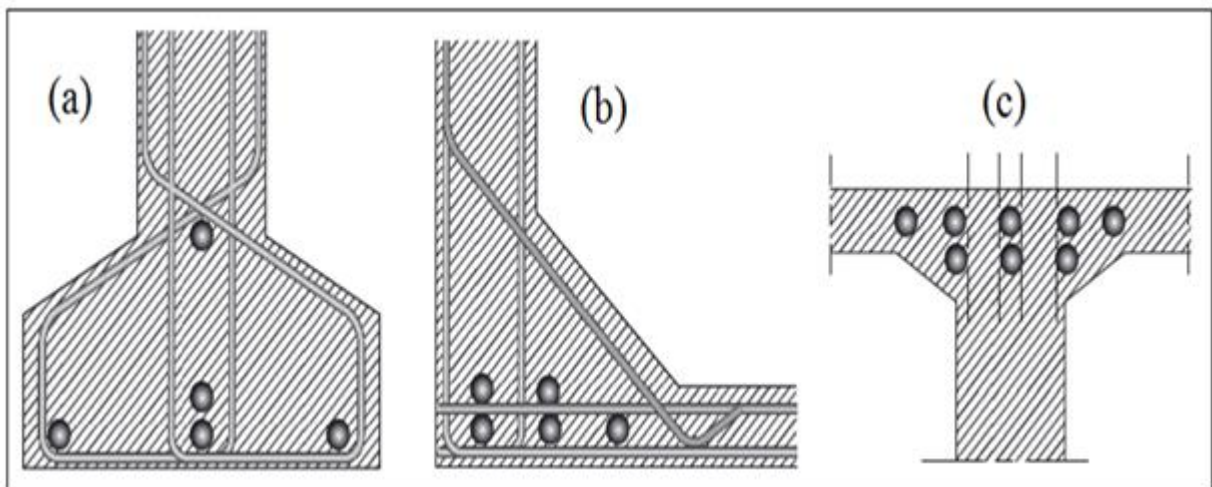


Figure 4: Transverse arrangements.

### **4.3.2 Longitudinal Arrangements**

Longitudinally, it is necessary to avoid conduit whistles. For high stresses, it is advisable to place gussets to deviate and turn the cables.

### **4.3.3 Concrete Covers**

The concrete cover of a sheath relative to any formed surface is at least 5 cm. For a half sheath relative to any unformed surface, the cover is a minimum of 3 cm.

For straight cables in thin slabs (upper or lower deck slabs of bridges), unintended deviations create void pressures that may cause delamination of the slab and rupture during grouting.

A special case arises for cables outside the concrete: particular precautions must be taken during grouting.

### **4.3.4 Concrete Cover for Passive Reinforcement**

The cover of any reinforcement must be at least:

- 1 cm for walls located in enclosed, covered spaces not exposed to condensation.
- 3 cm for formed or unformed walls exposed (or susceptible to exposure) to aggressive actions, weather, condensation, or liquid contact.
- 5 cm for marine structures or structures exposed to sea spray or saline fogs, as well as structures exposed to aggressive atmospheres.

### **4.3.5 Spacing of Prestressing Reinforcement**

In regular sections, the horizontal spacing  $e_h$  and vertical spacing  $e_v$  of active reinforcements must be at least 5 cm.

## **4.4. Fictitious Mean Cable**

The prestressing cables in each section form a system that can be quite complex. Therefore, for calculations, this system is often replaced by a fictitious mean cable that would have, in each section, the same effect as the cables actually installed. The eccentricity of the effective mean cable  $e_0$  lies between  $(-c' - \frac{M_{min}}{P})$  and  $(-c - \frac{M_{max}}{P})$ .

The segment through which the cable passes is called the passage segment.

## **4.5. Passage Zone**

This represents the zone bounded by all the passage segments along the entire length of the element.



Figure 5 : Passage zone

For the prestressing of an element to be possible, two conditions must be met: the existence of the passage segment and having sufficient concrete cover.

#### ***4.6. Calculation and Verification of Tangential (Shear) Stresses***

Tangential stresses correspond to the shear and torsional forces that the beam must resist. For prestressed beams:

- Shear is calculated based on the maximum shear force  $VV$  at the section.
- The allowable tangential stress in the concrete is limited, and excessive forces require reinforcement.
- The torsional effect, if present, increases the resultant tangential stress. This is taken into account during verification by vector addition.
- Calculations follow normative formulations (Eurocode 2) and can be performed manually or using specialized software.

#### ***4.7. Complementary Reinforcement for Tangential Stresses***

To address high tangential stresses:

- Passive reinforcement is strengthened, mainly with stirrups arranged vertically or obliquely, which provide resistance to shear.
- The arrangement and spacing of stirrups are calculated according to the code, based on the present stresses and minimum reinforcement ratios.
- Additional longitudinal reinforcement may be added to resist torsion-induced forces.
- This reinforcement ensures that the beam withstands not only bending but also transverse forces without damage, particularly near supports and concentrated load zones.

These measures optimize the strength, durability, and safety of prestressed beams under real and complex service loads.

## **5. PASSIVE REINFORCEMENT**

Passive reinforcements must be mandatorily present in the tensioned zones and also serve as skin reinforcement.

### **5.1. Skin Reinforcement**

These are primarily intended to limit premature cracking likely to occur before tensioning, under the effects of differential shrinkage and temperature variation. They are arranged around the perimeter of the members in two perpendicular directions, regardless of the serviceability limit state (SLS) class. A small spacing compatible with concreting should be chosen.

There are two types:

- Longitudinal skin reinforcement parallel to the mean fiber of a beam: There must be 3 cm<sup>2</sup> per linear meter of face. In typical cases, this corresponds to reinforcement bars of 10 mm diameter spaced at 25 cm. The total area of these reinforcements must not exceed 0.1% of the total concrete section area:  $A \leq B/1000$ .

### **5.2 Longitudinal Reinforcement in Tensioned Zones**

- Transverse skin reinforcement perpendicular to the mean fiber of a beam: There must be 2 cm<sup>2</sup> per linear meter of face. Typically, this corresponds to three 10 mm diameter bars or two 12 mm diameter bars per meter of face. Transverse skin reinforcement also helps prevent void pressure.

Tensioned zones in the concrete are fragile since concrete has poor tensile strength, which can lead to cracking. Just as in reinforced concrete, the non-fragility rule of the BAEL code applies in the tensioned zones of prestressed concrete: all tensioned concrete zones must be reinforced:

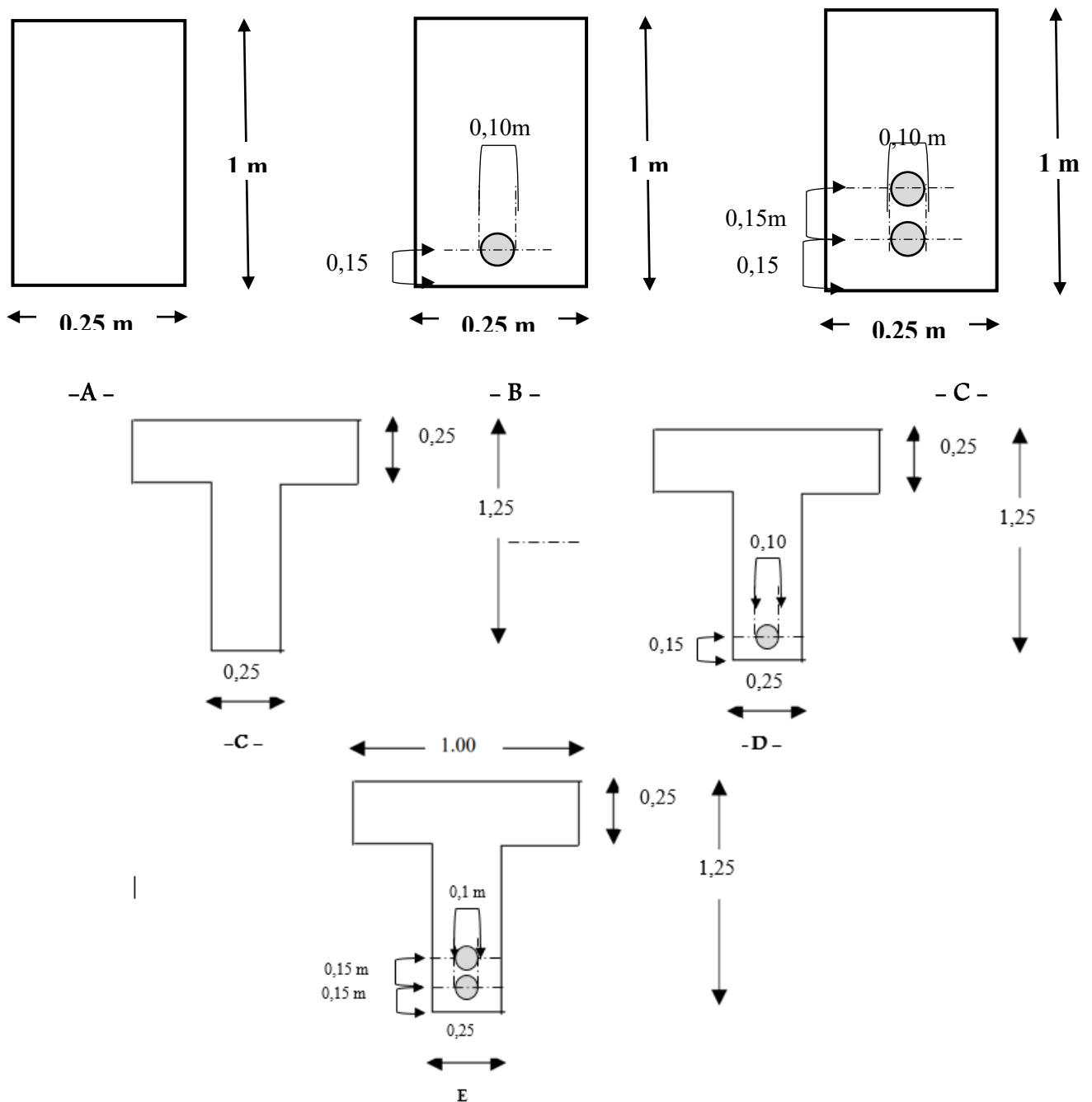
$$A_s = \frac{B_t}{1000} + \frac{N_{Bt}}{f_e} + \frac{f_{tj}}{\sigma_{Bt}} \quad (\text{IV-17}).$$

- $N_{bt}$ : resultant tensile forces defined by the stress diagram at the SLS.
- $B_t$ : area of the tensioned concrete part.
- $f_e$ : yield strength of the reinforcements used.
- $\sigma_{Bt}$ : absolute value of the maximum tensile stress in the section.

## 6. APPLICATIONS

Determine, for the following beams A B C D E F, the following geometric characteristics:

- The area of the section (B)
- The static moment (S)
- The distance to the top fiber ( $V_s$ ) and the distance to the bottom fiber ( $V_i$ )
- The moment of inertia (I)
- The section modulus (W)
- The radius of gyration (i)
- The section efficiency ( $\rho$ )



### ***Excercise***

Determine the dimensions and the cabling of a transfer beam for a 16-storey building, over a span of 30m between column axes. The following data are given

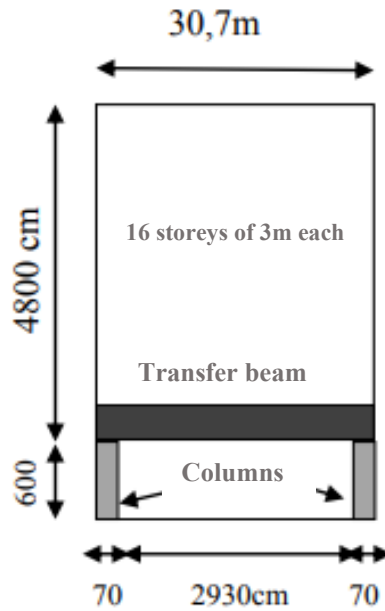
Classe II,  $f_{c28} = 30 \text{ MPa}$

$G = 0,502 \text{ MN/m}$ ,  $Q = 0,242 \text{ MN/m}$ .

Strands T15, TBR:  $\rho = 2,5\%$ ,  $\mu = 0,43$ , with:

Area:  $139 \text{ cm}^2$ ,  $f_{peg} = 1623 \text{ MPa}$  et  $f_{prg} = 1814 \text{ MPa}$ .  $\varphi = 0,0017\text{m}^{-1}$ ,  $f = 0,16 \text{ rd}^{-1}$ ,  $g = 5\text{mm}$ ,  $\epsilon_r = 2 \cdot 10^{-4}$

$\emptyset$  of the sheath : 105 mm for a cable 19T15.



## **CHAPTER 05 : STRUCTURAL RESISTANCE OF A BEAM AT THE ULTIMATE LIMIT STATE (ULS)**

### ***INTRODUCTION***

The ultimate limit state (ULS) resistance of a concrete beam is a critical aspect of structural design that ensures the safety and durability of buildings and infrastructure. The ULS corresponds to the maximum load-carrying capacity the beam can sustain before failure occurs. This failure can be caused by material yielding, crushing of concrete, or excessive deformation leading to collapse. Understanding and calculating the ULS helps engineers design beams that can withstand the most severe expected load combinations during the structure's lifetime.

Concrete and steel, the two main materials in reinforced concrete beams, have different mechanical behaviors. Concrete is strong in compression but weak in tension, whereas steel reinforcement is strong in tension and provides ductility to the beam. The interaction between these materials under stress must be carefully analyzed to prevent brittle failure.

To assess the beam's resistance, load combinations regulated by design codes combine permanent loads (such as self-weight), variable loads (such as live load or environmental influences), and accidental loads (such as seismic forces). Safety factors are applied to these loads to account for uncertainties and variability.

The design procedure involves calculating the internal forces within the beam's cross-section, primarily the compressive force in the concrete and the tensile force in the steel reinforcement. The moment of resistance is then found by balancing these forces, providing the maximum bending moment that the beam can resist.

Shear resistance and the verification of localized forces at supports and connections are also essential components of ULS design to prevent sudden brittle failures like shear cracks or punching shear.

Overall, the ULS design methodology ensures that concrete beams perform reliably under extreme conditions, keeping occupants safe and preserving the structural integrity throughout the service life of the structure.

## **1. LOAD COMBINATIONS AND MATERIAL BEHAVIOR**

### **1.1. Loads**

The design of a reinforced concrete beam must consider the various actions acting on it throughout the structure's lifespan. These loads fall into several main categories based on their nature and duration:

- **Permanent Loads (G):** These are constant loads over time. They mainly include the self-weight of the beam itself and fixed imposed loads (for example, weights of attached structural elements, finishes, partitions). These loads are generally well-controlled since they derive from the material properties and geometry.
- **Variable Loads (Q):** These loads vary during the life of the structure. They include live loads such as the weight of occupants or furniture in a building, wind, snow on the roof, or temporary usage loads. These are more uncertain and are treated with higher safety factors.
- **Accidental Loads:** These are exceptional but possible actions, such as seismic forces or impacts. Their consideration depends on the geographic location and functional use of the building, with specific rules for each type.

To ensure safety, design codes (such as BAEL in Algeria or Eurocode 2 in Europe) specify standardized load combinations with partial safety factors to account for uncertainties and design margins.

A typical combination used for ultimate limit state (ULS) design is:

$$\text{Combination} = 1.35 \cdot G + 1.5 \cdot Q \quad (\text{V-01}).$$

where G represents permanent loads and Q the leading variable load. These factors can vary depending on circumstances. Other combinations may include secondary or accidental loads.

The goal is to identify the most critical loading scenario for the beam.

### **1.2. Material Behavior**

The ULS design relies fundamentally on the mechanical properties of the materials composing the beam:

- **Concrete:** A composite material mainly resistant to compression. Its tensile strength is very low due to rapid cracking. At the ultimate limit state, concrete tensile resistance is neglected, and only the compressed zones are modeled. The maximum compression stress is represented by a simplified "parabola-rectangle" stress distribution that

describes the stress in the compressed zone. This simplified model facilitates calculations while closely approximating real behavior.

- Steel: Used for reinforcement, steel has a high tensile strength and exhibits ductile behavior, meaning it can undergo plastic deformation before failure, providing warning before collapse. Reinforcement bars are sized to reach yield strength without buckling, ensuring a safe and predictable beam response.

### ***1.3. Classical Assumptions in ULS Design***

Several classical assumptions simplify the calculations:

- Plane sections remain plane under bending, implying a linear strain distribution over the section height.
- No slip occurs between steel and concrete, ensuring perfect bond for force transfer.
- Concrete in tension is ignored at ULS due to widespread cracking.
- Material behavior is modeled by simplified diagrams: concrete's compression zone has a rectangular stress distribution, and steel behaves elastically-plastically.

## ***2. CALCULATION OF THE MOMENT RESISTANCE***

Calculating the moment resistance of a reinforced concrete beam at the ultimate limit state is a fundamental step in structural design. This calculation determines the maximum bending moment that the beam's cross-section can safely resist before failure. It is based on an internal equilibrium between compressive forces in the concrete and tensile forces in the steel reinforcement.

### ***2.1. General principle***

Under loading, the beam bends, generating compressive stresses toward the top fiber of the section and tensile stresses near the bottom where the reinforcement is placed. The position of the neutral axis depends on the balance of internal forces and the section's geometry and material properties.

To simplify calculations, a rectangular stress block representing the concrete's maximum compressive stress is typically used, derived from the simplified "parabola-rectangle" stress-strain diagram. This compressive stress, noted  $f_{cd}$ , takes into account the safety factors applied to the concrete's characteristic compressive strength.

## 2.2. Calculating internal forces

- The compressive force in the concrete,  $F_c$ , is calculated as:

$$F_c = \alpha \times f_{cd} \times b \times x \quad (\text{V-02}).$$

where:

- $\alpha$  is a coefficient (around 0.85) representing the average stress intensity,
- $b$  is the beam width,
- $x$  is the neutral axis depth (compression zone depth), to be determined.
- The tensile force in the steel,  $F_s$ , is:

$$F_s = A_s \times f_{yd} \quad (\text{V-03}).$$

where:

- $A_s$  is the tensile reinforcement area,
- $f_{yd}$  is the design tensile strength of steel (yield strength divided by a safety factor).

## 2.3. Determining the neutral axis depth $x$

Force equilibrium requires:

$$F_c = F_s \quad (\text{V-04}).$$

Solving this equation provides the neutral axis depth  $x$ , which in turn allows calculating the lever arm  $z$ .

## 2.4. Calculating the moment resistance $M_{Rd}$

The moment resistance is given by:

$$M_{Rd} = F_s \times z = F_c \times z \quad (\text{V-05}).$$

where the lever arm  $z$  is the distance between force application points in compression and tension. It can be approximated by:

$$z = d - \frac{\lambda_2}{2} \quad (\text{V-06}).$$

where  $d$  is the effective depth (distance from the extreme compression fiber to the tensile reinforcement), and  $\lambda$  is a factor ranging between 0.8 and 1 depending on code provisions.

### 2.5. Assumptions and limitations

- Concrete's tensile resistance is neglected at the ultimate limit state.
- Steel is assumed to reach its yield point without fracturing.
- Plane sections remain plane, ensuring a linear strain distribution.

#### Practical example

Consider a rectangular beam with:

- Width  $b = 30\text{cm}$ ,
- Effective depth  $d = 50\text{cm}$ ,
- Concrete strength class C30/37 ( $f_{cd} = 17\text{MPa}$ ),
- Steel yield strength  $f_{yd} = 42.5\text{MPa}$ ,
- Tensile reinforcement area  $A_s = 2010\text{ mm}^2$ .

By applying the formulas, one can calculate  $x$  and then the ultimate moment resistance  $M_{Rd}$ .

### 3. VERIFICATION OF SECTIONS SUBJECTED TO TANGENTIAL FORCES (SHEAR)

A beam subjected to shear forces must undergo the following verifications:

- throughout all beam zones regarding serviceability and ultimate limit states;
- in simple support and beam end zones, additional checks regarding the equilibrium of the shear strut and possibly the lower node must be performed.

Prestressing introduces new factors for the calculation of prestressed elements. In addition to dead and live loads, prestress contributes as:

$$V = V_g + V_q + V_p. \quad (\text{V-07}).$$

For a prestressing force inclined at an angle  $\alpha$  relative to the beam's mean fiber, its effect decomposes into a normal force  $N = P \cdot \cos \alpha$  and a perpendicular force  $V_p = -P \cdot \sin \alpha$ , potentially negative. Therefore, the effective shear force to consider is reduced and defined as:

$$V_{red} = (V_g + V_q) - P \cdot \sin \alpha \quad (\text{V-08})$$

- Ultimate Limit State (ULS) Justification

ULS verifications of a beam include assessing both the transverse reinforcement and the compressed struts' resistance.

The first step is to determine the angle  $\beta_u$  formed by the concrete struts with the beam's mean fiber, defined by:

$$\tan(2\beta_u) = \frac{2 \tau_u}{\sigma_{xu} - \sigma_{tu}} \quad (\text{V-09})$$

with a lower bound of  $30^\circ$  for  $\beta_u$ .

PBEL regulations then define the ultimate shear stress  $\tau_{u,lim}$ , corresponding to the maximum mobilizable resistance of active and passive transverse reinforcement, given by the formula:

$$\tau_{u,lim} = \frac{A_t}{b_n s_t} f_e \frac{\sin(\alpha + \beta_u)}{\sin \beta_u} + \frac{F_{tu}}{b_n s_t} \frac{\sin(\alpha' + \beta_u)}{\sin \beta_u} \quad (\text{V-10})$$

Where:

- $A_t$  is the total cross-sectional area of passive transverse reinforcement,
- $s_t$  and  $s_t'$  are the spacings between these reinforcements measured along the beam's mean fiber,
- $f_e$  is the steel yield strength,
- $\alpha$  and  $\alpha'$  are the angles of reinforcement relative to the beam fiber (from  $45^\circ$  to  $90^\circ$ ),
- $F_{tu}$  is the resisting force of the prestressed transverse reinforcement,
- $b_n$  is the net beam width.

Partial safety factors  $\gamma_p = \gamma_s$  are 1.15 for fundamental combinations and 1.00 for accidental combinations. Compliance with the shear stress limit  $\tau_u \leq \tau_{u,lim}$  must be ensured.

For partially tensioned sections, an additional term  $\frac{F_{tj}}{3}$  may be added to  $\tau_{u,lim}$  to account for the fact that part of the shear force is balanced by the compressed portion of the beam.

In common cases where the web only has passive reinforcement perpendicular to the mean fiber, the following inequality applies:

$$\frac{A_t}{b_n s_t} f_e \gamma_s \left( \tau_u - \frac{f_{tj}}{3} \right) \tan \beta_u \quad (\text{V-11})$$

PBEL regulations further specify constructive requirements:

- A minimum amount of web reinforcement is required in all beam zones, defined by the condition:

$$\frac{A_t}{b_n s_t} f_e \sin \alpha + \frac{F_{tu}}{b_n s_t} \sin \alpha' \geq 0.4 \text{ Mpa} \quad (\text{V-12}).$$

- The spacing  $s_t/s'_t$  of prestressed transverse reinforcements must not exceed  $0.8h$ , where  $h$  is the beam's total height.
- The spacing  $s_t/s'_t$  of passive transverse reinforcement must be at most the smallest of these values:  $0.8h$ ,  $3b_0$  (with  $b_0$  as the minimum gross web thickness), or 1 meter, to prevent excessive brittleness of the beam web concrete.

#### **4. LOCALIZED FORCES AND END ZONE VERIFICATIONS**

Localized forces within a structure, such as those induced by prestressing cable anchors or concentrated supports, require special attention. These forces cause intense and concentrated stresses which can generate complex local phenomena, including concrete plasticization or high stress concentrations.

Calculations of these forces must consider the specific conditions near anchors and support zones. A fundamental concept is the diffusion of forces within a so-called "local prism" volume, which extends from the point of application of a concentrated load or anchor. The length of this prism depends on the section's geometric and material properties.

Various methods exist to model this force diffusion, including:

- the elastic method, assuming linear and homogeneous behavior,
- the strut-and-tie method, which analyzes equilibrium through distributed forces,
- the analytical or semi-analytical method, combining equilibrium principles with geometric hypotheses to define an impacted prism.

It is crucial to ensure that the concentrated force does not cause local concrete failure or excessive crushing, which could compromise the structure's durability and stability.

Calculation rules specify regularization lengths and limit stresses, as well as special reinforcement provisions in anchor end zones to secure safe load transfer to the supporting structure.

## APPLICATION EXERCISES

### Exercise 1

A simply supported beam with span  $L = 7$  m carries uniform loads: permanent actions  $g_k = 17.6$  kN/m (self-weight included) and imposed actions  $q_k = 8.0$  kN/m.

1. Determine the design internal forces at ULS using the load combination:  $1.35 \cdot G + 1.5 \cdot Q$ .
2. Plot the ULS bending moment and shear force diagrams  $M(x)$  and  $V(x)$ ; report  $M_{Ed,max}$  and  $V_{Ed,max}$ .
3. Provide the design support reactions and check consistency by integrating the diagrams.

### Exercise 2

Rectangular section  $b \times h = 30$  cm  $\times$  60 cm; nominal cover  $c_{nom} = 35$  mm; steel FeE500; concrete C25/30. Span  $L = 6$  m; design midspan moment  $M_{Ed} = 0.28$  MN·m.

1. Estimate the effective depth  $d$  and the initial lever arm  $z \approx 0.9 \cdot d$ ; design the tensile reinforcement  $A_s$  to resist  $M_{Ed}$ .
2. Verify the failure domain (under-reinforced section) and the depth of the compression block; if needed, discuss the benefit of a small compression reinforcement  $A_s'$ .
3. Propose a practical bar layout (diameter, number, spacing) compatible with cover and standard detailing rules.

### Exercise 3

beam  $b \times h = 30$  cm  $\times$  60 cm, span  $L = 7$  m, same loading as Exercise 1. Bearing on a neoprene pad with a bearing length of 20 cm.

1. Determine  $V_{Ed}$  at ULS near the supports and check shear resistance without transverse reinforcement; conclude whether stirrups are required.
2. Design the stirrups (diameter, spacing  $s$ ) so that  $V_{Rd,s} \geq V_{Ed}$ , respecting the maximum code spacing.
3. Check localized bearing at the support (local crushing/bearing zone) and, if needed, propose tie/confinement reinforcement at the support.

## CHAPTER 06: LOSSES IN PRE-STRESSING

### INTRODUCTION

Pre-stressing cables have ultimate capacities ranging from 200 to 15,000 kN and are initially tensioned to relatively high stress levels, i.e., 60% to 80% of their tensile strengths. The corresponding pre-stressing force acting in the concrete element can be significantly lower than the initial force  $\sigma_{p0}$  applied by the jacks. This difference arises from instantaneous and delayed tension losses in the cable, caused by:

- Friction of the reinforcement within their ducts.
- Slippage during anchorage blocking (certain types of anchors).
- Tension loss due to instantaneous concrete deformations, also called elastic shortening loss of concrete.
- Concrete shrinkage
- Concrete creep
- Relaxation of prestressing steel

#### *a. Instantaneous losses:*

These occur in a relatively short time at the moment of tensioning and pre-stressing, resulting from the technology or material properties. They are also called losses at execution.

In the case of post-tensioning, the prestressing reinforcements undergo instantaneous tension losses which are:

- Tension loss due to friction.
- Tension loss due to anchorage slip back.
- Tension loss due to instantaneous concrete deformations.

At a section with abscissa "X" of the reinforcement, the total value of these instantaneous losses is noted  $\Delta\sigma_{pi}(x)$ .

After deduction of instantaneous losses, the residual tension at abscissa "X", called initial tension  $\sigma_{pi}(x)$ , is given by:

$$\sigma_{pi}(x) = \sigma_{p0} - \Delta\sigma_{pi}(x) \quad (\text{VI-01}).$$

- b. Deferred losses:** These occur over a more or less long period after the structure has been prestressed and result from the time evolution of material characteristics when subjected to permanent actions. In the case of post-tension prestressing, the prestressing reinforcements undergo deferred tension losses which are:

- Tension loss due to concrete shrinkage.
- Tension loss due to concrete creep.
- Tension loss due to relaxation of prestressing steel.

At time "t" and at a section with abscissa "X" of the reinforcement, the total value of these deferred tension losses is noted  $\Delta\sigma_{pd}(t, x)$ .

$\sigma_{p0}$  : Initial tension

$$\sigma_{pi}(x) = \sigma_{p0} - \Delta\sigma_i$$

### 1. INITIAL TENSION

The prestressing forces vary along the reinforcements and over time. They are evaluated from the probable value of the initial tension, noted «  $\sigma_{p0}$  ». They must also not exceed the lesser of the following:

- Min ( 0,80  $f_{prg}$  , 0 ,90  $f_{peg}$  ) for post-tension.
- Min (0,85  $f_{prg}$  , 0,95  $f_{peg}$  ) for pre-tension.

$F_{prg}$  : Guaranteed rupture strength of prestressing reinforcement.

$F_{peg}$  : Guaranteed elastic strength of prestressing reinforcement.

At any point of a prestressing reinforcement, the tension is:  $\sigma_p = \frac{P}{A_p}$  . Initially, its value is:  $\sigma_{p0} = \frac{P_0}{A_p}$  .

Since prestressing forces vary along the reinforcements and over time, they are evaluated from the value of initial tension after deduction of the various tension losses. Following these losses, the tensile stress can only decrease; the tensioning test guarantees the cable's service resistance.

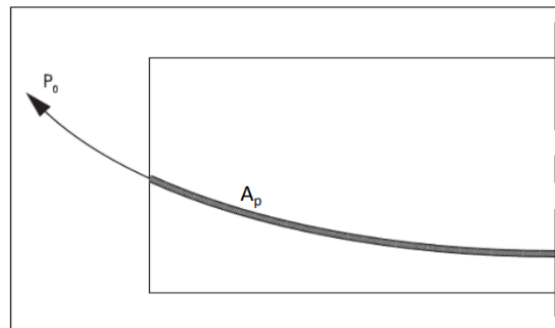


Figure1. Initial tension

### 2. INSTANTANEOUS LOSSES AT CABLE TENSIONING

#### 2.1. Loss by friction (between cable and duct)

When the cable is tensioned initially, its displacement inside the duct is hindered by friction

effects. These can be caused by duct curvature or parasitic undulations, knowing that the duct cannot be perfectly straight during assembly or concrete pouring.

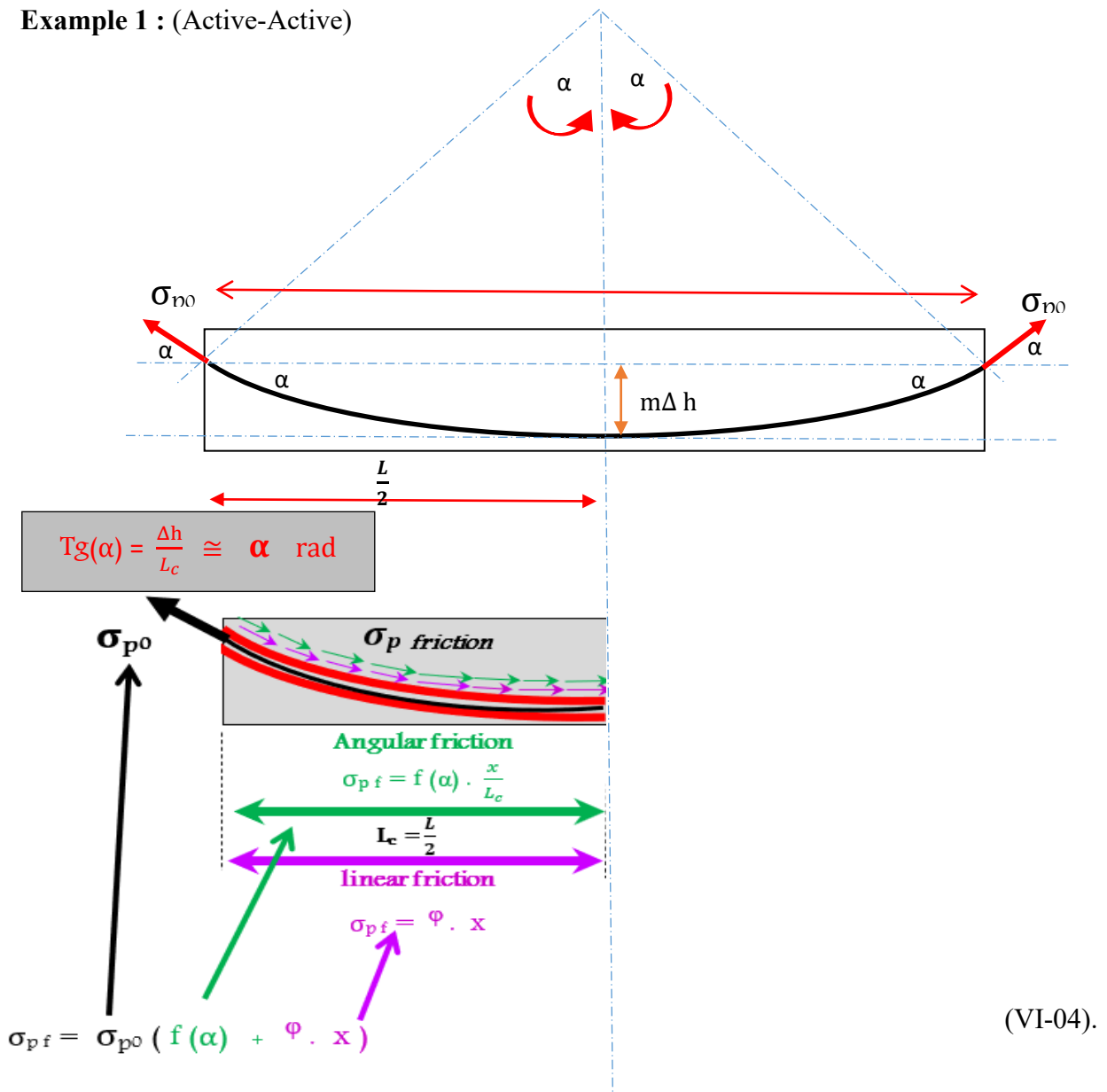
The prestress loss due to friction is estimated by the formula:

$$\Delta\sigma_{frot}(x) = \sigma_{p0} - \sigma_p(x) = \sigma_{p0}(1 - e^{-(f\alpha + \varphi x)}) \quad (VI-02)$$

If the exponent is small, the following relation may be assumed:

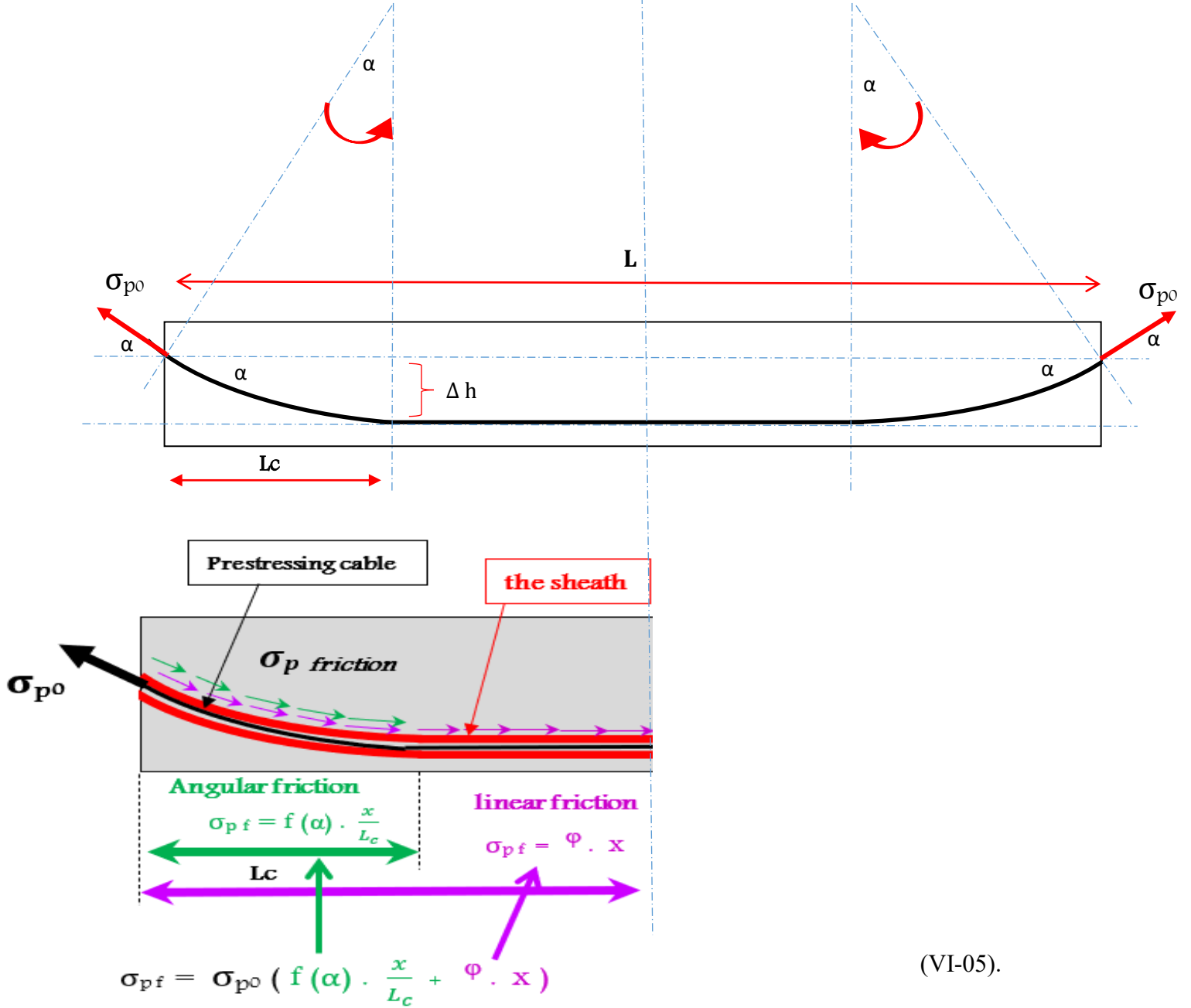
$$\Delta\sigma_{frot}(x) \cong \sigma_{p0}(f\alpha + \varphi x) \quad (VI-03)$$

**Example 1 :** (Active-Active)



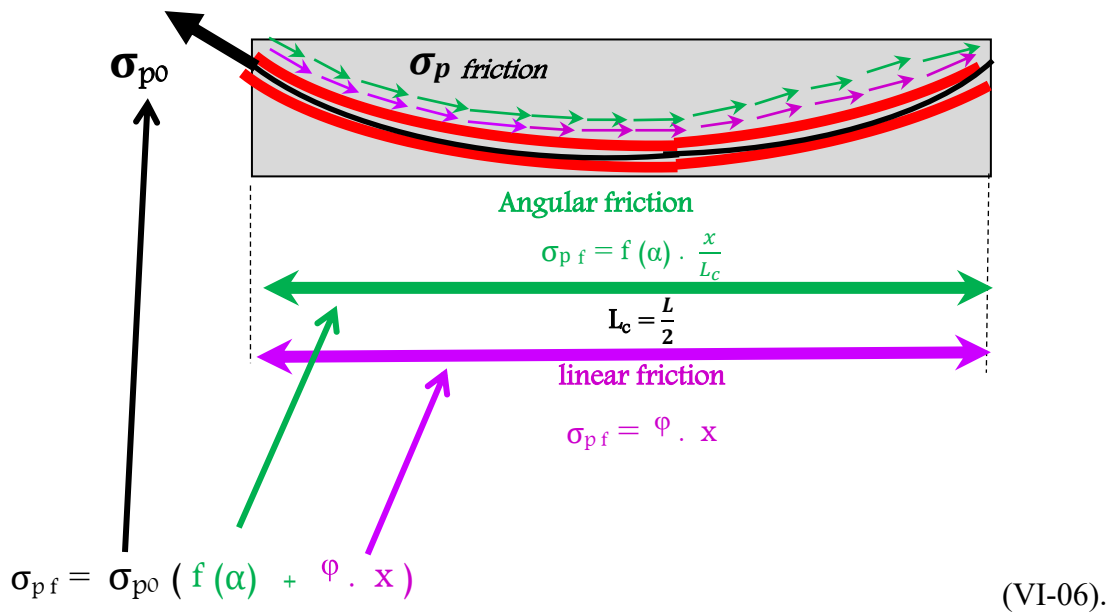
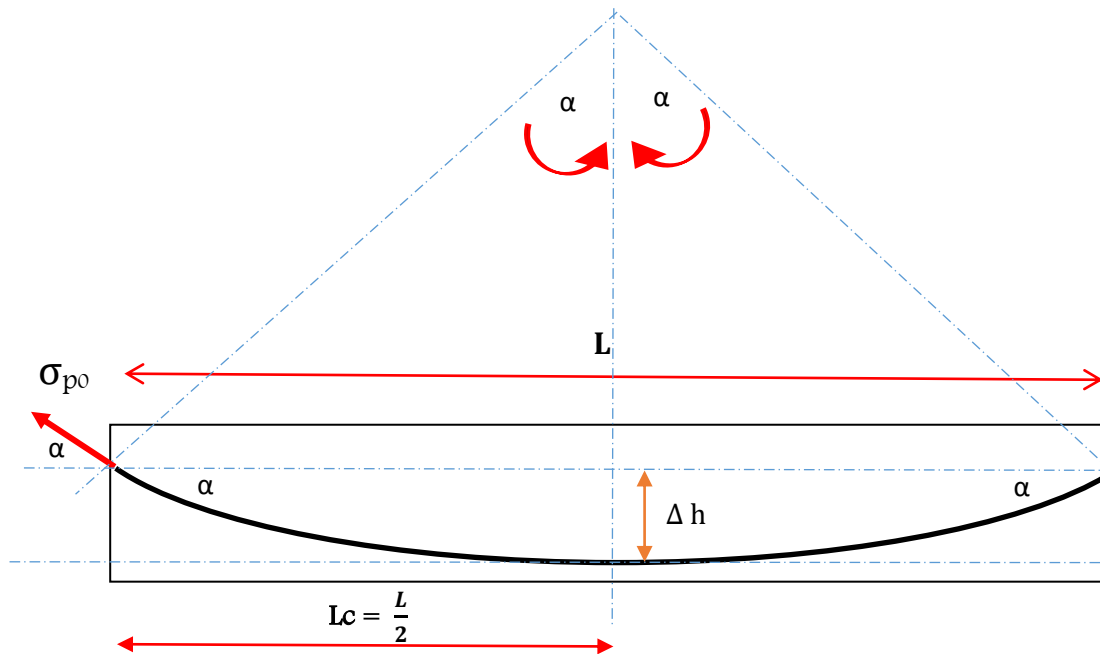
**Figure 2:** (Example (Active-Active)) Effect of friction angular on stress loss in prestressing cables.

**Example 2 : (Active-Active)**



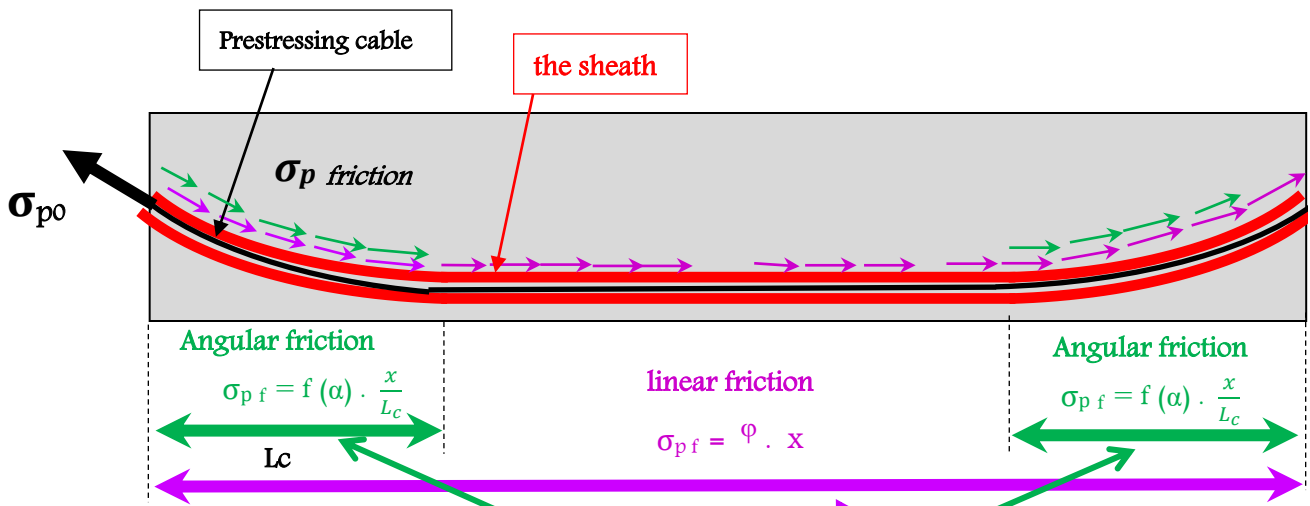
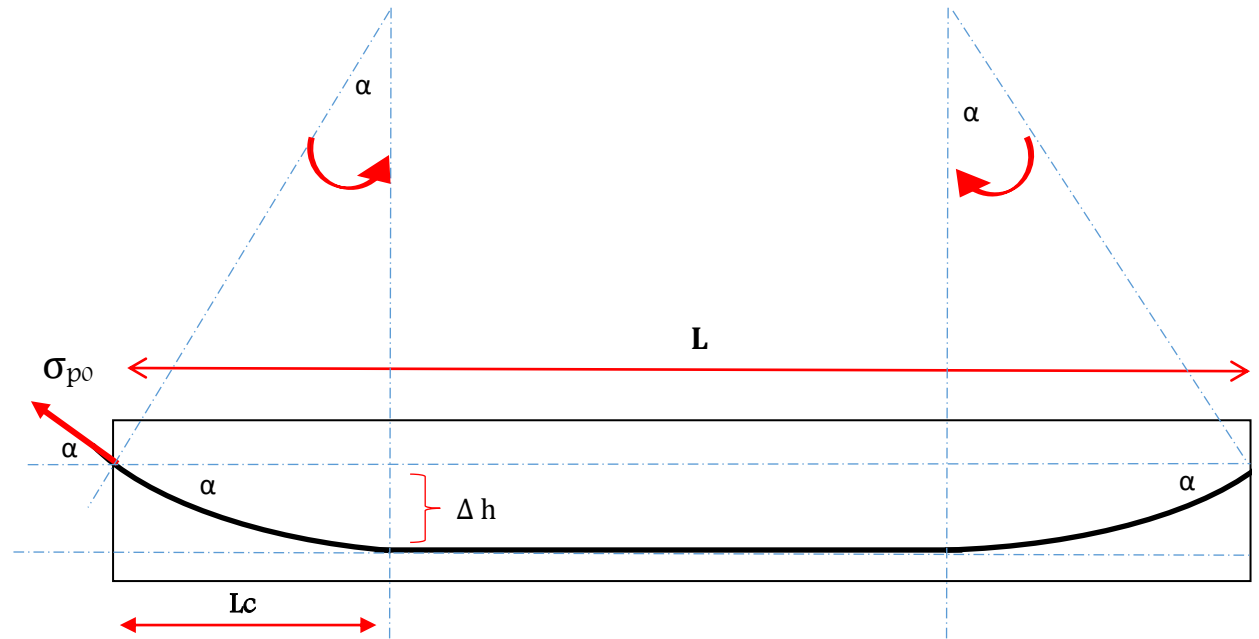
**Figure 3:** (Example (Active-Active)) Effect of friction (angular and linear) on stress loss in prestressing cables.

**Example 03 : (Active-Passive)**



**Figure 4:** (Example (Active–Passive)) Effect of friction angular on stress loss in prestressing cables

**Example 04 : (Active-Passive)**



$$\sigma_{pf} = \sigma_{po} \left( f(\alpha) \cdot \frac{x}{L_c} + \phi \cdot x \right) \quad (VI-07).$$

**Figure 5:** (Example (Active–Passive)) Effect of friction (angular and linear) on stress loss in prestressing cables.

The applied initial tension  $\sigma_{p0}$  decreases between points 0 and  $x = L$ , its new value is given by:

$$\sigma_p = \sigma_{p0} \left( f(\alpha) \cdot \frac{x}{L_c} + \varphi \cdot x \right) \quad (\text{VI-08}).$$

where  $f$  en ( $\text{rd}^{-1}$ ) et  $\varphi$  en ( $\text{m}^{-1}$ )

are friction coefficients with typical values listed in the attached table.

The tension loss due to friction is estimated by:

$$\Delta\sigma_{pi}(x) = \sigma_{p0} - \sigma_p(x) \quad (\text{VI-09}).$$

**Remark:**

Even if the cable is straight, this does not prevent the existence of parasitic undulations. In this case, there is an approximate formula for calculating the friction losses, namely

$$\Delta\sigma_{p \text{ frottement}}(x) = \sigma_{p0} (f\alpha + \varphi \cdot x) \quad (\text{VI-10})$$

**Table1: Average values of the friction coefficients  $f$  and  $\varphi$**

Case	Nature of Reinforcement	$f(\text{rd}^{-1})$		$\varphi (\text{m}^{-1})$
		$3 \leq R \leq 6 \text{ (m)}$	$R \geq 6 \text{ (m)}$	
Cables not crossing joints or construction joints	Drawn wires, round and smooth	$(22 - R) / 100$	0.16	0.002
	Strands	$(24 - R) / 100$	0.18	
Cables crossing many joints or concreting joints	Drawn wires, round and smooth	$(24 - R) / 100$	0.18	0.003
	Strands	$(26 - R) / 100$	0.20	

**2.2. Tension loss by anchorage slip-back:**

This tension loss results from the slip (sliding) of the anchorage, which depends on the anchorage devices used. Just after tensioning, the anchors are locked. However, a small anchorage slip is observed due to two phenomena:

- The existing play in the anchorage allows slight slipping before final locking.
- The force applied by the jack causes deformation of the anchorage parts.

These two phenomena cause shortening of the cable, noted  $g$ , which leads to tension loss. Technical data sheets for prestressing methods define, for each type of anchorage, the probable value of  $g$ , which can be:

- Small in the case of wedging or screwing anchors (about 5 mm), as a consequence of deformation of the anchorage body and settling of wedges or threads.
- Larger in the case of clamping anchors (about 12 mm). The strands and wedges undergo displacement inward into the part, and even more so when the jack is not equipped with a hydraulic wedge system (which forcibly drives the wedges into their conical housing before releasing pressure in the jack chamber).

The inward movement into the concrete is resisted by friction of the cable on its sheath, as during tensioning but in the opposite direction. Its influence thus diminishes from the anchorage until it cancels at a distance  $d$  from it (see next figure), beyond which tension remains unchanged.

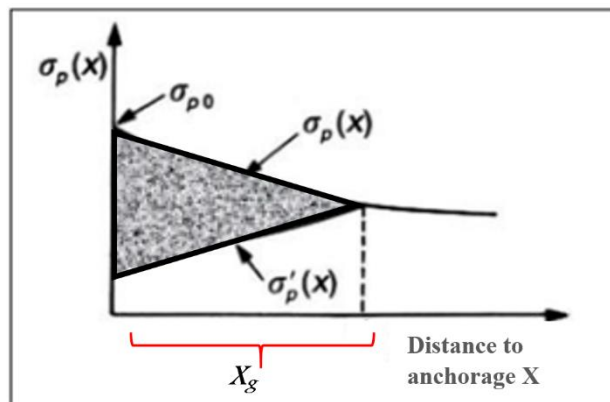


Figure 6. Tension loss due to anchorage slip.

Before anchorage slip, the relative elongation of the prestressing reinforcement at abscissa :

$$\varepsilon_p(x) = \frac{\sigma_p(x)}{E_p} \quad (\text{VI-11})$$

After anchorage slip, the tension at abscissa  $x$  ( $x < d$ ) is only:  $\sigma'_p(x)$ , and the relative elongation of the prestressing reinforcement is

$$\dot{\epsilon}_p(x) = \frac{\dot{\sigma}_p(x)}{E_p} \quad (\text{VI-12})$$

The influence length of the tension loss due to anchorage slip, called "point d," is given by the formula:

$$X_g = \sqrt{\frac{g \cdot E_p}{P}} \quad (\text{VI-13})$$

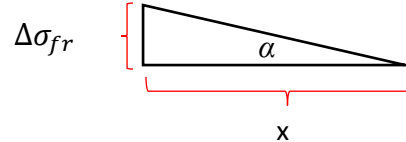
$g$ : between 2-6 mm (provided by the supplier).

$$E_p = 1,9 \times 10^5 \text{ MPa}$$

$$P = \text{The slope} = \tan \alpha = \frac{\Delta \sigma_{frott}(x)}{x}$$

$$\text{For } X_g < \frac{L}{2}: \quad \Delta \sigma_{P \text{ slip}}(x) = 0$$

$$\text{For } X_g \geq \frac{L}{2}: \quad \Delta \sigma_{P \text{ slip}}(x) = 2 \cdot P \cdot X_g$$



### 2.3. Loss due to instantaneous deformation of concrete (Cable relaxation)

When a beam has several cables tensioned one after another, the cables are tensioned sequentially. Each time a cable is tensioned, a shortening of the concrete occurs, meaning a drop in tension in the previously tensioned cables.

Thus, it can be shown that:

$$\Delta \sigma_{pi}(x) = \left(\frac{n-1}{2n}\right) \sigma_b(x) \cdot \frac{E_p}{E_{bi}} \quad (\text{VI-14})$$

- $n$ : number of cables
- $\sigma(x)$ : normal compressive stress of concrete under long-term actions
- $E_{bi}$ : instantaneous modulus of concrete at day  $j$
- $E_p$ : Young's modulus of prestressing reinforcement

#### Remark:

- If the prestressed member contains only one cable, or if all cables are tensioned simultaneously, elastic losses are zero ( $\Delta \sigma_{acc}(x) = 0$ ). Indeed, the elastic shortening occurs simultaneously with the tensioning of the cables, since the jack presses against the member.
- The BPEL91 regulation proposes :

$$\Delta \sigma_{pi}(x) = \frac{1}{2} \sigma_b(x) \cdot \frac{E_p}{E_{bi}} \quad (\text{VI-15})$$

### **\*\* Total Instantaneous Tension Loss \*\***

The previously treated losses occur at the moment of tensioning the active reinforcements. They are cumulative and defined in the space reference frame by:

$$\Delta\sigma_i = \Delta\sigma_p \text{ (friction)} + \Delta\sigma_p \text{ (instantaneous concrete deformation)} + \Delta\sigma_p \text{ (instantaneous concrete deformation)} \quad (\text{VI-16})$$

### **3. DELAYED TENSION LOSSES**

#### **3.1. Tension Loss Due to Concrete Shrinkage**

Shrinkage is the spontaneous shortening deformation of concrete during its hardening, in the absence of any constraint. This deformation is mainly due to the progressive evaporation of unbound water in the cement hydration reaction. The value of this shortening is given by a formula in chapter II:

$$\underline{\epsilon}_r(t) = \underline{\epsilon}_r^* r(t), \quad (\text{VI-17})$$

where  $\underline{\epsilon}_r$  is the final shrinkage value, and  $r(t)$  is a function describing the evolution of shrinkage over time.

In post-tensioning, tensioning takes place after the concrete has hardened. The resulting shrinkage after tensioning is only:

$$\underline{\epsilon}_r(t) = \underline{\epsilon}_r(r(t) - r(t_1)), \quad (\text{VI-18})$$

Where,

$t_1$ , is the age of the concrete at the time of tensioning.

Shrinkage shortening causes a progressive decrease in tension in the prestressing reinforcements. This is the tension loss due to shrinkage  $\Delta\sigma_{ret}$ , whose final value is given by the formula:

$$\Delta\sigma_{ret} = E_p \cdot \underline{\epsilon}_r(t) = E_p \cdot \underline{\epsilon}_r[r(t) - r(t_1)] \quad (\text{VI-19})$$

$$r(t) = \frac{t}{t+9rm} \quad (\text{VI-19}')$$

$rm$ : the average radius of the section and  $\frac{S}{P}$  en (m) (ratio of the cross-sectional area to its perimeter)

S: the cross-sectional area

P: The perimeter

It indicates the average path traveled by water during its evaporation, which conditions the evolution of shrinkage over time t.

- $\Delta\sigma_{ret}$  : Tension loss due to shrinkage
- $\underline{\epsilon}_r$  : final shrinkage
- T : time (age of concrete)

Very often,  $r(t_1)$  can be neglected compared to 1, which leads to the following simplified formula:

$$\Delta\sigma_{ret} = E_p \cdot \underline{\epsilon}_r \quad (VI-20)$$

### 3.2. Tension Loss Due to Concrete Creep

By definition, creep is the progressive shortening of concrete under constant stress. This phenomenon is also related to the migration of water inside the concrete. In a prestressed concrete member, the prestressing reinforcements undergo the same shortening as the concrete due to its creep, resulting in a loss of tension. The BPEL provides the final tension loss due to concrete creep over time,  $\Delta\sigma_{flu}(t)$ , by the formula:

$$\Delta\sigma_{flu} = E_p \cdot 2 \underline{\epsilon}_r = E_p \cdot 2 \frac{\sigma_b}{E_{ij}}$$

$$\Delta\sigma_{flu}(t) = 2 \frac{E_p}{E_{ij}} \cdot \sigma_b = 2 \frac{E_p}{E_{ij}} \left( \frac{\sigma_b^F + \sigma_b^M}{2} \right) = (\sigma_b^F + \sigma_b^M) \frac{E_p}{E_{ij}} \quad (VI-21)$$

$$/ \quad \sigma_b = \left( \frac{\sigma_b^F + \sigma_b^M}{2} \right)$$

With:

- $\sigma_b^{Final}$  Being the final permanent stress in the concrete at the average cable location.
- $\sigma_b^{Max}$  The maximum permanent stress in the concrete at the average cable location at tensioning.
- J: The age of the concrete at the time of tensioning.

### 3.3. Tension Loss Due to Steel Relaxation

Relaxation can be defined as the release of tension in prestressing reinforcements at constant length. It is a phenomenon that depends on temperature, time, and the stress in the steel.

The relaxation evolution law over time is quite complex; the stabilized one is taken into consideration.

It is defined by:

$$\Delta\sigma_{\text{relax}} = 6 * \frac{\rho_{1000}}{100} \left( \frac{\sigma_{pi}}{f_{prg}} - \mu_0 \right) \sigma_{pi} \quad (\text{Selon règles BPEL}) \quad (\text{VI-22})$$

$\sigma_{pi}$ : Initial stress in the prestressing reinforcements (after instantaneous losses).

$$[\sigma_{pi} = \sigma_{p0} - \sigma_{p \text{ instantaneous}} = \sigma_{p0} - \sigma_{p \text{ friction}} - \sigma_{p \text{ anchorage slip}} - \sigma_{p \text{ elastic}}]$$

$\rho_{1000}$ : Relaxation coefficient of prestressing steel at 1000 hours (in %).

$f_{prg}$ : Guaranteed rupture limit stress of prestressing steel.

$\mu_0$ : A coefficient taken as:

- 0.43 for very low relaxation reinforcements (TBR).
- 0.30 for normal relaxation reinforcements (RN).
- 0.35 for other reinforcements.

**\*\*\*\*\* Total Delayed Tension Loss \*\*\*\*\***

The formula given for the tension loss due to relaxation of prestressing steels assumes that the length of these reinforcements is constant. However, the relaxation loss is reduced by the effect of shortening due to shrinkage and creep of the concrete.

To account for this interaction, the BPEL proposes to reduce the relation by a factor of 5/6 when calculating the total delayed loss.

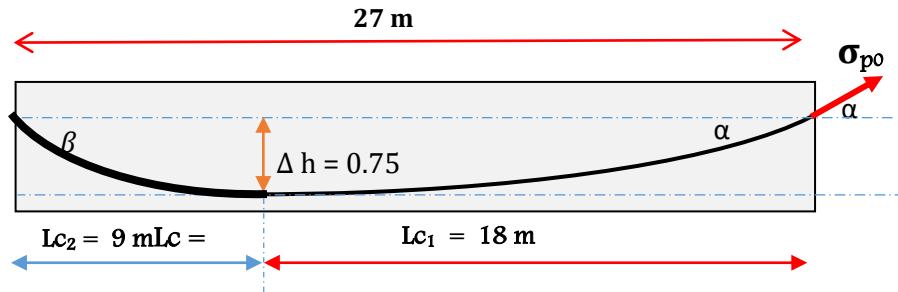
Thus, the final delayed loss is taken equal to: Here is the translation in English for the requested text:

$$\Delta\sigma_{pd}(x) = \Delta\sigma_{\text{shrinkage}} + \Delta\sigma_{\text{creep}} + \frac{5}{6} \Delta\sigma_{\text{relax}} \quad (\text{VI-23}).$$

#### 4. APPLICATIONS EXERCISES

**Exercise1** : Given a post-tensioned beam with a length of 27.00 m. For the midspan section, determine:

1. The initial stress, knowing that ( $f_{prg} = 1860$  MPa;  $f_{peg} = 1680$  MPa)
2. The friction loss at:  $x = 0$  ;  $x = \frac{L}{4}$  ;  $x = \frac{L}{2}$  ;  $x = \frac{3L}{4}$  ;  $x = L$
3. Plot the curve  $\Delta\sigma$  as a function of :  $x = L$



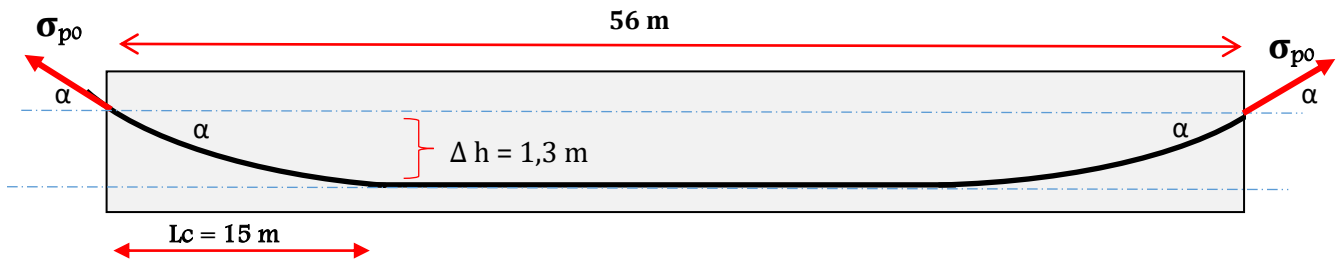
**Data :**

$f = 0.18 \text{ rd}^{-1}$      $\varphi = 0.002 \text{ m}^{-1}$      $g = 6 \text{ mm}$      $\sigma_{bc} = 12.4 \text{ MPa}$      $\sigma_b^M = 15.3 \text{ MPa}$      $\sigma_b^F = 11,1 \text{ MPa}$      $E_p = 190000 \text{ MPa}$   
 $f_{c28} = 35 \text{ MPa}$      $\varepsilon_r = 3.10^{-4}$      $\rho_{1000} = 2,5 \%$      $\mu_o = 0.43$      $E_{ij} = 11000 \sqrt{f_{c28}}$

**Exercise2**

Given a post-tensioned beam with a length of 56.00 m, prestressed by 4 tendons of 7T13. For the midspan section, determine:

- The initial stress, knowing that ( $f_{peg} = 1880$  MPa;  $f_{prg} = 1660$  MPa)
- The instantaneous losses.
- The time-dependent (deferred) losses.



**Data :**

$f = 0.18 \text{ rd}^{-1}$      $\varphi = 0.003 \text{ m}^{-1}$      $g = 6 \text{ mm}$      $\sigma_{bc} = 12.4 \text{ MPa}$      $\sigma_b^M = 15.3 \text{ MPa}$      $\sigma_b^F = 11,1 \text{ MPa}$      $E_p = 190000 \text{ MPa}$   
 $f_{c28} = 35 \text{ MPa}$      $\varepsilon_r = 3.10^{-4}$      $\rho_{1000} = 2,5 \%$      $\mu_o = 0.43$      $E_{ij} = 11000 \sqrt{f_{c28}}$

## **GENERAL CONCLUSION**

This course material provides a comprehensive and structured overview of pre-stressed concrete, covering the fundamental concepts, implementation methods, practical aspects, and recent developments in the field.

It highlights the major advantages of pre-stressed concrete in enabling the construction of stronger, lighter, and more durable structures to meet the increasing demands of modern construction.

By detailing the integration of pre-stressing techniques and special high-performance materials, and stressing the importance of tension losses and limit state verifications, the document equips students with essential knowledge for designing, sizing, and executing innovative and robust civil engineering projects.

The emphasis on practical examples and Algerian construction sites reflects the growing potential of pre-stressed concrete for national development. This work is designed to strengthen the scientific and technical mastery of future engineers and promote the transfer of advanced technology within the civil engineering sector.

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