



Department of Civil Engineering

Course Notes

Module Coordinator: Dr. BOUKHATEM Ghania

Title

Foundations and Geotechnical Structures

Course intended for students of

Licence Degree (specialization and level):

3rd Year LMD

Civil Engineering Licence

Year : 2025/2026

“No structure can be built securely without first understanding the ground on which it stands.”

Preface

Geotechnical engineering plays a fundamental role in civil engineering projects, since every structure rests on soil whose behavior directly influences its stability and durability. Understanding the properties and mechanisms governing soil behavior is therefore essential to ensure the safety and reliability of engineering works.

This book is intended as a pedagogical resource for engineering students and practitioners in the field. It provides a concise overview of the fundamentals of applied soil mechanics, covering limit equilibrium concepts, the theories of Rankine and Boussinesq, and the basic principles for the design of retaining walls and shallow foundations. It also addresses slope stability and the methods used for soil evaluation and reinforcement.

The methods presented are based on the fundamental principles of geotechnical engineering and are consistent with current design regulations, particularly Eurocode 7. However, their practical application requires careful adaptation to site-specific ground conditions and the applicable national standards.

Finally, I expressed sincere gratitude to colleagues and students whose discussions and valuable feedback have contributed to improving this work, with the hope that this book will serve as a useful guide for future engineers.

Table of contents

List of figures

List of tables

List of terms

Preface

Introduction 1

Chapter I : Limit Equilibrium States

1- Introduction	4
2- Lateral pressure-deformation relationship.....	5
3- Earth Pressure At Rest.....	6
3-1- Granular soil.....	8
3-2- Cohesive soil	9
4- Calculation of thrust and stop	11
4-1- Rankine Method	11
4-1-1- Granular soil.....	12
4-1-1-1- Unloaded backfill	12
4-1-1-2- Loaded backfill	13
4-1-1-3- Effect of Water Table on a Backfill	14
4-1-1-4- Effect of Stratified Soils in the Backfill	14
4-1-1-5- Inclined backfill	14
4-1-2- Cohesive soils.....	15
4-2- Boussinesq's Theory	17
4-2-1- Floor-wall friction	18
4- Conclusion	19
Applications	20
Exercise problems.....	25

Chapter II : Retaining walls

1- Introduction	27
2- Types of retaining walls	28

2-1- Gravity wall.....	29
2-2- Walls constructed in the ground	30
2-3- Composite retaining structures	31
3- Preliminary sizing.....	32
4- Failure modes	33
5- Drainage and water evacuation systems	34
6- Calculation of retaining walls	35
6-1- Efforts soliciting a retaining wall	35
6-2- The vertical forces	36
6-3- The horizontal forces	36
6-4- Punching stability	37
6-5- Sliding stability	38
6-6- Overtuning stability.....	39
7- Conclusion	40
Applications	40
Exercise problems.....	49

Chapter III : Shallow and deep foundations

1- Introduction	53
2- Shallow foundations.....	54
2-1- Definition.....	54
2-2- Types of shallow foundations	54
2-3- Behavior of a footing under centered vertical load.....	56
2-4- Bearing capacity theory	57
3- Bearing capacity	59
3-1- Rectangular or circular foundation	60
3-1-1- Square foundation with side B	60
3-1- 2- Circular foundation with diameter B	61
3-1- 3- Rectangular foundation with width B and length L	61
3-1-4 Eccentric and inclined loads.....	61
4- Long-term and short-term bearing capacity.....	62
4-1- Cohesive Soils	62
4-2- Granular Soils	62

5- Foundations on two-layer soils	62
6- Calculations of the allowable bearing capacity.....	63
7- Conclusion	63
Applications	64
Exercise problems	65

Chapter IV: Slope stability

1- Introduction	67
2- Causes of landslides	68
3- Classification of earth movement.....	69
3-1-Slides	69
3-1-1- Basic Types of Landslides	69
3-1-2- Compound Slides	70
3-1-3- Translational Slides	70
3-1-4- Flows	70
4- Problems posed.....	70
4-1- Factor of safety	71
4-2- Analysis	71
5- Analysis of an infinite slope.....	72
5-1- Infinite slope without seepage	72
5-2- Infinite slope with seepage.....	73
6- Analysis of a finite slope	74
6-1- Global method.....	74
6-2- Method of slices.....	75
6-2-1- Fellenius method	76
6-2-2- Bishop's method	77
8- Reinforcement methods	78
9- Conclusion	79
Applications	80
Exercise problems	66
Conclusion	85

Bibliography

List of figures

List of Figures chapter I

Fig. I.1. Force exerted by the soil and the reaction of the wall.....	5
Fig.I. 2. Relationship between lateral pressure and lateral displacement.....	6
Fig. I. 3. Subsurface stresses for the at-rest condition	7
Fig. I.4. Elastic equilibrium state at a point	8
Fig. I.5. Active equilibrium state at a point	9
Fig. I.6. Passive equilibrium state at a point.....	9
Fig. I.7. Long-term active and passive equilibrium states.....	11
Fig. I. 8. Short-term active and passive equilibrium states.....	11
Fig. I.9. Sliding wedge.....	12
Fig. I.10. Thrust on an uncharged soil mass	12
Fig. I.11. Loaded soil mass thrust	13
Fig. I.12. Thrust on an inclined soil mass.....	14
Fig. I. 13. Thrust in cohesive soil.....	16

List of figures chapter II

Fig II.1: Stone Walls.....	29
Fig II.2: Gabion Walls	29
Fig II.3: Reinforced Concrete Cantilever Wall	30
Fig II.4: Buttressed Wall.....	30
Fig II.5: Gravity wall with precast elements.....	30
Fig II.6: Sheet pile walls made of metal profiles	31
Fig II.7: Cofferdams for the construction of a bridge pier	31
Fig II.8: Earth structures reinforced with geotextiles.....	31
Fig II.9: Slope stabilized with nails.....	32
Fig II.10: Typical dimensions of a cantilever wall.....	32
Fig II.11: Typical dimensions of gravity walls and inverted T-walls	33
Fig II.12: Different failure modes	34
Fig II.13: Heel	34
Fig II.14: Drainage devices behind a retaining wall	35
Fig II.15: Acting forces on a gravity wall according to Rankine.....	37
Fig II.16: Acting forces on a gravity wall according to Coulomb	37

Fig II.17: Forces involved in sliding stability	38
Fig II.18: Overturning safety	39

list of figures chapter III

Fig III.1.: Shallow Foundations	54
Fig III.2: Strip foundations	55
Fig III.3: Isolated footings	55
Fig III.4: Raft slab	55
Fig III.5: Footing under centered vertical load	56
Fig III.6: Load-Settlement Curve	57
Fig III.7: Active and Passive Wedge during failure	58
Fig III.8: Failure zones	58
Fig III.9: Coefficients N_q , N_γ , and N_c (bearing capacity factors) as a function of the internal friction angle	60
Fig III.10. Case of an inclined load	61

List of figures chapter IV

Fig IV.1: Activities that decrease or increase the probabilities of slides	68
Fig IV.2: Types of mass movements on clay slopes	69
Fig IV.3: Rotational Slides	70
Fig IV.4: The shear stresses in Rotational Slides	71
Fig IV.5: Forces acting on a slice without seepage	72
Fig IV.6: Forces acting on a slice with seepage	73
Fig IV.7: Failure line of a finite slope	74
Fig IV. 8: Division of a slope into slices	75
Fig IV.9: Graphical representation of the forces	76
Fig IV.10: Reduction of the height of a slope	78
Fig IV.11: Reduction of the slope	78
Fig IV.12: Construct berms	79
Fig IV.13: Install a drainage layer	79
Fig IV.14: Create drainage trenches	79

List of tables

Chapter I

Table I.1: Ordre de grandeur du coefficient de poussée du sol au repos..... 8

Table I.2: Caquot–Kerisel tables of the active earth pressure coefficient K_a for some common cases 17

Table I.3: Floor-wall friction values 19

Chapter II

Table II.1: ϕ and C values of some soils 28

Chapter III

Table III.1: Bearing capacity factors (according to TERZAGHI) 59

List of terms

1. Soil parameters

γ : total unit weight of the soil

γ' : effective unit weight of the soil

ϕ' : effective internal friction angle of the soil

c_u : undrained shear strength of the soil

δ : soil–wall friction angle

c_a : soil–wall adhesion coefficient

2. Stresses

σ_v : vertical total stress

σ'_v : effective vertical stress

σ'_h : effective horizontal stress

σ_{ha} : horizontal active total stress

σ_{hp} : horizontal passive total stress

K_0 : coefficient of at-rest earth pressure (generally < 1)

τ_{max} : shear stress

q_u : bearing capacity stress

q_{adm} : allowable stress

3. Geometrical parameters

α : inclination of the wall with the horizontal

β : inclination of the ground surface with the horizontal

B : base width of the foundation

4. Forces and reactions

F_h : ground thrust force

F_p : possible passive resistance

R : vertical ground reaction

5. Stability

F_R : safety factor Overturning

F_S : safety factor Sliding

M_m : driving moment that promotes failure

M_s : stabilizing moment opposing failure

Introduction

The course *Foundations and Geotechnical Structures (FOG)* is designed to provide students with an in-depth understanding of the fundamental principles and methods of geotechnical engineering as applied to civil engineering structures. This course combines theoretical concepts and practical applications, enabling students to develop the skills necessary to design, analyze, and verify the stability of various types of soil-supported structures.

At the end of this module, the student will be able to:

- Analyze and calculate the stability of geotechnical structures, including shallow and deep foundations, retaining walls, and slopes.
- Understand soil strength and earth pressure concepts and apply limit equilibrium principles to determine potential failure conditions.
- Apply design and verification methods for geotechnical structures using theoretical formulas, safety factors, and regulatory guidelines.
- Develop practical skills for reading, interpreting, and using geotechnical data in the calculation and design of structures.

To effectively follow this course, students should have acquired knowledge in the following subjects:

- Soil Mechanics 1 and 2 (MDS1, MDS2): consolidation, compressibility, shear strength concepts.
- Strength of Materials 1 and 2 (RDM1, RDM2): stress, deformation, and elastic limit concepts.
- Reinforced Concrete 1 (BA1): understanding of materials and structural design principles.

This prior knowledge will enable students to tackle advanced applied geotechnical concepts, interpret soil test results, and understand soil-structure interaction.

The course is organized into four main chapters, scheduled over the semester to balance theory and practical work:

Chapter 1: Limit States (3 weeks)

- Introduction to lower and upper Rankine limit states
- Calculation of earth pressure and passive resistance coefficients
- Boussinesq equilibrium analysis for general loading cases
- Prandtl equilibrium for surcharge-induced pressures
- Determination of failure planes using Mohr's circle for active and passive pressure cases
- Case studies and practical examples for calculating active and passive forces

Chapter 2: Retaining Structures (4 weeks)

- Definition, function, and classification of retaining structures
- Earth pressures: active, passive, and surcharge loads
- Verification of global and local stability of retaining walls
- Design and dimensioning of concrete walls, gabions, and other types of structures
- Tutorials and exercises on calculating safety factors and earth pressures

Chapter 3: Shallow Foundations (4 weeks)

- Definition and classification of shallow foundations
- Soil bearing capacity theory and calculation methods (Terzaghi, Meyerhof)
- Effects of eccentric and distributed loads
- Influence of foundation depth, width, and soil conditions on design
- Tutorials: practical calculation of bearing capacity and stability verification

Chapter 4: Slope Stability (4 weeks)

- Introduction to slope stability concepts and safety factors

- Analytical and graphical methods: global method and slice method
- Application to cohesive and non-cohesive soils, effective and residual states

- Influence of groundwater and hydraulic conditions on stability
- Case studies: natural slopes, excavations, and artificial embankments

The course combines:

- Theoretical lectures to present fundamental principles and calculation formulas.
- Tutorials (TD) to apply concepts to practical problems and develop computational skills.

Evaluation Method

- Continuous assessment: 40 %
 - Practical exercises, tutorials, and mini-projects
- Final exam: 60 %
 - Assessment of theoretical knowledge and practical application skills

I. Limit Equilibrium States

Chapter I. Limit Equilibrium States

1. Introduction

In civil engineering, accurately estimating lateral earth pressure is critical for the structural integrity of retaining walls, sheet piles, basement walls, and cofferdams. These pressures primarily arise from soil and water acting against a supporting structure. The magnitude of this pressure is influenced by several variables, including soil properties, wall roughness, and the displacement of the structure (Das & Sobhan, 2008; Budhu, 2000).

Depending on how the wall moves relative to its backfill, the soil may remain in a state of elastic equilibrium with minimal movement, or it may reach a plastic state when the shear stresses approach the ultimate shear strength of the soil (Craig, 2007).

In a soil mass, the upper layers exert vertical stresses on the lower layers. These vertical stresses generate horizontal stresses within the soil mass, commonly referred to as earth pressures. The analysis of these lateral stresses is a fundamental topic in soil mechanics and geotechnical engineering (Terzaghi, 1943).

Classical earth pressure theories generally assume a plane strain condition and a rigid–perfectly plastic soil behavior, where shear deformation occurs at constant stress once the soil reaches its failure state (Budhu, 2000; Das & Sobhan, 2008).

The understanding of earth pressure is essential for many practical applications, particularly in the analysis of slope stability and the design of geotechnical structures such as embankments, retaining walls, sheet pile walls, and other earth-retaining systems. These structures must be designed to safely resist the pressures exerted by the soil mass behind them (Coduto, Yeung & Kitch, 2004).

For a soil mass behind a retaining structure, three main equilibrium states are generally distinguished:

- The **at-rest state**,
- The **active (thrust) state**,
- The **passive (resistance) state**.

These states depend mainly on the magnitude and direction of the wall displacement relative to the backfill soil.

2. Lateral pressure-deformation relationship

Consider a soil mass supported against a rigid and smooth wall, Fig I.1 The equilibrium of the system is ensured by the action exerted by the soil and the reaction of the wall, these two forces having the same magnitude.

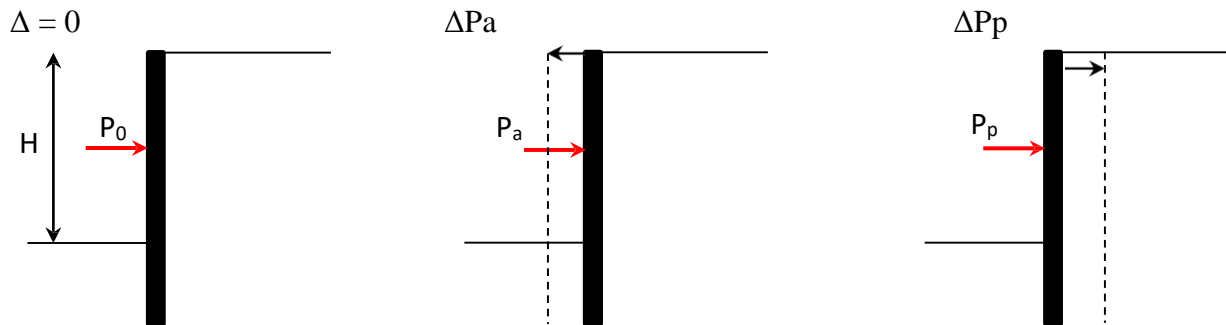


Fig I.1. Force exerted by the soil and the reaction of the wall.

The Relationship between lateral pressure and lateral displacement in Fig I.2 is as:

Case 1: No displacement ($\delta = 0$)

When the wall shows no displacement, the soil mass remains in equilibrium, and the stress exerted corresponds to the at-rest earth pressure..

Case 2: Displacement outward ($\delta \neq 0$)

If the wall moves slightly outward, the pressure exerted by the soil gradually decreases until it reaches a limiting value F_a . Beyond this value, the soil mass fails by horizontal sliding. The force F_a corresponds to the active earth pressure

Case 3: Displacement inward ($\delta \neq 0$)

Conversely, if the wall moves inward, the pressure exerted by the soil gradually increases until it reaches a limiting value F_p . Beyond this value, the soil mass fails by vertical movement. The force F_p corresponds to the passive earth pressure

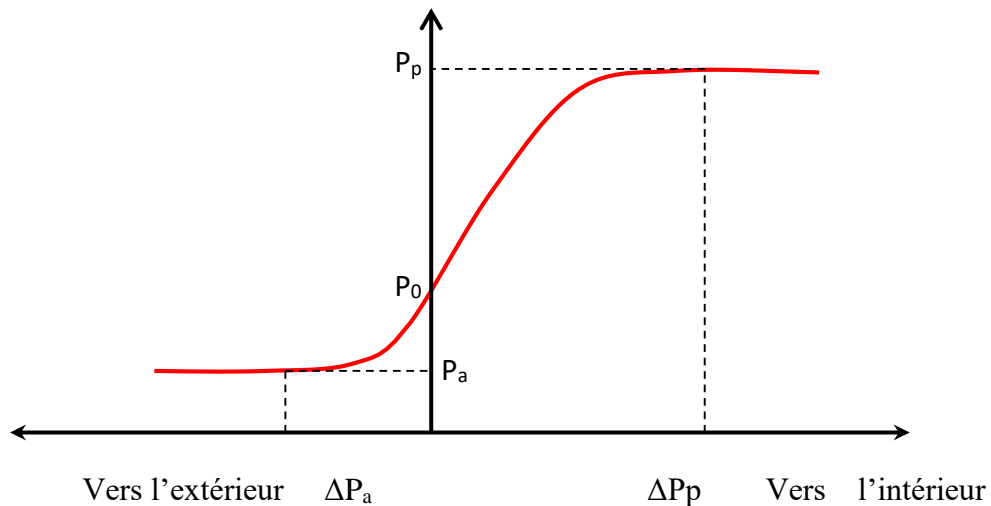


Fig I.2. Relationship between lateral pressure and lateral displacement.

Equilibrium States :

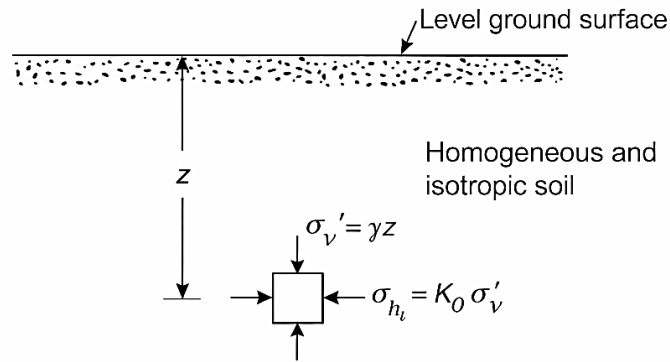
- Lower limit equilibrium called active or thrust equilibrium.
- Upper limit equilibrium called passive or resistance equilibrium
- Elastic equilibrium, which is the soil state without lateral deformation.

3. Earth Pressure At Rest

During the formation of a soil deposit, the soil mass at a point is acted on by the vertical geostatic stress, σ_v , of the overburden. This vertical stress causes a vertical compression of the soil and at the same time produces a lateral strain. This lateral strain is completely restrained due to formation of all-round lateral stress of equal magnitude. With time, the vertical compression and lateral creep strains become zero, and a stable state of stress is created. Because of zero strain, a situation of effective vertical and horizontal stresses is attained. This state of equilibrium is called the at-rest condition or K_0 -condition. Consider an element of soil in such a homogeneous and isotropic soil bounded by a level ground surface. The effective horizontal and vertical stresses are shown in Fig. I.3. For the at-rest condition, the ratio of horizontal to vertical stress is called the coefficient of lateral stress at rest or lateral stress ratio at rest or coefficient of earth pressure at rest K_0 ;

The calculation of earth pressure and earth resistance is linked to the type of equilibrium, which corresponds to a certain stress distribution

The relationship between σ'_h and σ'_v is expressed in terms of effective stresses Fig I.3; that is:



(a) Sub-surface stresses in the soil mass

Fig I.3. Subsurface stresses for the at-rest condition

These stresses are represented by a Mohr's circle along with the shear strength envelope in Fig. I.4. The location of Mohr's circle well below the failure envelope indicates a stable equilibrium condition. An increase in stresses would still keep the soil in an elastic equilibrium until the stresses are increased further to cause a failure, and the soil is then said to be at plastic or limiting equilibrium. Mohr circles for these two conditions are shown in Fig. I.4. In general, for many situations, $K_0 < 1$, except in over-consolidated clays (OCC) where K_0 may be as high as 3.0. For normally consolidated clays (NCC), $K_0 < 1$, and for sand deposits, K_0 varies from 0.40 to 0.50. It is impossible to determine K_0 by measuring σ_{h0}' in situ. Therefore, certain correlations have been suggested. Brooker and Ireland (1965) have suggested for K_0 in terms of the plasticity index I_p and effective friction angle ϕ' ; that is,

$$K_0 = M - \sin\phi'$$

where $M = 1$ for NCC and non-cohesive soils

$M = 0.95$ for OCC for over-consolidation ratio (OCR) > 2 For NCC,

K_0 is also given as $K_0 = a + b I_p$

where

$a = 0.40$ and $b = 0.007$ for $0\% < I_p < 40\%$

$a = 0.64$ and $b = 0.001$ for $40\% < I_p < 80\%$

that is

K_0 : At-rest earth pressure coefficient (generally < 1). The coefficient K_0 can be calculated by:

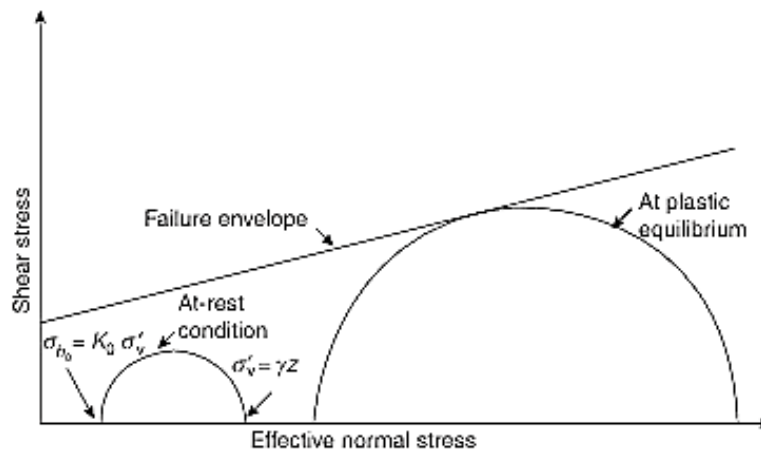
$$K_0 = 1 - \sin \phi \text{ (granular soils).}$$

$$K_0 = 0,44 + 0,0042 IP \text{ (cohesive soils).}$$

The values of Table I.1 below can be retained as an order of magnitude of the coefficient K_0 of thrust of the ground at rest: is independent of the state of saturation of the massif. It is constant for the same soil layer and the same density.

Tableau I.1. Ordre de grandeur du coefficient de poussée du sol au repos (Bouafia .2009).

Type de sol	Sable	Argil e	Argile très molle et vase	Roche à très grande profondeur
Valeur de K_0	0.5	0.7	1.0	≥ 1



(b) Stress related to failure envelope for the at-rest condition

Fig I.4. Elastic equilibrium state at a point.

3.1.Granular soil

Active equilibrium (FigI. 5)

When the wall moves outward, the soil undergoes lateral expansion. In this case:

The vertical stress remains constant $\sigma_v = \gamma Z$

The horizontal stress σ_h decreases until it reaches σ_{ha}

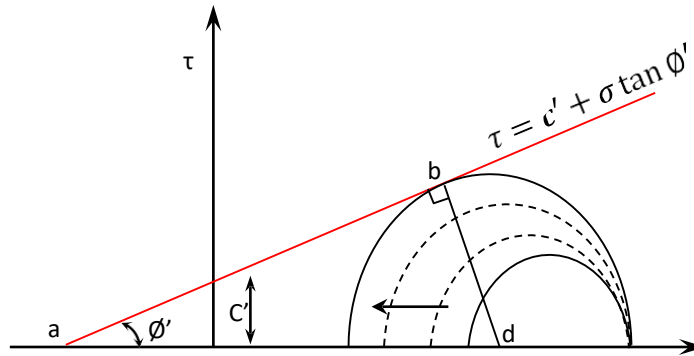


Fig I.5. Active equilibrium state at a point.

We are dealing with an active earth pressure.

The vertical stress remains constant, and the horizontal stress will decrease until it reaches σ_{ha} .

At failure: $\sigma_1 = \sigma_v$ et $\sigma_3 = \sigma_{ha}$

The failure plane makes an angle $\theta = 45 - \phi/2$

$$\sigma_{ha} = \sigma_v \text{tg}^2 (45 - \phi/2)$$

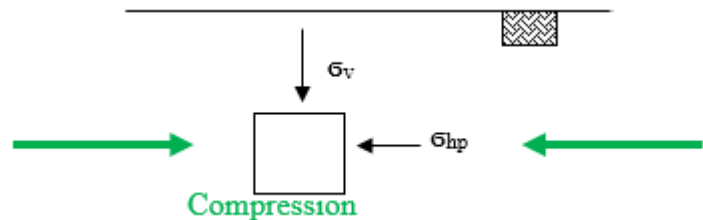
$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi}$$

K_a : active earth pressure coefficient

Passive equilibrium (Fig I.6)

σ_{hp} : "increases

σ_v : constant



We are dealing with a passive earth pressure.

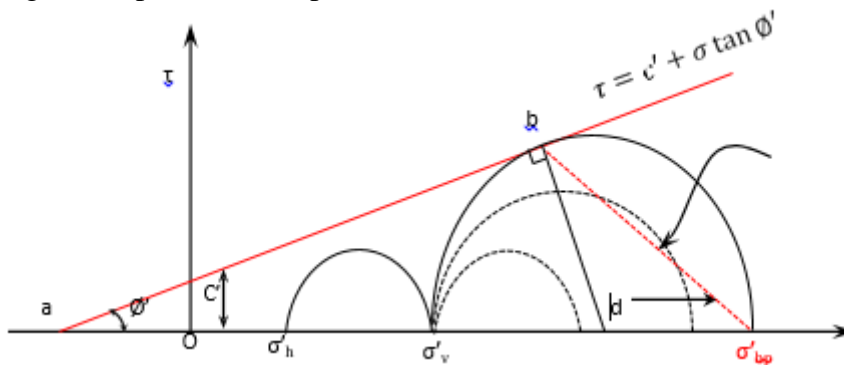


Fig I.6. Passive equilibrium state at a point.

The horizontal stress will increase until it reaches σ'_{hp} .

The failure planes make an angle with the major principal plane, which is the vertical plane, of:

$$\theta = 45 + \varphi/2$$

$$\sigma_{hp} = \sigma_v \tan^2 (45 + \varphi/2)$$

$$K_p = \frac{1 + \sin \varphi}{1 - \sin \varphi} = \frac{1}{K_a}$$

Now we recognize three lateral stresses depending on the strain or displacement experienced by the backfill soil,

$\sigma'_h = K_0 \sigma'_v$	At-rest condition	Zero displacement
$\sigma'_{ha} = K_a \sigma'_v$	Active condition	Movement causing expansion
$\sigma'_{hp} = K_p \sigma'_v$	Passive condition	Movement causing compression

The horizontal displacement required to attain the active state is substantially less than that required to obtain the passive state. The active state is a condition of loosening strains, where the frictional resistance is mobilized to reduce the force necessary to hold the soil in position. As the soil cannot stretch

3.2. Cohesive soil

For cohesive soils, two types of behavior will be considered:

- 1) In the long term: The stresses in terms of effective stresses and the strength parameters are C' , φ'
- 2) In the short term: The stresses in terms of total stresses and the strength parameters are C_u , φ_u

Elastic equilibrium

- 1) In the long term:

$$\left\{ \begin{array}{l} \sigma'_v = \gamma Z \\ \sigma'_h = K_0 \sigma'_v = K_0 \gamma Z \end{array} \right.$$

2) In the short term :

$$\left\{ \begin{array}{l} \sigma_v = \gamma Z \\ \gamma' = \gamma - \gamma_w \\ \sigma_h = K_0 \gamma' Z + U \end{array} \right.$$

Active and passive equilibrium (Fig I.7)

1) In the long term :

$\sigma'_{ha} = K_a \sigma'_v - 2C' \sqrt{K_a}$ Active state.

$\sigma'_{hp} = K_p \sigma'_v + 2C' \sqrt{K_p}$ Passive state.

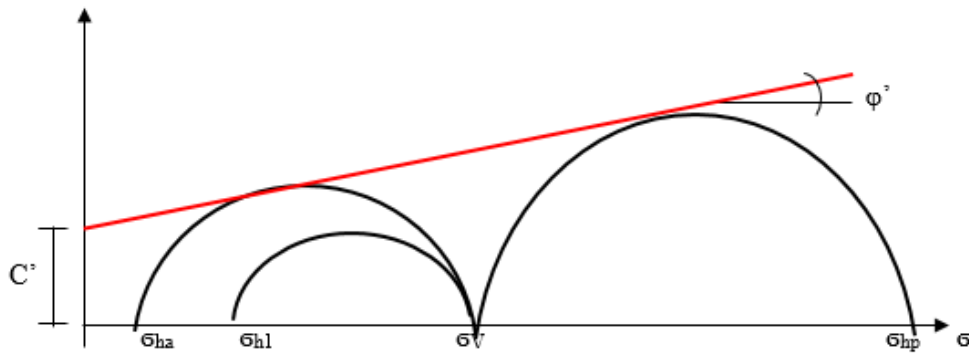


Fig I.7. Long-term active and passive equilibrium states.

2) In the short term (Fig I.8) :

$\sigma_v = \gamma Z$

$K_a = 1 \rightarrow \sigma_{ha} = \sigma_v - 2C_u$ Active state ;

$K_p = 1 \rightarrow \sigma_{hp} = \sigma_v + 2C_u$ Passive state

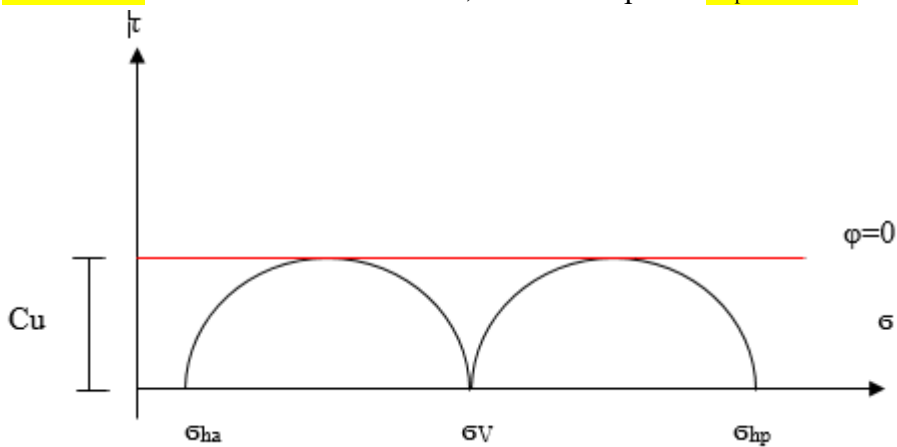


Fig I.8. Short-term active and passive equilibrium states.

4. Calculation of thrust and stop

Rankine’s earth pressure theory and its applications discussed in the previous section were based on the assumption that the surface of the retaining wall is frictionless Fig I.9.

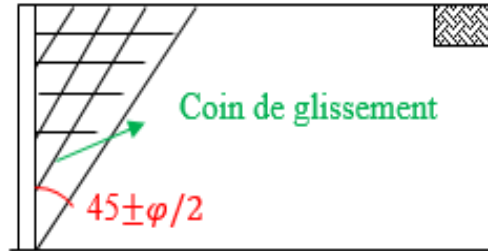


Fig I.9. Sliding wedge.

4.1. Rankine Method

In the course of various attempts at designing of earth-retaining structures, several earth pressure theories have been suggested since 1687, (Rankine, 1857). Coulomb’s and Rankine’s are perhaps the two best-known theories and are frequently referred to as classical earth pressure theories.

The Rankine method is based on the following assumptions :

The entire sliding wedge is at the state of limit equilibrium.

The friction between the soil and the retaining wall (or screen) is not taken into account.

4.1.1. Granular soils

4.1.1.1. Unloaded backfill (Fig I.10)

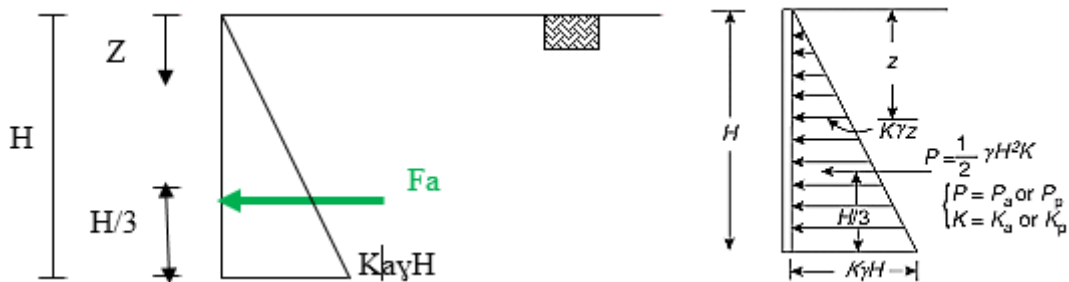


Fig I.10. Thrust on an uncharged soil mass

$\sigma_{ha} = K_a \gamma Z$ (Thrust)

$\sigma_{hp} = K_p \gamma Z$ (Passive)

➤ Active and passive earth pressure forces :

$F_a = \int \sigma_{ha} dZ$ $F_a = 1/2 K_a \gamma H^2$

$$F_p = 1/2 K_p \gamma H^2$$

The point of application of F_a and F_p is located at $H/3$ from the base of the wall.

4.1.1.2. Loaded backfill (Fig I.11)

Consider a dry, non-cohesive level backfill (Fig. 11.8a) with a uniform surcharge load q applied all over the surface. It may be assumed that the vertical effective stress is increased by the amount of surcharge. Then, at any depth z ,

$$\sigma_v = q + \gamma Z$$

$$\sigma_{ha} = K_a \sigma_v$$

$$\sigma_{hp} = K_p \sigma_v$$

La force de poussée et butée :

$$F_a = \int \sigma_{ha} dZ$$

$$F_a = 1/2 K_a \gamma H^2 + K_a q H$$

$$F_p = 1/2 K_p \gamma H^2 + K_p q H$$

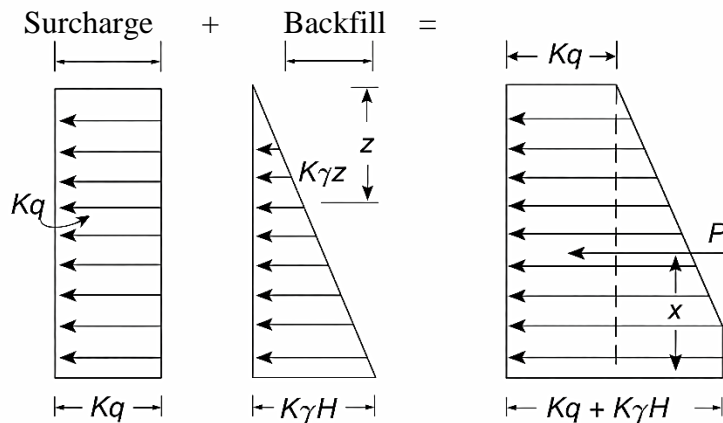
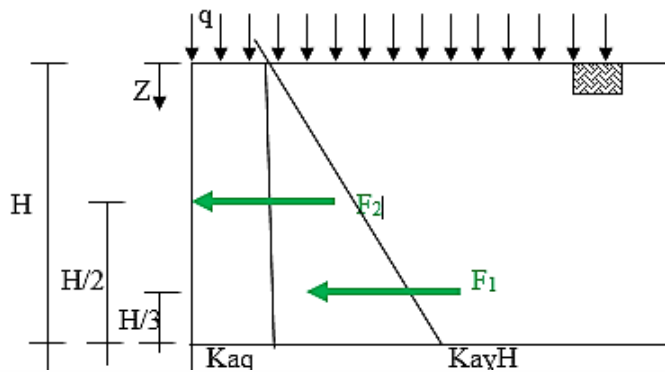
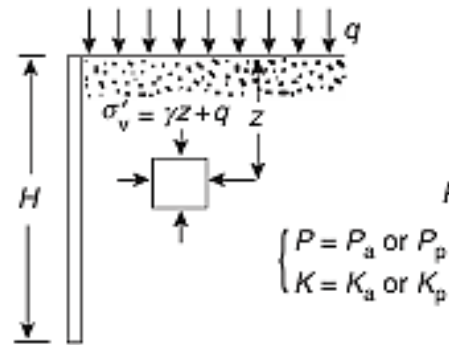


Fig I.11. Loaded soil mass thrust.

The point of application of F_a (or F_p) is the resultant of the moments with respect to the base.

4.1.1.3. Effect of Water Table on a Backfill

Consider again a non-cohesive level backfill with the water table at the surface. The vertical stress at any depth z can be split into two, viz., one due to soil grains and the other due to water; that is

$$P_a = K_a \gamma H + K_w \gamma_w H \quad K_w = 1$$

4.1.1.4. Effect of Stratified Soils in the Backfill

Consider two dry, non-cohesive soils in the level backfill. This is similar to the case of partial submergence. Here the angles of shearing resistances are different in the two layers, whereas in the previous case they were the same in both the layers. If ϕ'_1 and ϕ'_2 are the angles of shearing resistances, γ_1 and γ_2 the unit weights in the top and bottom layers of heights H_1 and H_2 , and K_{a1} , K_{p1} and K_{a2} , K_{p2} the lateral coefficients for the respective layers, then

$$P_{a1} = K_{a1} \gamma_1 H_1$$

$$P_{a2} = K_{a2} (\gamma_1 H_1 + \gamma_2 H_2)$$

4.1.1.5. Inclined backfill (Fig I.12)

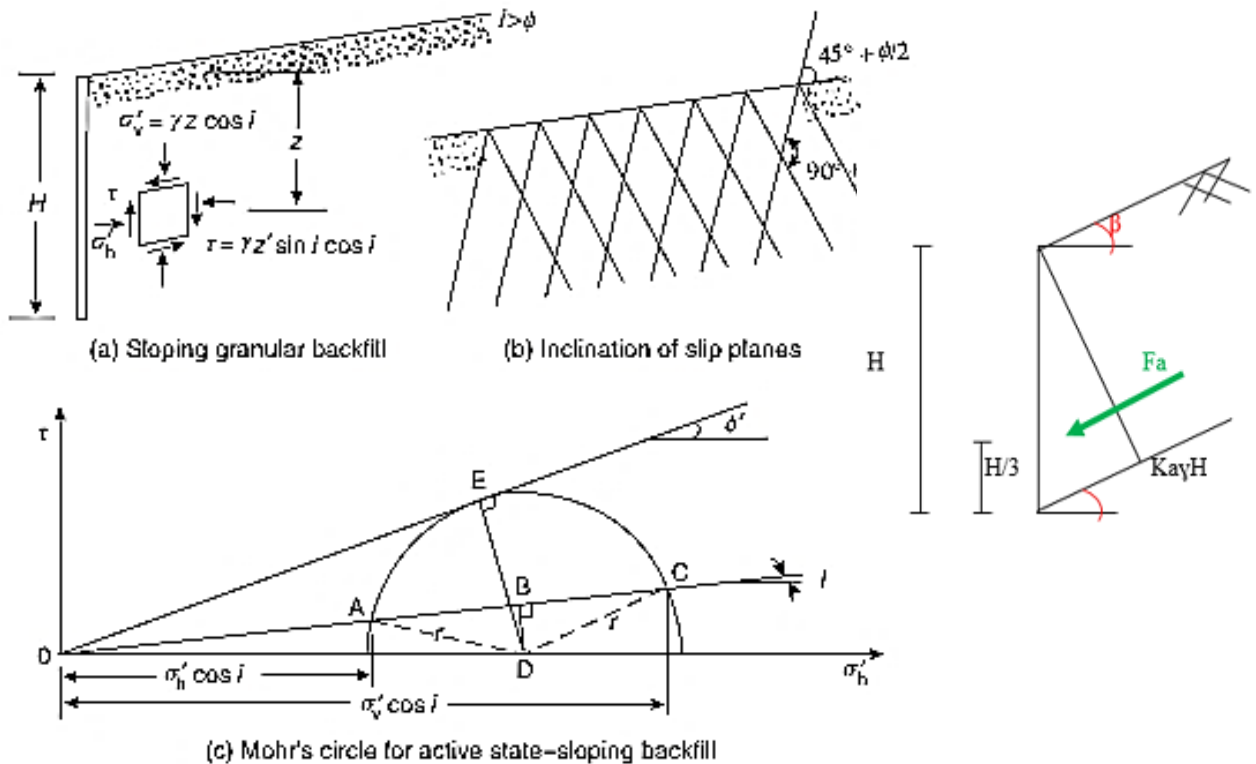


Fig I.12. Thrust on an inclined soil mass.

$$K_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$

$$K_p = \cos\beta \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}$$

$$F_a = 1/2 K_a \gamma H^2$$

$$F_p = 1/2 K_p \gamma H^2$$

- **Special cases**

- For a cohesionless soil ($c'=0$), in the case of a horizontal ground surface:

$$\sigma'_h = K_a \cdot \sigma'_v$$

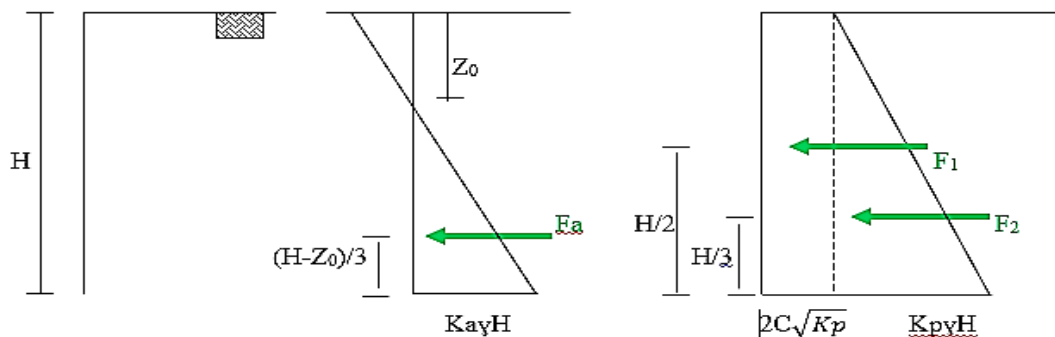
with active earth pressure coefficient: $K_a = \tan^2(45^\circ - \phi/2)$

$$\sigma'_h = K_p \cdot \sigma'_v$$

with passive earth pressure coefficient: $K_p = \tan^2(45^\circ + \phi/2)$

4.1.2. Cohesive soils (FigI. 13)

$$\sigma_{ha} = K_a \gamma Z - 2C\sqrt{K_a}$$



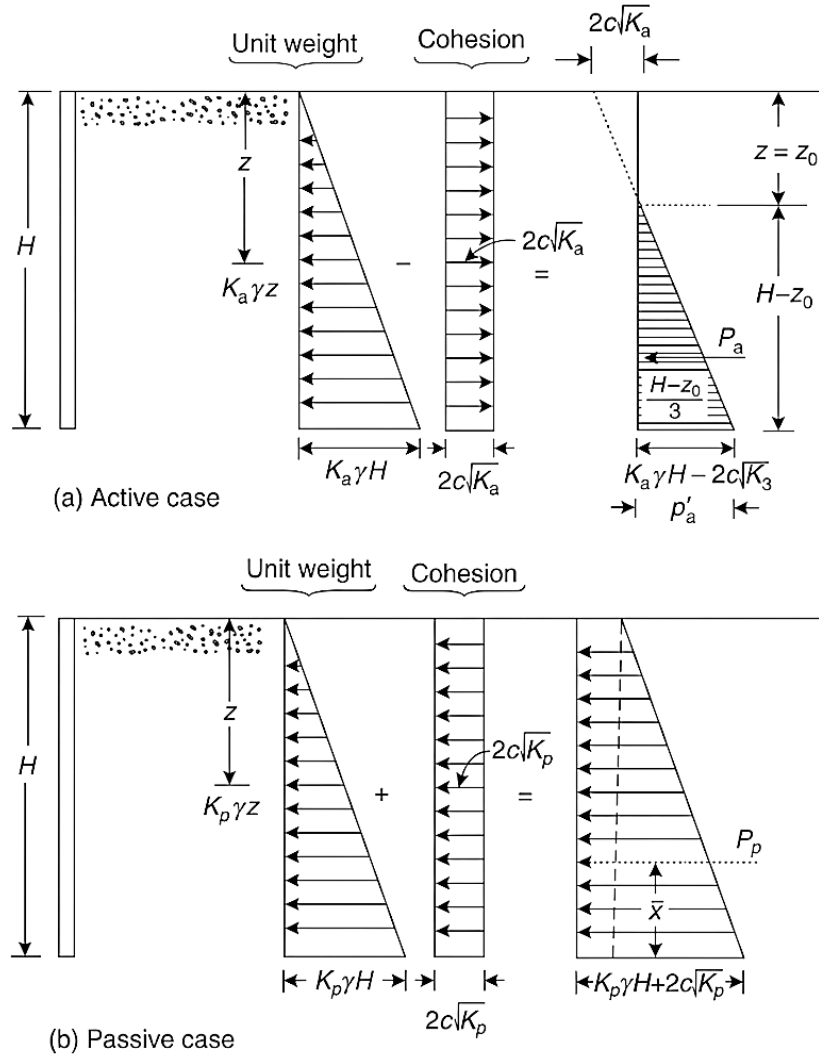


Fig I.13. Thrust in cohesive soil

The soil up to a depth of z_0 will be in a state of tension and will neither impart any pressure on the wall nor provide support. When the tension is released, tension cracks will develop from the surface up to a depth of z_0 . From a practical point of view, the tension zone is ignored, and the active thrust is calculated only for the height $(H-z_0)$ from the base

$$H=0 \rightarrow Z_0 = \frac{2c\sqrt{K_a}}{K_a \gamma} \rightarrow F_a = \int_{Z_0}^H \sigma_h a \, dZ$$

$$F_a = \frac{1}{2} K_a \gamma (H^2 - Z_0^2) - 2c\sqrt{K_a} (H - Z_0)$$

The soil is in tension up to Z_0 (which is neglected), therefore:

$$A \frac{1}{3}(H - Z_0) \rightarrow F_a = \frac{1}{2} K_a \gamma (H^2 - Z_0^2)$$

$$F_p = \int_0^H \sigma_h p \, dZ \rightarrow F_p = 1/2 K_p \gamma H^2 + 2C\sqrt{K_p}$$

$$(F_p = F_1 + F_2) \quad F_1 \approx 1/2 H; \quad F_2 \approx 1/3 H.$$

4.2. Boussinesq’s Theory

Rankine’s theory does not account for the friction that exists between the soil and the wall.

For example, in the case of a soil mass with a horizontal surface and a vertical retaining wall, Rankine’s theory assumes that the friction between the wall and the soil is zero, meaning that the wall is perfectly smooth.

In 1882, Boussinesq improved Rankine’s theory by considering the real interaction between the soil and the retaining structure, that is, by introducing the soil–wall friction angle δ . The stresses applied to the retaining wall are therefore inclined at an angle δ with respect to the normal to the wall.

Although Boussinesq properly formulated the problem, it was not fully solved until 1948 by Caquot and Kerisel.

The results are presented in the Caquot, Kerisel, and Absi tables, which provide the active and passive earth pressure coefficients for purely frictional, weighty soils: K_a and K_p .

Table I.2 Caquot–Kerisel tables of the active earth pressure coefficient K_a for some common cases (Schlosser, F).

		β/φ	0	0.4	0.6	0.8	1
		δ/φ					
$\varphi = 30^\circ$	0	$\lambda = 0^\circ$	0,333	0,386	0,428	0,500	0,850
		$\lambda = 10^\circ$	0,398	0,470	0,528	0,634	-
	2/3	$\lambda = 0^\circ$	0,300	0,352	0,395	0,469	0,822
		$\lambda = 10^\circ$	0,366	0,440	0,499	0,602	-
	1	$\lambda = 0^\circ$	0,308	0,363	0,409	0,488	0,866
		$\lambda = 10^\circ$	0,378	0,458	0,534	0,634	-
0	$\lambda = 0^\circ$	0,271	0,316	0,353	0,419	0,767	
	$\lambda = 10^\circ$	0,336	0,403	0,456	0,548	-	

$\varphi = 35^\circ$	2/3	$\lambda = 0^\circ$	0,247	0,291	0,329	0,397	0,756
		$\lambda = 10^\circ$	0,314	0,383	0,439	0,538	-
	1	$\lambda = 0^\circ$	0,260	0,309	0,349	0,423	0,819
		$\lambda = 10^\circ$	0,333	0,409	0,472	0,583	-
$\varphi = 40^\circ$	0	$\lambda = 0^\circ$	0,218	0,254	0,286	0,342	0,676
		$\lambda = 10^\circ$	0,282	0,341	0,388	0,472	-
	2/3	$\lambda = 0^\circ$	0,202	0,239	0,271	0,330	0,683
		$\lambda = 10^\circ$	0,269	0,331	0,382	0,475	-
	1	$\lambda = 0^\circ$	0,219	0,261	0,297	0,364	0,766
		$\lambda = 10^\circ$	0,295	0,366	0,425	0,533	-

Avec :

φ : Internal friction angle of the backfill;

β : Inclination of the ground surface (slope) relative to the horizontal;

λ : Inclination of the wall with respect to the vertical;

δ : Inclination of the earth pressure with respect to the normal to the wall.

4.2.1. Floor-wall friction

The friction angle φ between the soil and the wall depends on the wall's roughness relative to the soil particles for its value, and on the direction of relative movement between the soil and the wall for its sign. In the absence of displacement between the soil and the wall, $\varphi = 0$.

It therefore depends both on the roughness of the facing and the internal friction angle φ of the soil. As a first approximation, this friction angle can be determined based on the surface condition of the facing. In common cases of rough concrete or masonry walls, a value of $2/3 \cdot \varphi$ is the one to be adopted.

Table 1.3. Floor-wall friction values.

Wall surface condition	Soil–wall friction angle
Very smooth or lubricated surface	$\delta = \varphi$
Slightly rough surface (smooth concrete, treated concrete)	$\delta = \frac{1}{3}\varphi$
Rough surface (concrete, shotcrete, masonry, steel)	$\delta = \frac{2}{3}\varphi$
Sheet pile walls	$\delta \geq \frac{2}{3}\varphi$
Fictitious inclined back faces of cantilever walls	$\delta = \varphi$

5. Conclusion

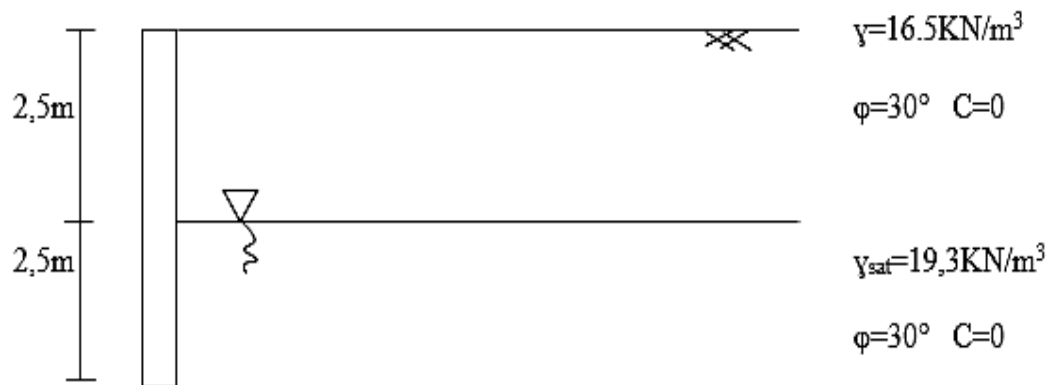
This chapter presented the fundamental principles governing the limit equilibrium states of soils and the resulting lateral earth pressures acting on retaining structures. The different stress conditions within a soil mass namely the at-rest, active, and passive states were introduced and analyzed in relation to wall displacement and soil behavior.

The theoretical approaches used to estimate earth pressures, particularly the classical theories of Rankine and Boussinesq, were discussed along with their underlying assumptions and practical applications. The influence of several important factors such as soil type, cohesion, surcharge loads, groundwater conditions, and stratification of backfill soils was also examined.

Through analytical formulations and illustrative applications, this chapter highlighted the importance of accurately evaluating lateral earth pressures in the design of geotechnical structures such as retaining walls. A proper understanding of these concepts is essential to ensure the stability, safety, and durability of earth-retaining systems in civil engineering practice.

Applications**Application 01**

For the retaining wall shown in the figure below, determine the at-rest lateral earth pressure per unit length of the wall. Also, determine the point of application of the resultant force.

**Solution Application 01**

$$\sigma_v = \gamma h \quad U = \gamma_w h w \quad \sigma'_v = \sigma_v - U$$

$$\sigma'_h = K_0 \sigma'_v \quad (K_0 = 1 - \sin \phi)$$

$$\sigma_h = \sigma'_h + U$$

1st layer

$$\sigma_v = \gamma h = 16.5 \times 2.5 = 41.21 \text{ kN/m}^2$$

$$U = 0$$

$$\sigma'_v = 41.21 \text{ kN/m}^2$$

$$K_0 = 0.5$$

$$\sigma'_{h1} = 0.5 \times 41.21 = 20.62 \text{ kN/m}^2$$

$$\sigma_{h1} = \sigma'_{h1} = 20.62 \text{ kN/m}^2$$

2nd layer

$$\sigma_{v2} = \gamma_1 h_1 + \gamma_2 h_2 = 41.25 + 48.25$$

$$\sigma_{v2}=89,5\text{KN/m}^2$$

$$U_2=h_2\rho_w=2,5\times 10=25\text{KN/m}^2$$

$$\sigma'_{v2}=\sigma_{v2}-U_2=89,5-25$$

$$\sigma'_{v2}=64,5\text{KN/m}^2$$

$$\sigma'_{h2}=K_0 \sigma'_{v2}=0,5\times 64,5$$

$$\sigma'_{h2}=32,25\text{KN/m}^2$$

$$\sigma_{h2}=\sigma'_{h2}+U_2=32,25+25$$

$$\sigma_{h2}=57,25\text{KN/m}^2$$

$$F_1=\frac{\sigma_{h1}xh_1}{2}=\frac{20,62\times 2,5}{2}=25,77\text{KN/ml}$$

$$F_2=\sigma_{h1}xh_2=20,62\times 2,5=51,55\text{KN/ml}$$

$$F_3=\frac{(\sigma_{h2}-\sigma_{h1})xh_2}{2}=\frac{(57,25-20,62)\times 2,5}{2}=45,78\text{KN/ml}$$

$$F=F_1+F_2+F_3=>F=123\text{KN/ml}$$

$$Z_1=h_1/3+h_2=(2,5/3)+2,5=3,33\text{m}$$

$$Z_2=h_2/2=2,5/2=1,25\text{m}$$

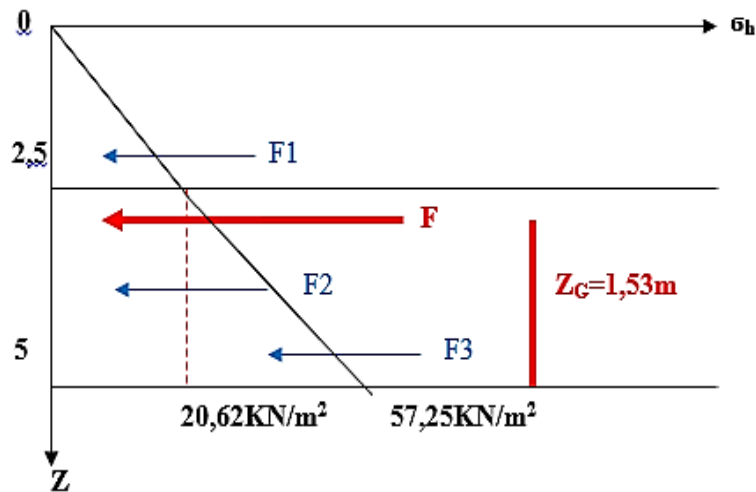
$$Z_3=h_2/3=2,5/3=0,83\text{m}$$

$$FZ=F_1Z_1+F_2Z_2+F_3Z_3$$

$$Z=(F_1Z_1+F_2Z_2+F_3Z_3)/F$$

$$Z_G=(25,77\times 3,33+51,55\times 1,25+45,78\times 0,83)/123$$

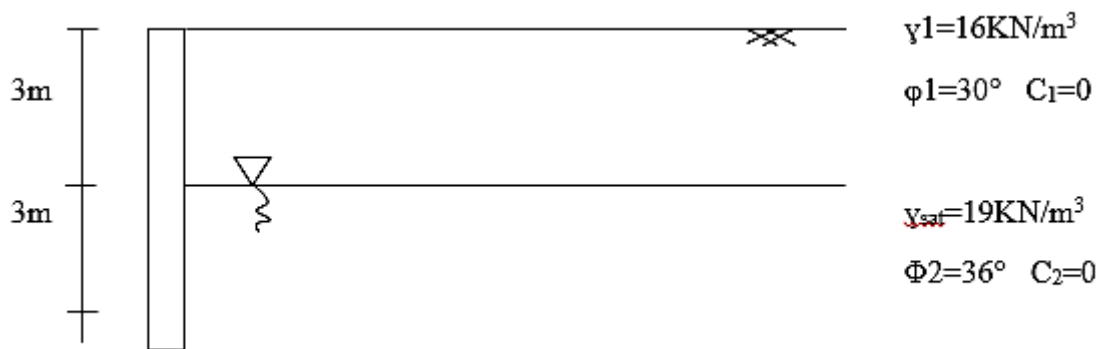
$$Z_G=1,53\text{m}$$



Application 2

Determine the earth pressure according to Rankine per unit length of the wall.

Determine its point of application.



Solution Application 2

$$K_{a1} = \frac{1 - \sin\phi}{1 + \sin\phi} = 1/3 = 0,33$$

$$K_{a2} = 0,25$$

$$\sigma_{h11} = K_{a1} \gamma_1 H_1 = 15,84 \text{ kN/m}^2$$

$$\sigma_{h12} = K_{a2} \gamma_1 H_1 = 12 \text{ kN/m}^2$$

A 6m:

$$\sigma'_{2h} = (\gamma_1 h_1 + (\gamma_2 - \gamma_w) h_2) K_{a2}$$

$$= (16,3 + (19 - 10) * 3) 0,25$$

$$\sigma'_h = 18,75 \text{ KN/m}^2$$

$$\sigma_h = \sigma'_h + U = 18,75 + 30$$

$$\sigma_h = 48,75 \text{ KN/m}^2$$

$$F_1 = (\sigma_{h1} * h_1) / 2 = (15,84 * 3) / 2 = 23,76 \text{ KN/m}$$

$$F_2 = \sigma_{h2} * h_2 = 12 * 3 = 36 \text{ KN/m}$$

$$F_3 = ((\sigma_h - \sigma_{h2}) * h_2) / 2 = ((48,75 - 12) * 3) / 2 = 55,125 \text{ KN/m}$$

$$F = F_1 + F_2 + F_3 = 114,88 \text{ KN/m}$$

$$Z_1 = h_1 / 3 + h_2 = 3 / 3 + 3 = 4 \text{ m}$$

$$Z_2 = h_2 / 2 = 3 / 2 = 1,5 \text{ m}$$

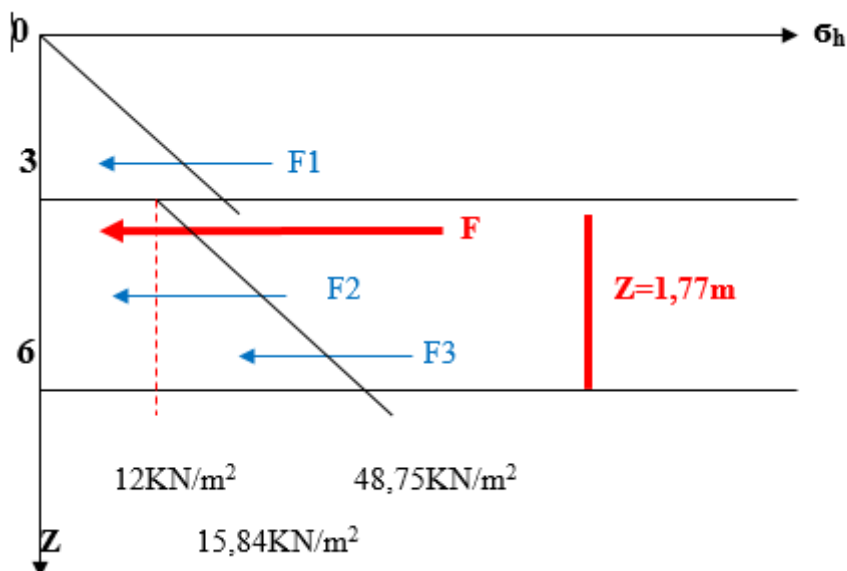
$$Z_3 = h_2 / 3 = 3 / 3 = 1 \text{ m}$$

$$FZ = F_1 Z_1 + F_2 Z_2 + F_3 Z_3$$

$$Z = (F_1 Z_1 + F_2 Z_2 + F_3 Z_3) / F$$

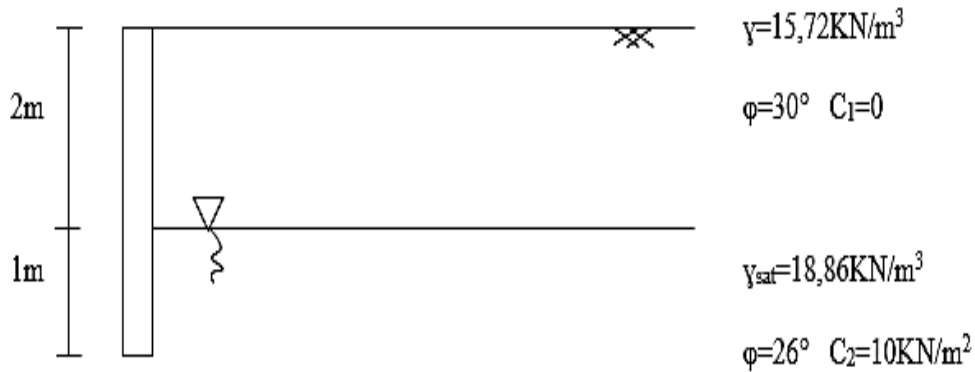
$$Z = (23,76 * 4 + 36 * 1,5 + 55,125 * 1) / 114,88$$

$$Z = 1,77 \text{ m}$$



Application 03

Determine the passive earth pressure per unit length of the wall.



Solution Application 3

$$\sigma_{hp} = K_p \sigma_v + 2C\sqrt{K_p}$$

$$K_{p1} = \frac{1 + \sin\varphi_1}{1 - \sin\varphi_1} = 3 \quad K_{p2} = \frac{1 + \sin\varphi_2}{1 - \sin\varphi_2} = 2,56$$

A 2m:

$$\sigma_{h1} = K_{p1}(\gamma_1 h_1) = 94,32\text{KN/m}^2$$

$$\sigma_{h1} = K_{p2}(\gamma_1 h_1) = 80,48\text{KN/m}^2$$

A 3m:

$$\sigma_{h2} = K_{p2}(\gamma_1 h_1 + \gamma'_2 h_2) + \gamma_w h_2 + 2C_2\sqrt{K_{p2}}$$

$$= 2,56(15,72 \cdot 2) + (18,86 - 10) + 10 \cdot 1 + 2 \cdot 10 \cdot \sqrt{2,56}$$

$$\sigma_{h2} = 145,16\text{KN/m}^2$$

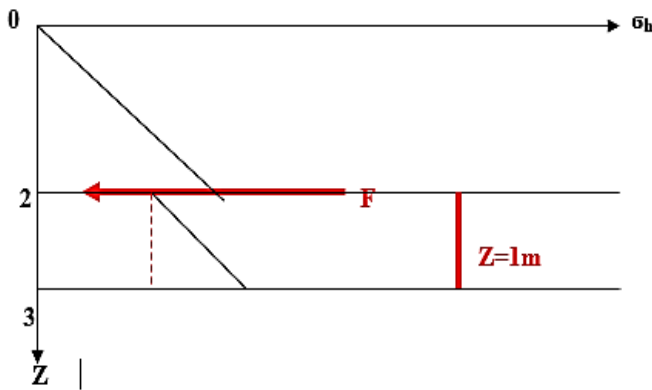
$$F_1 = 93,32 \text{ KN/ml} \quad Z_1 = 1,66\text{m}$$

$$F_2 = 80,48 \text{ KN/ml} \quad Z_2 = 0,5\text{m}$$

$$F_3 = 32,34 \text{ KN/ml} \quad Z_3 = 0,33\text{m}$$

$$F = 206,14\text{KN/ml}$$

$$Z = 1\text{m}$$



Exercise problems

Exercise 1

A retaining wall 6.5 m high supports an over-consolidated clay backfill with a plasticity index of 32% and an over-consolidation ratio of 2.3. Determine the lateral force per unit length of wall and the location if the yield of the wall is completely prevented. The unit weight of the soil is 17.6 kN/m³.

Exercise 2

A 4 m high smooth vertical wall retains a mass of dry loose sand. Compute the total lateral force per metre acting against the wall if the wall is prevented from yielding. The sand has a 30° angle of internal friction and unit weight of 14.8 kN/m³. Also, estimate the lateral force per metre run of the wall if sufficient yield of the wall is permitted so as to develop the active Rankine state.

Exercise 3

A vertical frictionless pressure face of an 8 m high retaining wall supports a non cohesive 5° sloping backfill. The unit weight of the soil is 18 kN/m³, and the angle of shearing resistance is 32°. Draw a Mohr circle representing the state of stresses, and hence, compute the lateral passive resistance per linear length of the wall.

Exercise 4

A wall 15 m high has to be designed so as to retain dry sand. Under loose condition the sand has a void ratio of 0.65 and ϕ' of 32°, and under dense condition the void ratio and ϕ' are 0.41 and 43°, respectively, and $G = 2.65$. Compute the resultant lateral pressures for active and

passive cases for both the density conditions. Recommend a suitable resultant lateral force if the wall has to be designed for (i) the active case and (ii) the passive case.

Exercise 5

A dockside retaining wall 10 m high retains a non-cohesive backfill with a horizontal surface level with the top of the wall. The properties of the backfill material are, $G = 2.65$, $e = 0.55$, and $\phi = 32^\circ$. An additional superimposed load of 20 kN/m^2 is induced at the surface of the backfill due to construction of warehouses and dockyard traffic. Compute the lateral thrust on the wall when the water table is (i) 2 m below the level surface, (ii) 5 m below the level surface, and (iii) at the bottom of the wall. Neglect wall friction.

Exercise 6

A dry granular level backfill of a 6.3 m high retaining wall weighs 16.2 kN/m^3 . The active thrust on the wall is believed to be 75 kN/m length of the wall. It is intended 422 Soil Mechanics and Foundation Engineering to increase the height of the wall and, at the same time, to keep the force on the wall within permissible limits. The backfill to a depth of 2.8 m from the top is removed. The removed portion is replaced by a material such as cinder with $\gamma = 8.2 \text{ kN/m}^3$. If the portion of the additional height is also to be filled with cinder, estimate the additional height of the wall without increasing the initial active thrust. Neglect the wall friction, and assume that both the backfill soil and the cinder have the same friction angle.

II. Retaining Walls

Chapter II: Retaining Walls

1. Introduction

Retaining structures are structures designed to hold back soil (earth, rock, or fill) and sometimes water. The material is considered retained by the structure when it is maintained at a slope steeper than the natural angle it would adopt in the absence of any support. Retaining structures therefore include various types of walls and support systems in which structural elements are subjected to forces exerted by the retained soil mass (Sitek, 2018).

Many engineering situations require the use of retaining structures. Typical examples include:

- Buildings with multiple basement levels,
- Underground parking facilities,
- Shallow metro or underground transportation systems,
- Construction on steeply sloped terrain,
- Roads and highways in cut sections.

In natural conditions, soils tend to stabilize according to their mechanical properties. When there is a difference in elevation between two points A and B on a terrain, the angle formed between the line AB and the horizontal corresponds to the natural slope angle of the soil. This angle is closely related to the internal friction angle (ϕ) of the soil, which represents the maximum slope that a soil can maintain without structural support (Holtz, Kovacs, 2015).

Beyond a certain limit, the soil mass becomes unstable and may collapse or slide. In such situations, it becomes necessary to construct a structure capable of retaining the soil mass. This structure is called a retaining wall (Das, 2007). The difference in elevation between the upstream and downstream ground surfaces can be created either by placing fill material behind the structure or by excavating the soil in front of it. In practice, both operations are frequently performed simultaneously during construction (Hatzor, 2017). Another important parameter governing soil stability is soil cohesion (C), generally expressed in kilopascals (kPa). Cohesion represents the ability of soil particles to adhere to one another. Granular soils such as dry sand or gravel typically exhibit negligible cohesion ($C \approx 0$) and are therefore classified as cohesionless soils. In contrast, cohesive soils such as clays possess significant cohesion due to

electrochemical bonds between particles (Holtz et al., 2015). Stability checks of retaining structures should be performed in accordance with the recommendations of Eurocode 7 (EN 1997).

Typical values of the internal friction angle (ϕ) and cohesion (C) for different soil types are presented in Table II.1.

Table II.1. ϕ and C values of some soils.

Nature of the soil	C (kPa)	ϕ (°)
Gravel	0	40 à 45
Compact sand	0 à 10	30 à 40
Loose or not very compact sand	0 à 10	25 à 30
Clay	20	15 à 25

2. Types of Retaining Walls

There is a wide variety of ground support structures (Bieth Emmanuel, 2009) which are mainly distinguished by :

- * The morphology (massive reinforced or not, curtains and walls anchored or not, reinforced concrete work or not) ;

- The method of execution and field of use ;

- * The manufacturing materials,

- The mode of operation ;

The choice of the type of support structure depends on several factors [4] such as :

- * Cuttings or embankment or mixed ;

- Support height ;

- * Foundation soil ;

- * Availability of materials ;

* Exterior appearance ;

As the retaining structures have in common the thrust force exerted by the earth mass retained, this criterion will be retained for a description of the various retaining structures.

Three modes of operation can be distinguished (Salençon.1983):

* Cases where the thrust is taken up by the weight of the work ;

* Cases where the thrust is taken up by embedding the structure ;

* Cases where the thrust is taken up by anchors ;

2.1. Gravity Wall

Gravity-type walls provide slope and soil retention on account of their weight, which can consist of masonry, concrete mass, concrete in combination with soil weight, or the weight of earth mass alone (Salençon.1983). In addition to weight, they are aided by the passive resistance developed in front of the wall. They are all free to deflect at the top and thereby mobilize active earth pressure. Representative types of gravity-type walls are shown bellow. Massive walls are uneconomical because of the large wall material used for the dead weight. Reinforced concrete cantilever walls are more economical because the backfill itself is aimed to provide most of the required dead weight.

The self-weight of the wall plays an important role in retaining the supported material. Gravity walls can be constructed from different materials, such as masonry walls, concrete walls, and gabion walls, which are more flexible from Fig II.1 to FigII. 5.



Fig II.1. Stone Walls.



Fig II.2. Gabion Walls.



Fig II.3. Reinforced Concrete Cantilever Wall.

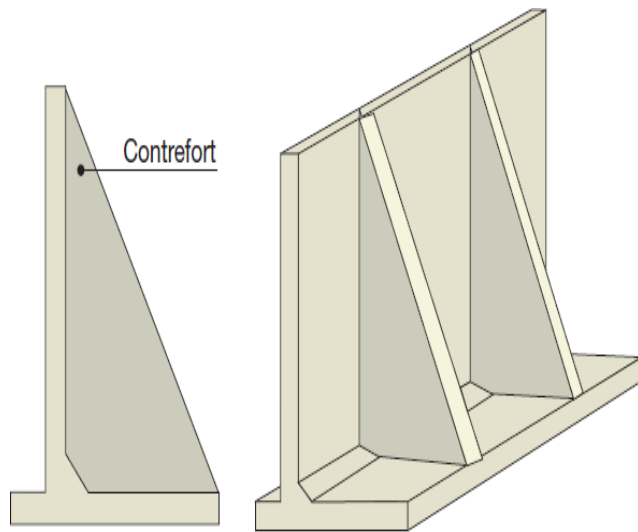


Fig II.4. Buttressed Wall.



Fig II.5. Gravity wall with precast elements.

2.2. Walls constructed in the ground

Juxtaposition of vertical elements whose stability is ensured by their anchors and possible braces and tie rods. These are relatively thin retaining structures made of steel, reinforced concrete, or wood. The bending resistance of these structures plays an important role in the retention, whereas their weight is insignificant. Examples include sheet pile walls formed of metal profiles fitted into each other (Fig. II.6).



Fig II.6. Sheet pile walls made of metal profiles.

2.3. Composite retaining structures

Structures composed of elements belonging to the two previous types. There is a very large number of walls of this kind. Examples include cofferdams made up of two sheet pile walls (Fig. II.7), geotextiles (Fig.II. 8), or earth structures reinforced with anchors or nails (Fig. II.9).



Fig II.7. Cofferdams for the construction of a bridge pier.



Fig II.8. Earth structures reinforced with geotextiles.

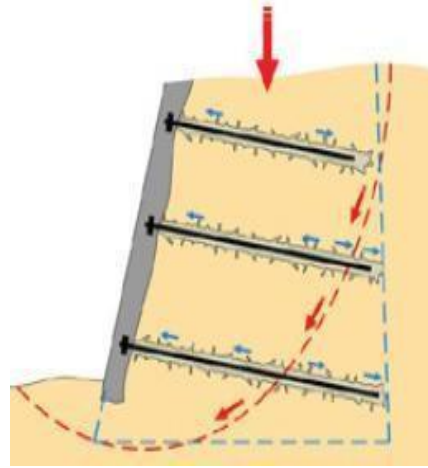


Fig II.9. Slope stabilized with nails.

In this chapter, we limit ourselves to the calculation of reinforced concrete retaining walls, in the form of L-shapes or inverted T-shapes.

3. Preliminary sizing

When designing retaining walls, it is important to preliminarily size the geometric characteristics of the wall as accurately as possible and then verify its stability.

If stability is not ensured, the wall dimensions must be adjusted until safety is guaranteed or confirmed. The figure below (Fig II.10 and II.11), shows a preliminary sizing of reinforced concrete cantilever walls.

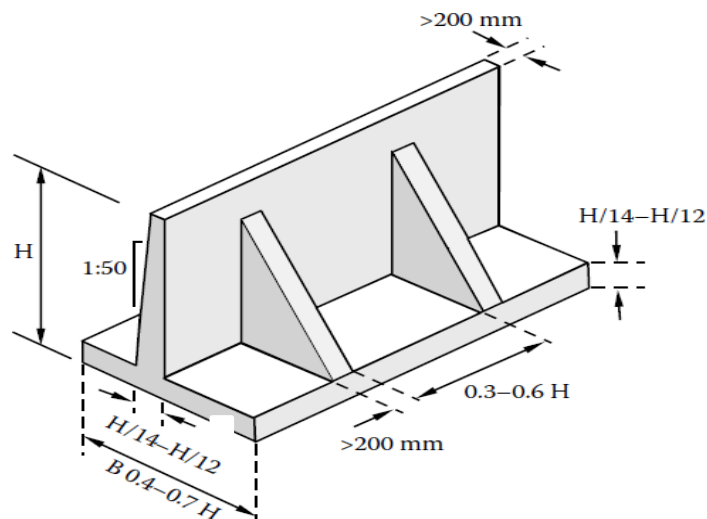


Fig II.10. Typical dimensions of a cantilever wall.

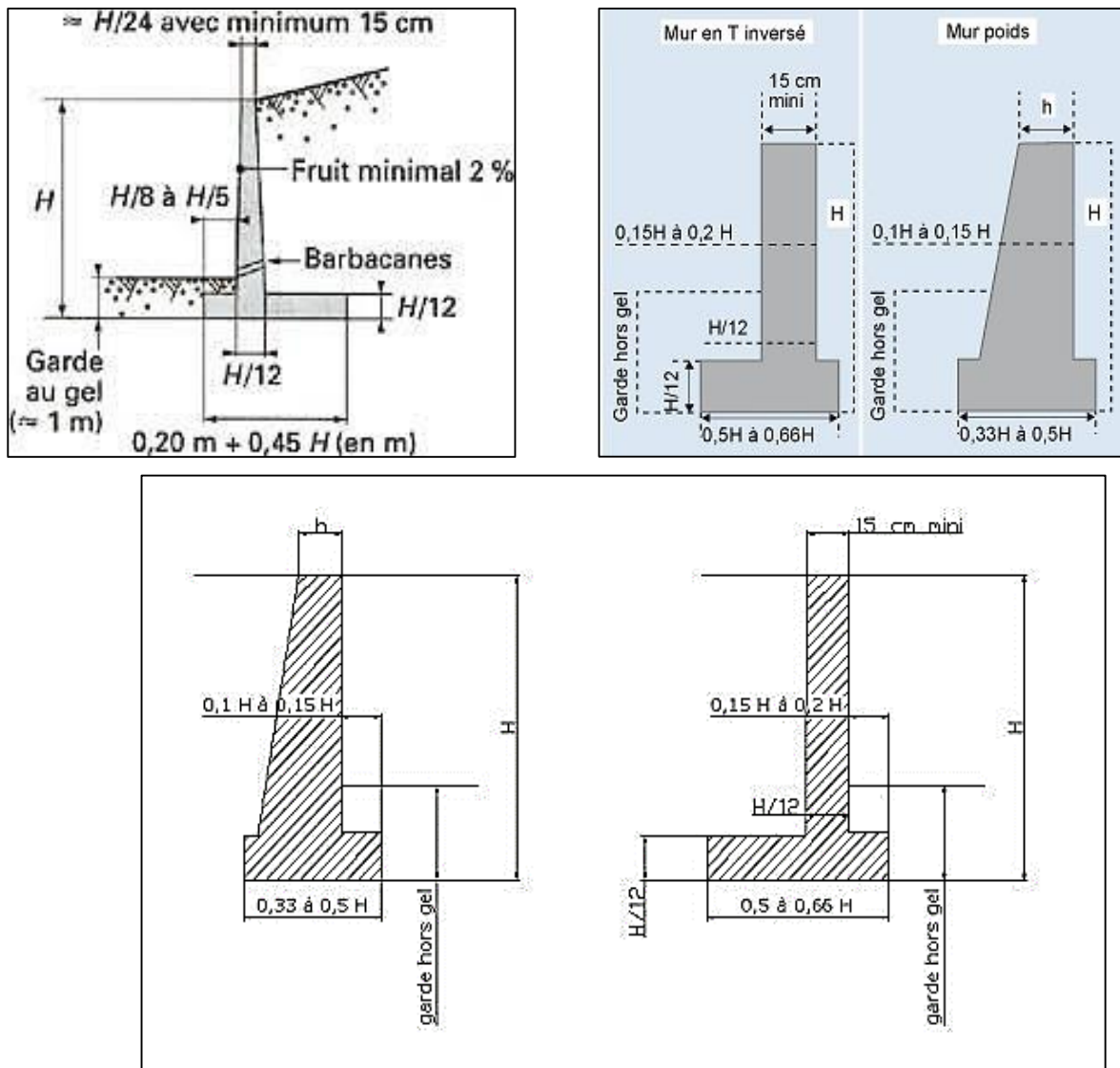


Fig II.11. Typical dimensions of gravity walls and inverted T-walls.

A fixed anchoring D is generally taken to be equal to :

$$D=1\text{ m si } H < 4\text{ m et } D=1.5 \text{ si } D > 4\text{ m ;}$$

4. Failure modes

Among the possible failure modes of retaining walls, the following can be distinguished (Fig II. 12):

- Sliding of the wall on its base.
- Overturning of the wall.
- Punching failure of the wall's foundation soil

Large sliding including the wall (global instability).

Failure of structural elements.

Check external stability : this concerns the first four types of failure (sliding, overturning, punching, global instability).

Check internal stability : this concerns the verification of the structural elements' strength of the structure.

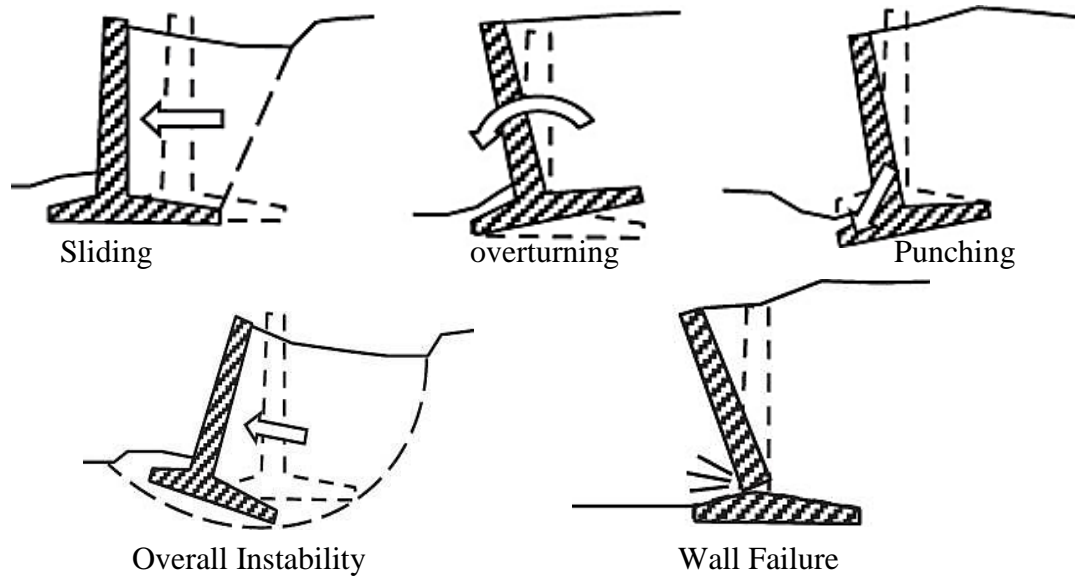


Fig.II.12. Different failure modes.

If safety is not ensured, a heel can be added (the bearing capacity will increase) (Fig II.13).

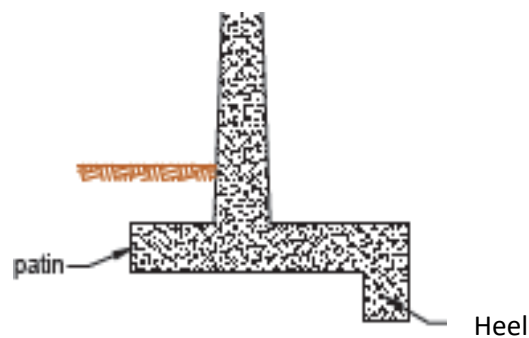


Fig II. 13. Heel.

5. Drainage and water evacuation systems

To avoid saturation behind the retaining wall and ensure a high safety factor, it is therefore necessary to provide proper drainage at the back of the wall in order to reduce the effect of water on the lateral earth pressure

Among the commonly used drainage devices, the following can be distinguished (see figure

below):

Weep holes which are slightly inclined pipes toward the downstream side and pass through the wall, allowing water behind the wall to be drained; the filter placed at the back of the wall, either directly against the wall either directly against the wall or on the natural sloping ground (Fig II.14).

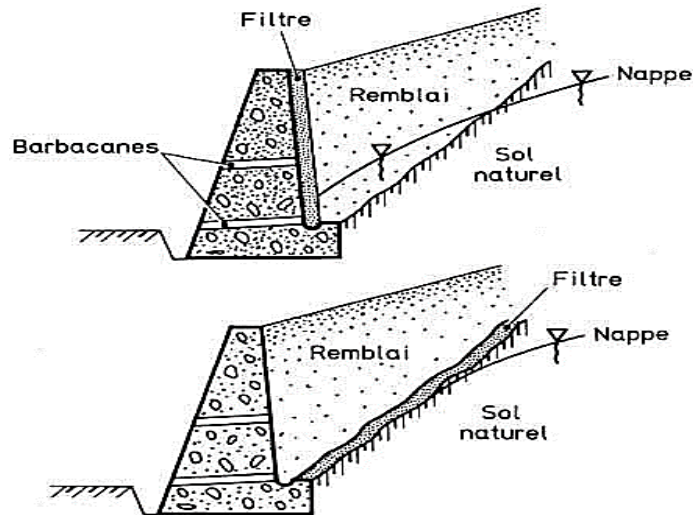


Fig. II.14. Drainage devices behind a retaining wall.

6. Calculation of retaining walls

To ensure the stability of the wall, it is necessary to:

Check stability against overturning about the lower edge.

Check overall sliding of the wall on its base.

Check shear failure of the foundation soil.

Check overall sliding (when the wall is massive).

6.1. Efforts solliciting a retaining wall

For the calculation of the reinforcement and therefore the internal stability of the retaining wall, the active thrust of the earth or of operating overloads are assumed to be exerted with an angle of inclination δ zero on the normal to the facing. When the facing upstream of the curtain is inclined by λ , the vertical component of the thrust is neglected. This simplification goes in the direction of security. The forces are illustrated in Fig. 15 and can be defined.

6.2. The vertical forces

W: Self-weight of the wall (W_r : for the curtain and W_s for the sole). In the case of retaining walls with buttresses, the self-weight of the buttresses can be taken into account in the calculation of the vertical load which must be related to 1 linear meter of the wall ;

V: Weight of the land surmounting the foundation upstream ;

V_p : Weight of the land located upstream of the foundation (usually neglected) ;

V_1 : Weight due to the operating load q ;

6.3. The horizontal forces

F_a : Thrust of the supported soils upstream of the wall and depending on the characteristics of the soil (volume weight, internal friction angle) and the height of the soils to be supported ;

F_p : Stop of the ground located downstream of the wall (usually neglected) ;

F_{aq} : Thrust due to operating load ;

To these forces can possibly be added :

1. Concentrated forces at certain points (anchor rods, for example) ;

2. If drainage is not ensured, then the presence of water will generate hydrostatic pressures upstream and downstream as well as sub-pressures under the sole ;

Check internal forces in the case of reinforced concrete walls to design and determine the steel reinforcement sections (Fig II.15).

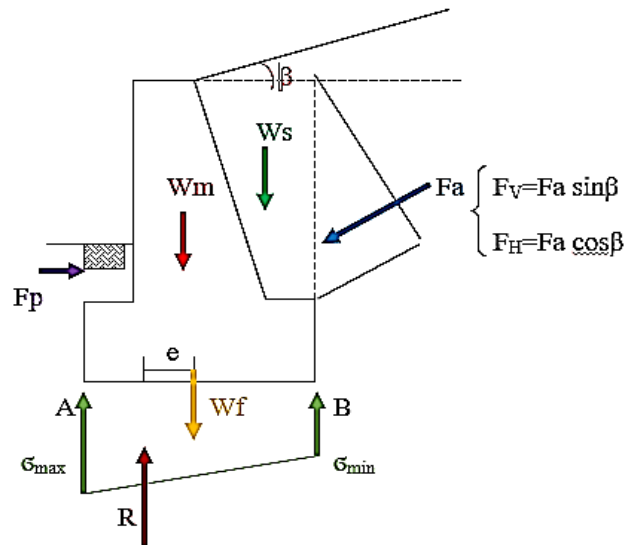


Fig.II. 15. Acting forces on a gravity wall according to Rankine.

If we calculate the moments about point A (Fig 16):

$$X = \frac{\Sigma M/A}{R}$$

$e = B/2 - X$ (B : base width).

if $e \geq B/6$ It is necessary to resize.

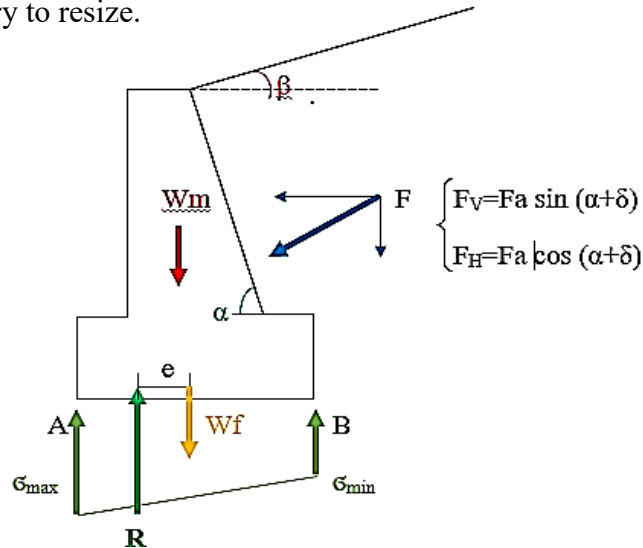
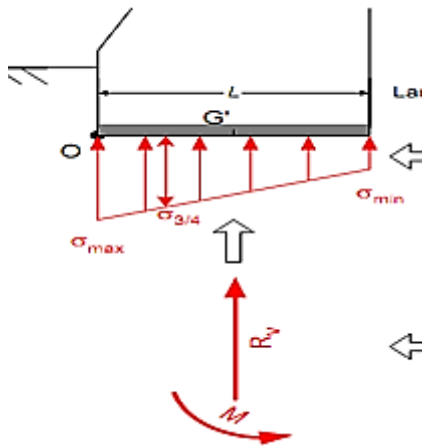


Fig. II.16. Acting forces on a gravity wall according to Coulomb.

6.4. Punching stability

The punching stability consists in checking that one is sufficiently far from the conditions of rupture of the foundation soil. In principle, its justification consists in verifying that the normal stress applied to the foundation soil remains less than a fraction of the breaking stress of the soil.



$$\sigma_{\max/\min} = \frac{R}{B} \left(1 \pm \frac{6e}{B} \right)$$

$$\sigma_{3/4} = \frac{2\sigma_{\max} + \sigma_{\min}}{4} \leq q_1$$

Note: the heel is neglected in the case of lightly embedded walls.

6.5. Sliding stability

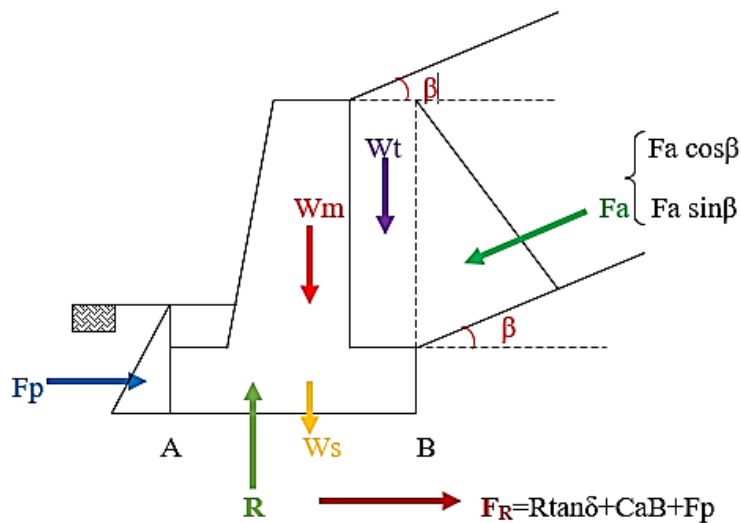


Fig.II. 17. Forces involved in sliding stability.

The forces resisting this sliding are due to adhesion and friction between the foundation and the ground (Fig II.17), as well as the heel if it is not neglected. For the wall to be stable, the sum of these forces must be greater than the horizontal component of the earth pressure. Let F_s be the

safety factor:
$$F_s = \frac{R_f + C_a B + F_p}{F_h}$$

$f = \tan \delta$ (friction between soil and base)

$\varphi < \delta < 2/3\varphi$

$0,5C < Ca < 0,7C$ (soil-base adhesion coefficient)

R: sum of the vertical forces.

If a thin layer remains bonded to the base, the characteristics δ and Ca will be equal to φ and C ; sliding stability is ensured if the safety factor is greater than 1.5, or greater than 2 when considering the heel.

If the safety factor is greater than 1.5, or greater than 2 when taking the heel into account

6.6. Overturning stability

The overturning of the wall is likely to occur around the axis passing through point A (fig 18). The risk of overturning is largely due to the eccentricity of the reaction, so the overturning stability is studied in two ways:

1/ By ensuring that the reaction R passes within the 1/3 central of the base, so we must verify that $e \leq B/6$.

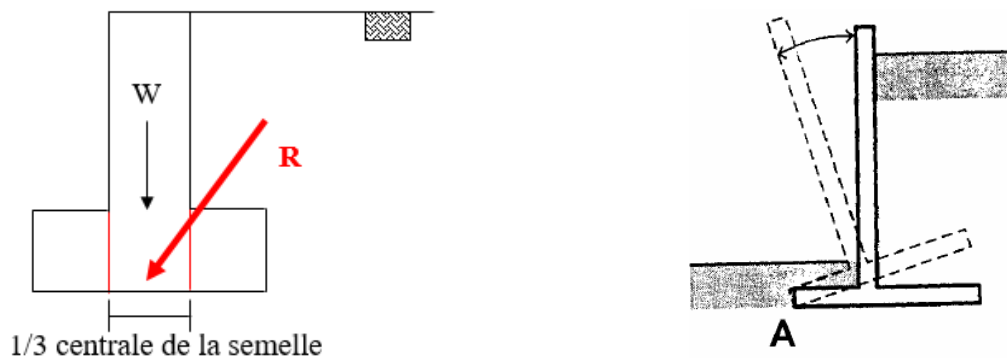


Fig.II. 18. Overturning safety.

2/ By comparing the moments of the forces about the axis passing through point A, we can determine the ratio given by the safety factor:

$$F_R = \frac{\Sigma M \text{ Resisting}}{\Sigma M \text{ Overturning}} \geq 1,5 \text{ (or 2 when taking the heel into consideration).}$$

7. Conclusion

This chapter provided an overview of retaining walls and their role in ensuring the stability of structures. Particular emphasis was placed on the evaluation of earth pressures and on the analytical models commonly used in geotechnical engineering to estimate the forces acting on the wall. In addition, the influence of several key parameters was considered, including soil cohesion, surcharge loads, and the presence of water. These factors play an important role in the behavior of the soil–structure system and must be carefully taken into account in accordance with the applicable design standards and regulations.

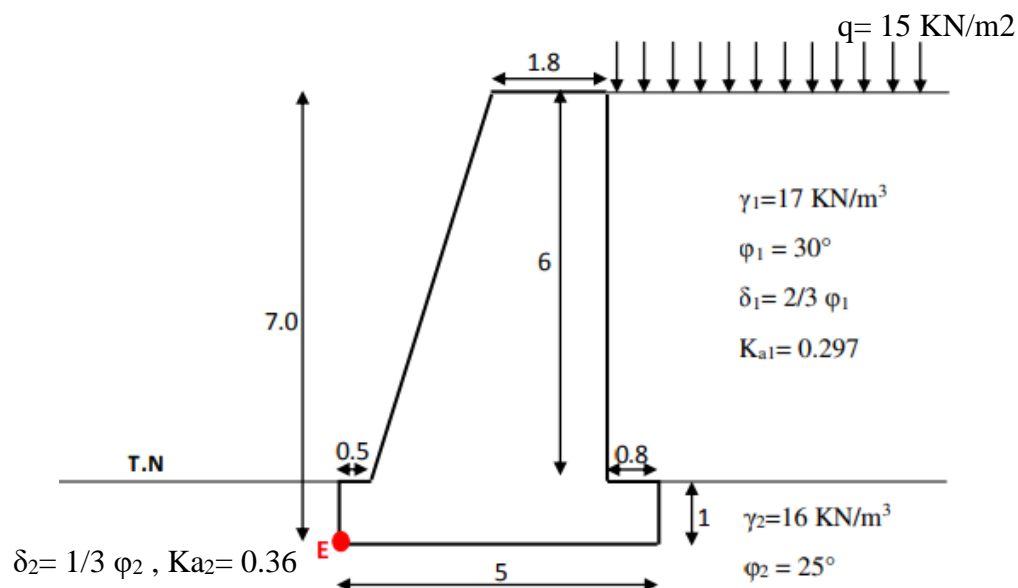
Overall, the analysis presented in this chapter shows that the safe performance of retaining structures depends not only on accurate design and detailed calculations, but also on appropriate construction practices, regular monitoring, and periodic maintenance. These combined measures are essential to ensure the long-term stability and serviceability of retaining walls in civil engineering projects. Stability checks of retaining structures should be performed in accordance with the recommendations of Eurocode 7 (EN 1997).

Applications

Application 1

Consider a gravity wall made of (non-reinforced) concrete, as shown in the figure:

$$\gamma_1 = 17 \text{ KN/m}^3 \quad \phi_1 = 30^\circ \quad \delta_1 = 2/3 \phi_1 \quad K_{a1} = 0.297$$



- 1- Calculate the pressures exerted and represent them graphically.
- 2- Calculate the resultant of the exerted pressures and the forces involved in equilibrium.
- 3- Graphically represent all the forces along with their lever arms.
- 4- Determine all overturning and stabilizing moments of the wall.
- 5- Verify the stability of the wall against sliding and overturning.
- 6- **Note:** Slope inclination angle $\beta = 0^\circ$, wall inclination angle $\alpha = 0^\circ$, the interface is considered very rough, soil cohesion is zero ($C = 0$).

Solution Application 1

1) Calculation of exerted pressures:

1 - Earth pressures

Layer 1:

$$\sigma_{a\gamma 1} = Ka_1 \times \gamma_1 \times z$$

$$\sigma_{a\gamma 1} = Ka_1 \times \gamma_1 \times z = 0.279 \times 17 \times 0 = 0 \text{ KN/m}^2$$

$$\sigma_{a\gamma 1} = Ka_1 \times \gamma_1 \times z = 0.297 \times 17 \times 6 = 30,294 \text{ KN/m}^2$$

$$\sigma_{aq 1} = Ka \times q = 0.297 \times 15 = 4,455 \text{ KN/m}^2$$

Layer 2:

$$\sigma_{a\gamma 2} = Ka_2 \times \gamma_2 \times z$$

$$\sigma_{a\gamma 2} = Ka_2 \times \gamma_2 \times z = 0.36 \times 16 \times 0 = 0 \text{ KN/m}^2$$

$$\sigma_{a\gamma 2} = Ka_2 \times \gamma_2 \times z = 0.36 \times 16 \times 1 = 5,76 \text{ KN/m}^2$$

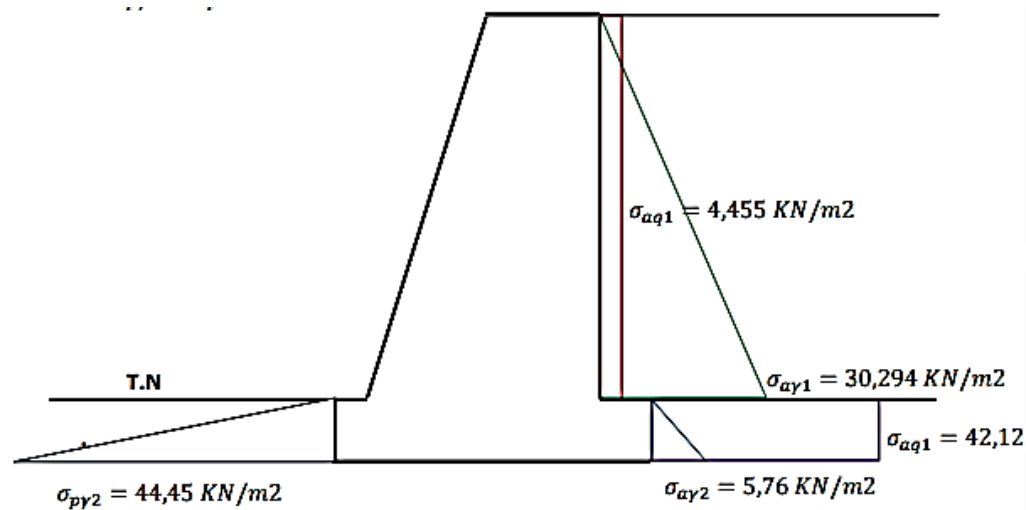
$$\sigma_{aq 2} = Ka_2 \times (q + \gamma_1 \times H_1) = 0.36 \times (15 + 17 \times 6) = 42,12 \text{ KN/m}^2$$

2 - Bearing pressure

$$\sigma_{p\gamma 1} = Kp \times \gamma_2 \times z$$

$$\sigma_{p\gamma 2} = Kp \times \gamma_2 \times z = 1/0.36 \times 16 \times 0 = 0 \text{ KN/m}^2$$

$$\sigma_{p\gamma 2} = Kp \times \gamma_2 \times z = 1/0.36 \times 16 \times 1 = 44,45 \text{ KN/m}^2$$



·3) Calculation of earth pressures on the wall

1- Earth pressure forces

Layer 1:

$$F_{\gamma 1} = \frac{1}{2} K_{a1} \times \gamma_1 \times H_1^2 = 0.5 \times 0.297 \times 17 \times 6^2 = 90,883 \text{ KN/ml}$$

$$\Rightarrow \{ a_{\gamma 1 X} = F_{\gamma 1} \cos \delta_1 = 90,883 \times \cos 20 = 85,40 \text{ KN/ml}$$

$$F_{\gamma 1 Y} = F_{\gamma 1} \sin \delta_1 = 90,883 \times \sin 20 = 31,08 \text{ KN/ml}$$

$$F_{aq1} = K_{a1} \times q \times H_1 = 0.297 \times 15 \times 6 = 26,73 \text{ KN/ml}$$

$$\Rightarrow \{ a_{q1 X} = F_{aq1} \cos \delta_1 = 26,73 \times \cos 20 = 25,11 \text{ KN/ml}$$

$$F_{aq1 Y} = F_{aq1} \sin \delta_1 = 26,73 \times \sin 20 = 9,14 \text{ KN/ml}$$

Layer 2:

$$F_{\gamma 2} = \frac{1}{2} K_{a2} \times \gamma_2 \times H_2^2 = 0.5 \times 0.36 \times 16 \times 12 = 2,8823 \text{ KN/ml}$$

$$\Rightarrow F_{\gamma 2 X} = F_{\gamma 2} \cos \delta_2 = 2,88 \times \cos 8.33 = 2,85 \text{ KN/ml}$$

$$F_{\gamma 2 Y} = F_{\gamma 2} \sin \delta_2 = 2,88 \times \sin 8.33 = 0,42 \text{ KN/ml}$$

$$F_{aq2} = K_{a2} \times (q + \gamma_1 \times H_1) \times H_2 = 0.36 \times (15 + 17 \times 6) \times 1 = 42,12 \text{ KN/ml}$$

$$\Rightarrow F_{aq2 X} = F_{aq2} \cos \delta_2 = 42,12 \times \cos 8.33 = 41,67 \text{ KN/ml}$$

$$F_{aq2 Y} = F_{aq2} \sin \delta_2 = 42,12 \times \sin 8.33 = 6,10 \text{ KN/ml}$$

2 - Bearing forces

$$F_{p\gamma 2} = 1/2 K_p \times \gamma_2 \times H_2^2 = 0.5 \times (10.36) \times 16 \times 12 = 22.22 \text{ KN/ml}$$

$$\Rightarrow F_{p\gamma 2X} = F_{p\gamma 2} \cos \delta_p = 22.22 \times \cos 8.33 = 21.98 \text{ KN/ml}$$

$$F_{p\gamma 2Y} = F_{p\gamma 2} \sin \delta_p = 22.22 \times \sin 8.33 = 3.22 \text{ KN/ml}$$

3- Weight of the wall

$$P_1 = 1.8 \times 6 \times 25 = 270 \text{ KN/ml}$$

$$P_2 = 5 - 0.5 - 0.8 - 1.8/2 \times 6 \times 25 = 142.5 \text{ KN/ml}$$

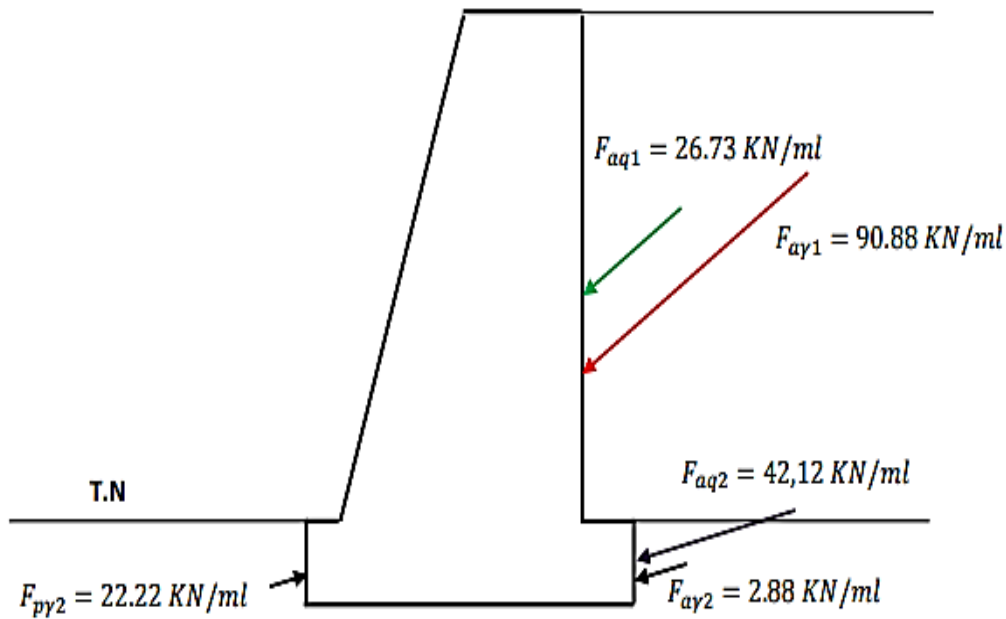
$$P_3 = 5 \times 1 \times 25 = 125 \text{ KN/ml}$$

4- Weight of the backfill

$$P_4 = 0.8 \times 6 \times 17 = 81.6 \text{ KN/m}$$

3) Calculation of lever arms

Force	Intensité (KN/ml)	Bras de levier/E (m)
$F_{a\gamma 1X}$	85.4	$6/3 + 1 = 3$
F_{aq1X}	25.11	$6/2 + 1 = 4$
$F_{a\gamma 2X}$	2.85	$1/3 = 0.33$
F_{aq2X}	41.67	$1/2 = 0.5$
$F_{p\gamma 2X}$	21.98	$1/3 = 0.33$
$F_{a\gamma 1Y}$	31.08	$5 - 0.8 = 4.2$
F_{aq1Y}	9.14	$5 - 0.8 = 4.2$
$F_{a\gamma 2Y}$	0.42	5
F_{aq2Y}	6.10	5
$F_{p\gamma 2Y}$	3.22	0
P_1	270	$5 - 0.8 - 1.8/2 = 3.3$
P_2	142.5	$0.5 + 1.9 \times 2/3 = 1.76$
P_3	125	$5/2 = 2.5$
P_4	81.6	$5 - 0.8/2 = 4.6$



4) Determination of overturning and stabilizing moments

1. Calculation of moments

Force	Intensité (KN/ml)	Bras de levier/E (m)	Moment/E (KN.m)
F_{ay1X}	85.4	3	-256,2
F_{aq1X}	25.11	4	-100,44
F_{ay2X}	2.85	0.33	-0,9405
F_{aq2X}	41.67	0.5	-20,835
F_{py2X}	21.98	0.33	+7,2534
F_{ay1Y}	31.08	4.2	+130,536
F_{aq1Y}	9.14	4.2	+38,388
F_{ay2Y}	0.42	5	+2,1
F_{aq2Y}	6.10	5	+30,5
F_{py2Y}	3.22	0	0
P_1	270	3.3	+914,1
P_2	142.5	1.76	+250,8
P_3	125	2.5	+312,5
P_4	81.6	4.6	+375,36

(+) : Stabilizing moment

(-) : Overturning moment

2. Verification of sliding at the base of the wall:

$$F_s = \frac{a \cdot b + N \cdot \text{tg} \delta_g}{T} \geq 1,5$$

If the adhesion coefficient a is neglected, we will have:

$$F_s = \frac{N \cdot \text{tg} \delta_g}{T} \geq 1,5$$

$$N = 31.08 + 9.14 + 0.42 + 6.10 - 3.22 + 270 + 142.5 + 125 + 81.6 = 662,62 \text{ KN/ml}$$

$$T = 85.4 + 25.11 + 2.85 + 41.67 - 21.98 = 133,05 \text{ KN/m}$$

$$\text{tg} \delta_g = 2/3 \text{ tg} \phi_2 = 2/3 \text{ tg} 25^\circ = 0,3105,1$$

$$F_s = \frac{N \cdot \text{tg} \delta_g}{T} \geq 1,5 \Rightarrow (662.62 \times 0.31) / 133.05 = 1,54$$

\Rightarrow Condition verified \Rightarrow No risk of sliding

3. Verification of wall overturning:

The safety condition against overturning is expressed as:

$$F_s = \frac{M_s}{M_r} \geq 1,5$$

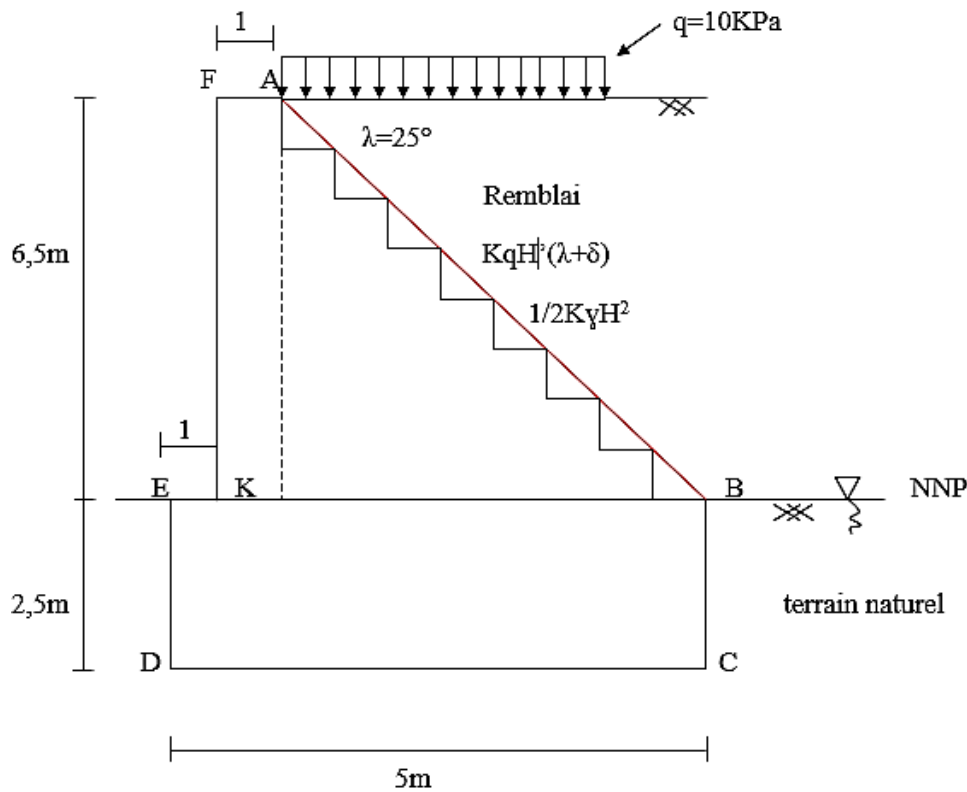
$$M_s = 7,25 + 130.53 + 38.38 + 2.1 + 30.5 + 914.1 + 250.8 + 312.5 + 375.36 = 2061,52 \text{ KN.m/ml}$$

$$M_r = 256.2 + 100.44 + 0.94 + 20.835 = 378,415 \text{ KN.m/ml} \geq 5,1 \text{ r s s M M F} \Rightarrow 2061.52 / 378.415$$

$$= 5,44 \Rightarrow F_s = \frac{M_s}{M_r} \geq 1,5 \quad \text{Condition verified} \Rightarrow \text{no risk of overturning.}$$

Application 2

We want to calculate the quay wall shown in the following figure, which specifies its dimensions (the series of offsets is approximated by a straight face AB, and it is assumed that the weight of the soil is not significantly different from that of the concrete in the triangular zones thus defined). The top surface of the foundation slab is leveled at the height of the water table, which is at the same level as the ground at the foot of the wall stem. The foundation slab is entirely embedded in the natural ground, which is fully submerged, whereas the wall stem is subjected to the thrust of a backfill above water level.



The following calculation assumptions will be adopted:

Concrete : $\gamma_{\text{Concrete}}=23\text{KN/m}^3$

Backfill : $\gamma=18\text{KN/m}^3$; $\varphi_1=30^\circ$; $C=0$

on AB ($\delta=\varphi_1$; $\lambda=25^\circ$) $K_{ax}=0,474$; $K_{aq}=0,522$; $q=10\text{KPa}$.

Natural ground : $\gamma'=11\text{KN/m}^3$; $\varphi_2=25^\circ$; $C=0$

on BC ($\delta=2/3 \varphi_2$) ; $K_{ax}=K_{aq}=0,364$, $\sigma_{adm}=350\text{KN/m}^2$

As a safety measure, the passive earth pressure on the upstream face of the foundation slab (on side ED) will be neglected. The following are required:

-the eccentricity of the resultant force on the base CD of the foundation,

-whether any tensile stresses occur!

-the safety factor against overturning with respect to point D.

-the sliding safety factor (it will be assumed that the coefficient of friction between the foundation soil and the concrete of the footing is equal to $\tan \varphi_2$).

Solution Application 02

$$1/W_1 = h_1 * a * c_{\text{concrete}} = 6,5 * 1 * 23 = 149,5 \text{KN/ml}$$

$$x = 1,5 \text{m} \quad y = 5,75 \text{m}$$

$$W_2 = (h_1 * a') / 2 * 8 c_{\text{concrete}} = 0,5 * 6,5 * 3 * 23 = 224,25 \text{KN/ml}$$

$$x = 3 \text{m} \quad y = 4,66 \text{m}$$

$$W_3 = h_2 * B * (8 c_{\text{concrete}} - 8_w) = 2,5 * 5 * (23 - 10) = 162,5 \text{KN/ml}$$

$$x = 2,5 \text{m} \quad y = 1,25 \text{m}$$

In the presence of β , δ and λ , there will be two components (H and V).

$$H' = AB = \sqrt{6,5^2 + 3^2} = 7,15 \text{m}$$

$$F_{1q} = K * q * H' = 0,522 * 10 * 7,15 = 37,32 \text{KN.ml}$$

$$F_{1qH} = F_{1q} \cos(\lambda + \delta) = 37,32 \cos 55 = 21,4 \text{KN.ml}$$

$$x = 3,5 \text{m} \quad y = 5,75 \text{m}$$

$$F_{1qV} = F_{1q} \sin(\lambda + \delta) = 30,57 \text{KN.ml}$$

$$x = 3,5 \text{m} \quad y = 5,75 \text{m}$$

$$F_{1\gamma} = 1/2 K \gamma H'^2 = 218,08 \text{KN/ml}$$

$$F_{1\gamma H} = F_{1\gamma} \cos(\lambda + \delta) = 125,1 \text{KN/ml}$$

$$x = 4 \text{m} \quad y = 4,66 \text{m}$$

$$F_{1\gamma V} = F_{1\gamma} \sin(\lambda + \delta) = 179,06 \text{KN/ml}$$

$$x = 4 \text{m} \quad y = 4,66 \text{m}$$

$$F_{2q} = K_q * q' * H_2 = 115,57 \text{KN/m}^2 \quad q' = \gamma_1 h_1 + q_1$$

$$F_{2qH} = F_{2q} \cos(2/3 \varphi_2) = 110,71 \text{KN/m}$$

$$x = 5 \text{m} \quad y = 1,25 \text{m}$$

$$F_{2qV} = F_{2q} \sin(2/3 \varphi_2) = 33,14 \text{KN/m}$$

$$x=5\text{m} \quad y=1,25\text{m}$$

$$F_{2x} = 1/2 K \gamma y^2 H^2 = 12,5 \text{KN/ml}$$

$$F_{2xH} = F_{2x} \cos(2/3 \varphi_2) = 11,97 \text{KN/m}$$

$$x=5\text{m} \quad y=0,83\text{m}$$

$$F_{2xv} = F_{2x} \sin(2/3 \varphi_2) = 3,58 \text{KN/m}$$

$$x=5\text{m} \quad y=0,83\text{m}$$

$$U_2 = 1/2 \gamma_w h_w^2 = 31,25 \text{KN/ml}$$

$$U_{2H} = U_2 \cos(2/3 \varphi_2) = 29,9 \text{KN/ml}$$

$$x=5\text{m} \quad y=0,83\text{m}$$

$$U_{2v} = U_2 \sin(2/3 \varphi_2) = 8,96 \text{KN/ml}$$

$$x=5\text{m} \quad y=0,83\text{m}$$

$$R = W_1 + W_2 + W_3 + F_{1qv} + F_{1xv} + F_{2qv} + F_{2x} + U_{2v}$$

$$R = 791,1 \text{KN/ml}$$

The eccentricity:

$$\Sigma M/D = 1473,86 \text{KN.m}$$

$$X = \frac{\Sigma M/D}{R} = \frac{1473,86}{791,1} = 1,86 \text{m}$$

$$\Sigma M_{\text{Stabilizing}} = 2353,155 \text{KN.m}$$

$$\Sigma M_{\text{Overturning}} = 881,019 \text{KN.m}$$

$$e = B/2 - X = 5/2 - 1,86$$

$$e = 0,64 < B/6 = 0,83$$

$e < B/6$ □ The load lies within the core (kern) \Rightarrow the section is fully compressed, therefore no tensile stresses occur.

$$2 / \sigma_{\text{max}} = R/A(1 + 6e/B) \quad \sigma_{\text{max}} = 279,74 \text{KN/m}^2$$

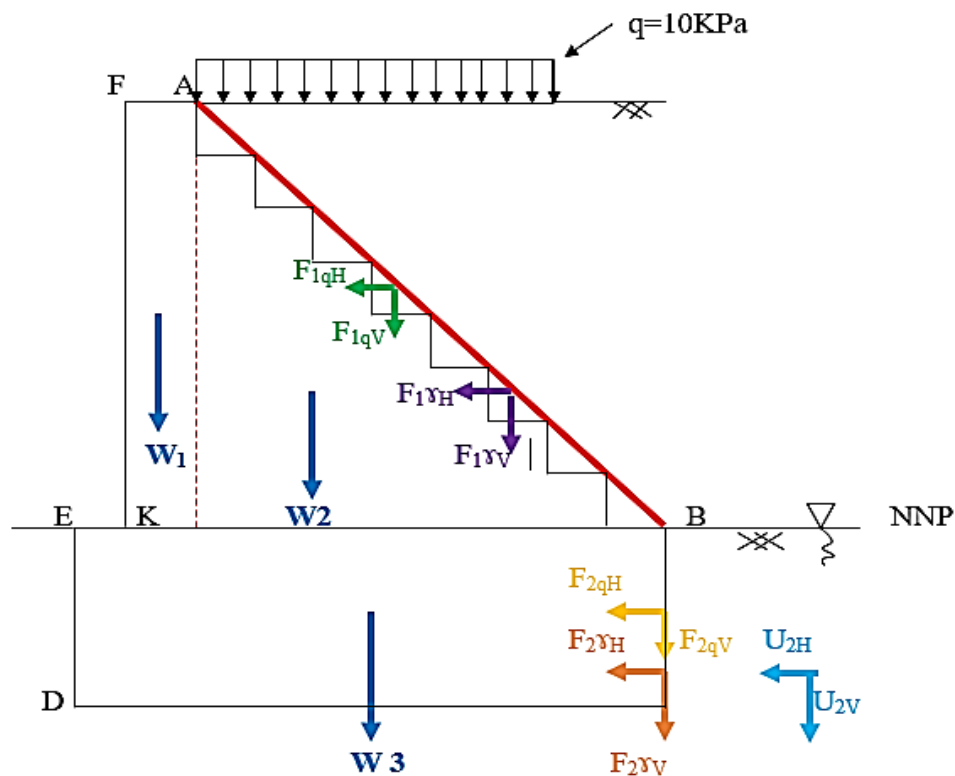
$$3/ F_R = \frac{\Sigma M_{\text{Stabilizing}}}{\Sigma M_{\text{Overturning}}} = \frac{2354,885}{881,019}$$

$F_R = 2,67 > 1,5$ There is no risk of overturning

$$4/ F_S = \frac{R \tan \delta + Ca\beta + F_p}{\Sigma F_h} = \frac{791,1 \tan 25}{299,08}$$

$F_H = 299,08 \text{ KN/ml}$

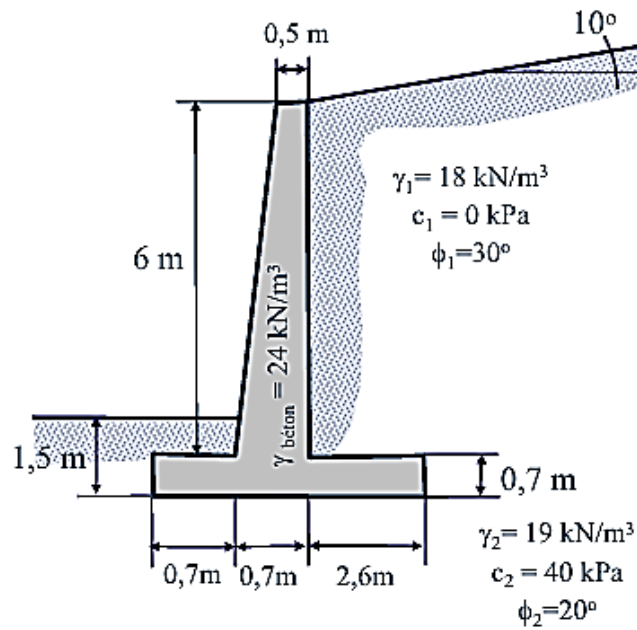
$F_S = 1,23 < 1,5$ The wall is unstable; it must be redesigned.



Exercise problems

Exercise 1

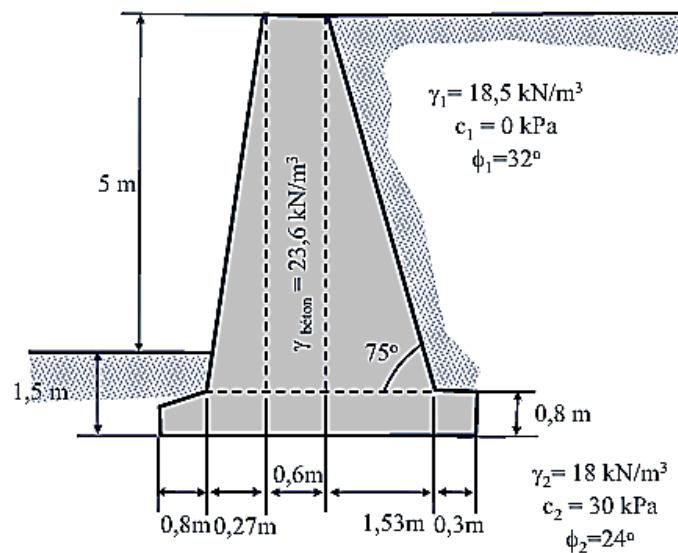
The section of a cantilevered retaining wall is shown in the figure below. Determine the factors of safety against overturning and sliding.



Exercise 2

The section of a retaining wall weight is shown in Figure below. Using Rankine's theory :

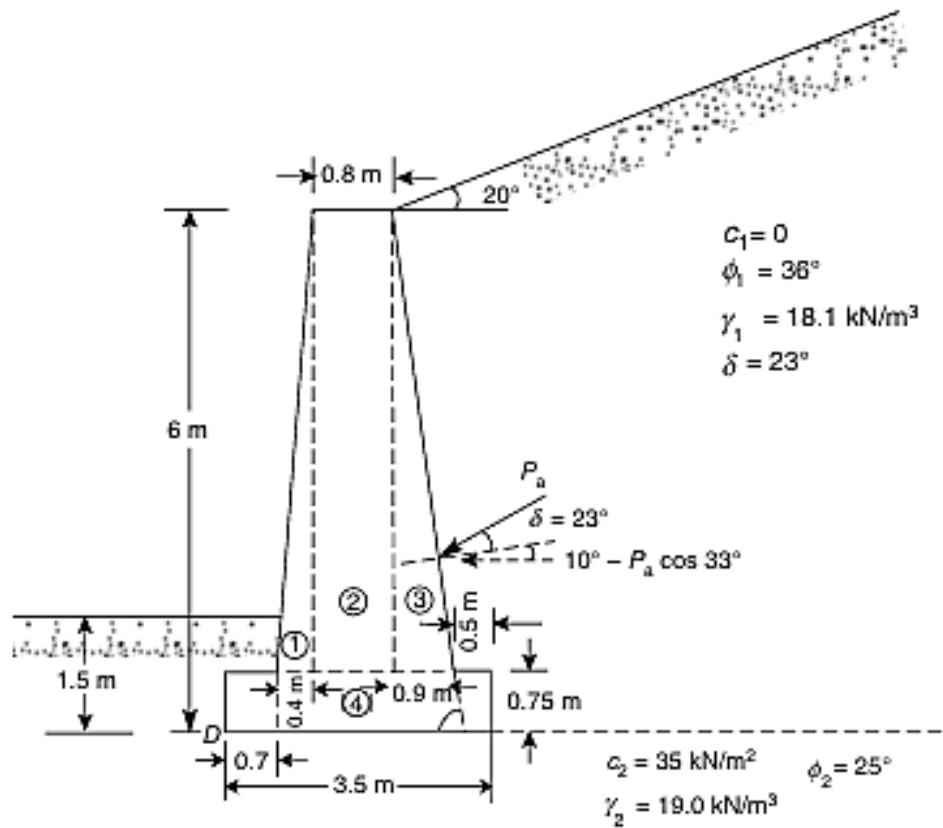
Determine the safety factors against overturning, sliding and the breakup.



Exercise 3

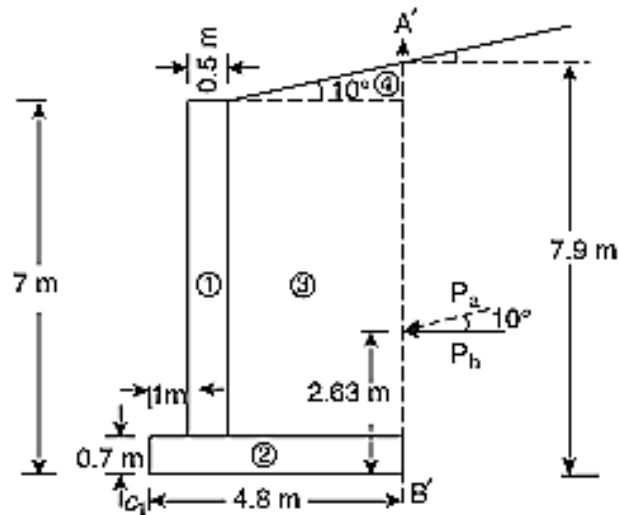
It is proposed to construct a gravity-retaining wall 6 m high, with a backfill sloping at an inclination of 20° with the horizontal. The base of the wall is to be placed 1.5 m below the ground surface. The properties of the backfill material are $c_1 = 0$, $\phi_1 = 36^\circ$, and $\gamma_1 = 18.1$ kN/m³, and the angle of wall friction is $\delta = 23^\circ$. The foundation soil is a cohesive friction soil

with $c_2 = 35 \text{ kN/m}^2$, $\phi_2 = 25^\circ$, and $\gamma_2 = 19.0 \text{ kN/m}^3$. Neglect wall friction in the front face of the wall. Unit weight of the wall material is 23.5 kN/m^3 . Proportion the dimensions of the retaining wall and check for safety against overturning and sliding. The water table is located at a greater depth.



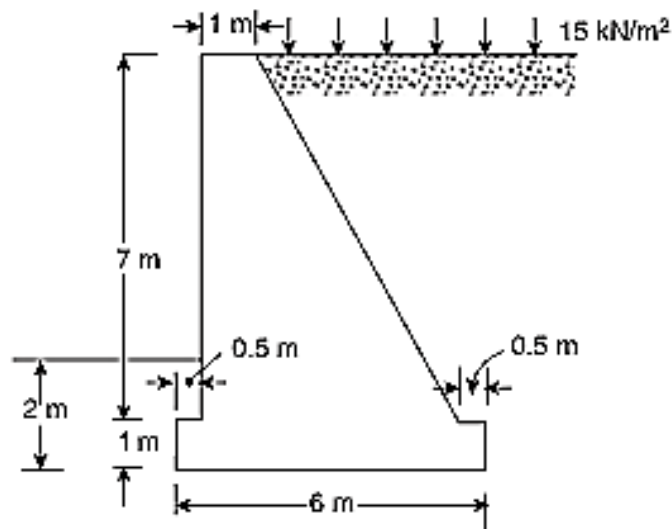
Exercise 4

For the cantilever retaining wall shown in Fig. 12.27, determine the maximum and minimum pressures under the base of the cantilever. The relevant shear strength parameters of the backfill and foundation soil are $c' = 0$, $\phi = 35^\circ$, and unit weight of the soil $\gamma = 17.5 \text{ kN/m}^3$. The unit weight of the wall material is 23.5 kN/m^3 . Find also the factor of safety against sliding, considering the reduced value of base friction as $2/3\phi^\circ$.



Exercise 5

Check the stability of the concrete retaining wall shown in Fig. 12.34. The backfill material is a mixture of sand and gravel with the following properties: $\gamma = 19.6 \text{ kN/m}^3$ and $\phi = 33^\circ$. The tangent of the coefficient of friction between the concrete and the soil is 0.48. The unit weight of concrete is 2.5 kN/m^3 . The retaining wall is placed on a very dense gravelly bed with an allowable soil pressure of 380 kN/m^2 .



III. Shallow Foundations

Chapter III: Shallow Foundations

1. Introduction

Structural foundations are generally classified into two main categories: shallow foundations and deep foundations. This classification is mainly based on the depth at which the foundation transfers structural loads to the supporting soil. Shallow foundations are constructed near the ground surface and transfer loads to soil layers located at relatively small depths, whereas deep foundations transfer loads to deeper and more competent soil or rock strata (Bowles, 2006; Coduto, 2007).

A shallow foundation is typically placed at a depth that is small compared with its width, and it directly supports the loads transmitted by the superstructure. In contrast, deep foundations, such as piles or drilled shafts, are used when surface soils are not capable of safely supporting the structural loads (Vesic, 1975). Although several practical criteria exist, there is no universally strict definition separating shallow and deep foundations; in general, foundations with an embedment depth less than or comparable to their width are considered shallow foundations (Bowles, 2006). When designing a shallow foundation for a given loading system, the foundation must satisfy several essential design requirements to ensure safety and serviceability. These requirements are typically related to foundation placement, bearing capacity, and settlement performance (Das, 2007).

The first requirement concerns foundation placement, which involves determining the appropriate location and depth of the foundation. This decision requires a thorough investigation of the construction site, including the previous use of the land and detailed information about subsurface soil conditions obtained through geotechnical site investigations (Rao, 2011). The selected depth should ensure that the foundation is placed on soil layers capable of providing adequate support and that future environmental or structural influences do not adversely affect its performance. The second requirement is safety against bearing capacity failure. The foundation must be proportioned so that the stresses transmitted to the supporting soil do not exceed the soil's shear strength. If the applied stress surpasses the soil's bearing capacity, a sudden shear failure may occur beneath the foundation, potentially leading to catastrophic structural collapse (Fang, 2017). Therefore, accurate knowledge of the geotechnical properties of the soil and rock layers is essential for determining the allowable

bearing capacity. The third requirement concerns tolerable settlement of the foundation. Even when the bearing capacity requirement is satisfied, excessive settlement may occur due to deformation of the soil mass under applied loads or due to consolidation of compressible soil layers. Excessive settlement may lead to structural damage or serviceability problems. Consequently, it is necessary to evaluate the expected magnitude and rate of settlement using appropriate soil parameters obtained from laboratory and field tests (Lambe & Whitman, 1969).

For these reasons, the design of shallow foundations requires a comprehensive understanding of soil behavior and the interaction between soil and structure. The design and verification of shallow foundations are commonly carried out according to modern geotechnical standards such as Eurocode 7 (EN 1997), the Fascicule 62 – Titre V, and the DTU provisions for building foundations.

2. Shallow Foundations

2.1. Definition

Shallow foundations (Fig III.11) are intended to transfer loads from the superstructure to the ground. A foundation is considered shallow if: $D/B < 6$ et $D < 3\text{m}$, selon le DTU 13.11

D: Depth of embedment of the footing

B: Width of the footing

L: Length of the footing

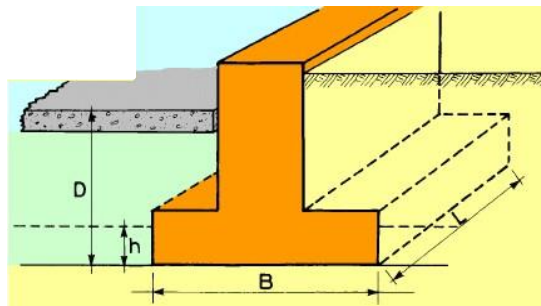


Fig III.1. Shallow Foundations

2.2. Types of Shallow Foundations

We distinguish:

Strip footings(Fig III.2), generally with a width B (at most a few meters) and a considerable length ($L/B > 10$) ;

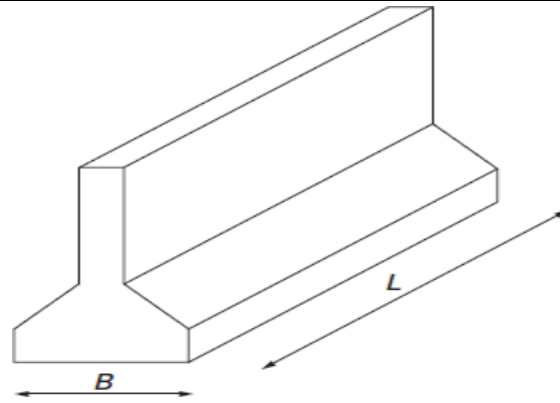


Fig.III. 2. Strip foundations.

Isolated footings, with both plan dimensions B and L typically no more than a few meters. ($L/B < 5$). This category includes square footings ($B/L = 1$) and circular footings (de Diamete B)

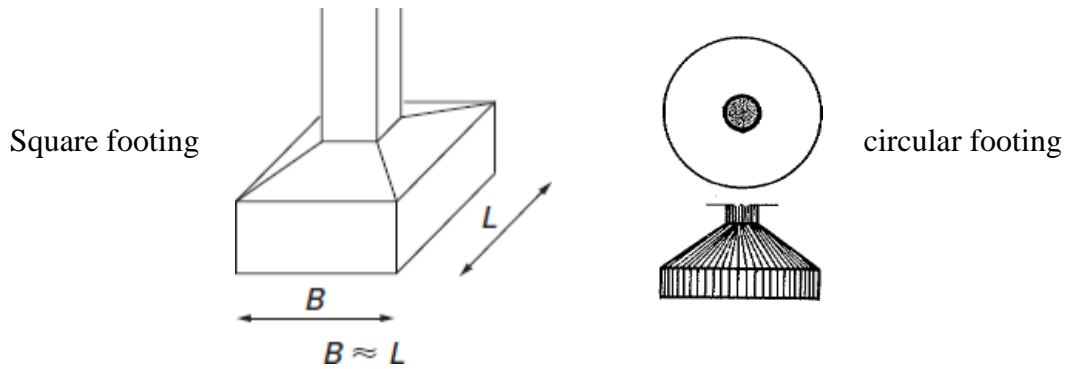


Fig III. 3. Isolated footings

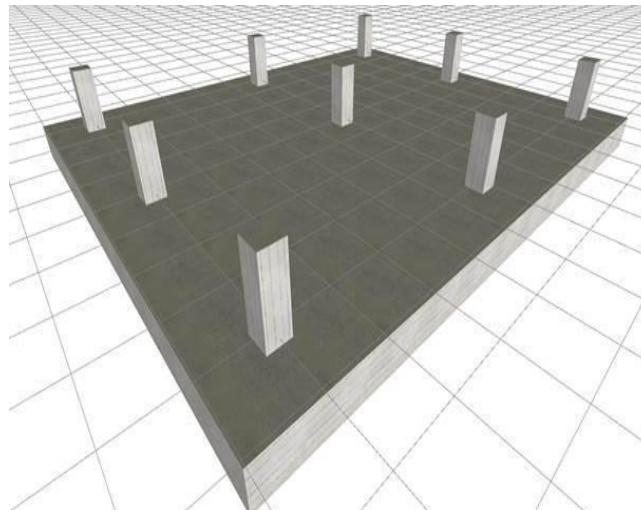


Fig III.4. Raft slab.

Rafts or slabs (FigIII. 4), with large dimensions B and L; this category includes mat foundations.

When selecting the type of foundation, the engineer must carry out five successive steps (Jhon,1985).

1. Determine the nature of the superstructure and the loads to be transferred to the foundation.
2. Obtain the necessary information about the bearing soil (geological profiles, geotechnical parameters internal friction angle ϕ , cohesion c , unit weight γ).
3. Examine the possibility of constructing on any type of foundation (shallow or deep) under the existing conditions, taking into account:
 - The bearing capacity of the soil to support the transmitted loads;
 - The harmful effects on the structure due to differential settlements.
4. Once one or two types of foundations have been selected based on preliminary studies, carry out more detailed studies. These studies may require a more precise determination of loads, geotechnical parameters, and foundation dimensions. It is also necessary to evaluate settlement to ensure the stability of the structure.
5. Estimate the cost of each type of foundation and select the type that offers the most acceptable balance between execution and cost.

2.3. Behavior of a footing under centered vertical load

A foundation is subjected to an increasing load Q (Fig III.5). After measuring the settlement s during loading, two zones are observed: the elastic zone and the plastic zone.

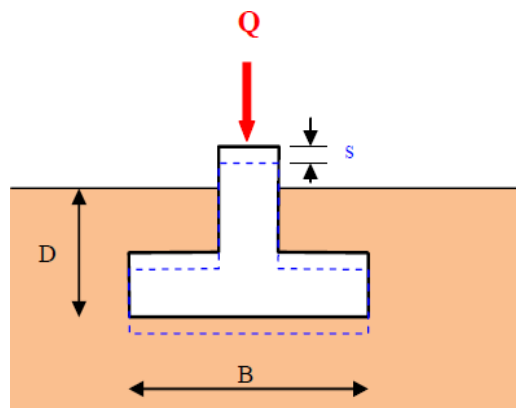


Fig .III.5. Footing under centered vertical load.

- At the beginning (Fig 6), the behavior is approximately linear (settlement is proportional to load Q): this is the **elastic zone**.

- Afterward, plastic zones develop and propagate in the soil beneath the foundation (**plastic zone**). Beyond a certain load Q_u , the soil can no longer support any additional load (this is called **failure** or **ultimate load**)? Fig.III.6.

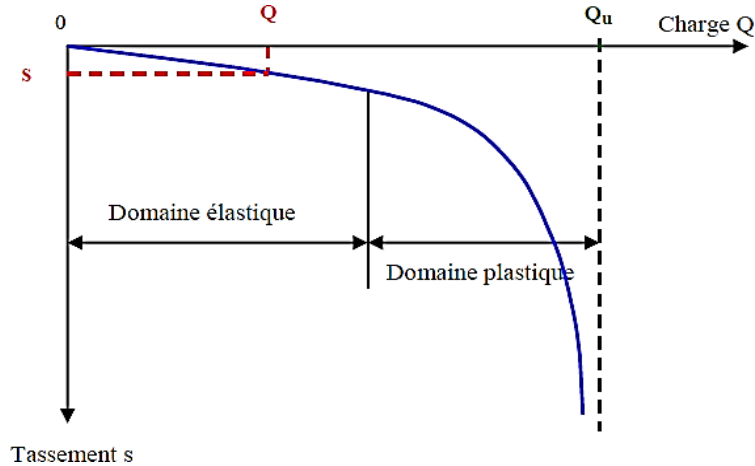


Fig.III.6. Load-Settlement Curve.

This load Q_u is called the ultimate load, which causes the failure of the foundation soil

- The bearing capacity stress of the footing is: $q_u = Q_u/A$. (A : Footing area)

Beyond the ultimate load, a soil failure occurs, and the foundation causes soil displacement (or soil heaving). The load capacity depends on the soil's cohesion C and friction angle ϕ , as well as the shape and dimensions of the foundation and the depth at which it rests.

2.4. Bearing Capacity Theory

A foundation of infinite length and width B is generally considered, exerting pressure on a homogeneous, isotropic, and weightless soil mass with a horizontal free surface, subjected to a pressure q_0 . The following figure illustrates the failure mechanism generated by the foundation load q .

The footing sinks into the soil, producing a plastic equilibrium state shaped like a Rankine active earth pressure zone. Under the footing, this zone is bounded by two initial slip planes inclined at an angle of $45+\phi/2$.

- The ultimate bearing capacity of a shallow foundation is defined as the maximum load that the supporting soil can carry.
- **Bearing capacity** is the ability of a soil to support loads.

Objective: To determine whether the soil can support the external loads or not.

Methods for calculating bearing capacity have been gradually developed since the early nineteenth century, such as those by Rankine and Pauker. The most commonly used methods are those of Terzaghi, Meyerhof, Hansen, and Vesic, who established a set of rules validated by experience and covering most common situations.

- **Note:** These methods are based on measuring the shear properties of the soil. (c et ϕ).

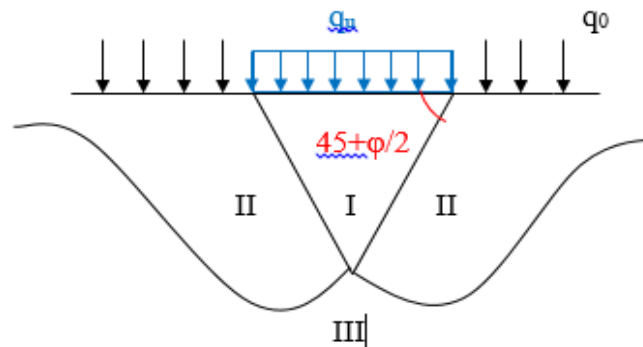


Fig .III.7. Active and Passive Wedge during failure.

In (Fig III.7, III.8) we see three zones :

Zone I: A rigid wedge ABC forms beneath the base of the footing, pushing the soil on both sides up to the surface.

Zone II: The soil in these areas is fully plasticized and is displaced laterally along the slip lines.

Zone III: A zone where stresses are lower and do not cause failure

In the general case, the solution given for this figure according to Prandtl is:

$$Q_u = \frac{c}{\tan \phi} [\tan^2 (45 + \phi/2) e^{\pi \tan \phi} - 1] + q_0 [\tan^2 (45 + \phi/2) e^{\pi \tan \phi}]$$

We set $N_q = \tan^2 (45 + \phi/2) e^{\pi \tan \phi}$

$$N_c = \frac{N_q - 1}{\tan \phi}$$

We will have:

$$q_u = C N_c + q_0 N_q$$

Since foundations are generally buried, the load q_0 is due to the weight of the soil located below the foundation level $q_0 = \gamma D$

or $q_u = C N_c + \gamma D N_q = q_c + q_D$ To account for the soil's self-weight, a factor must be added :

$q_{\gamma} = \frac{\gamma B}{2} N_{\gamma}$ Footing at a depth D below the soil surface.

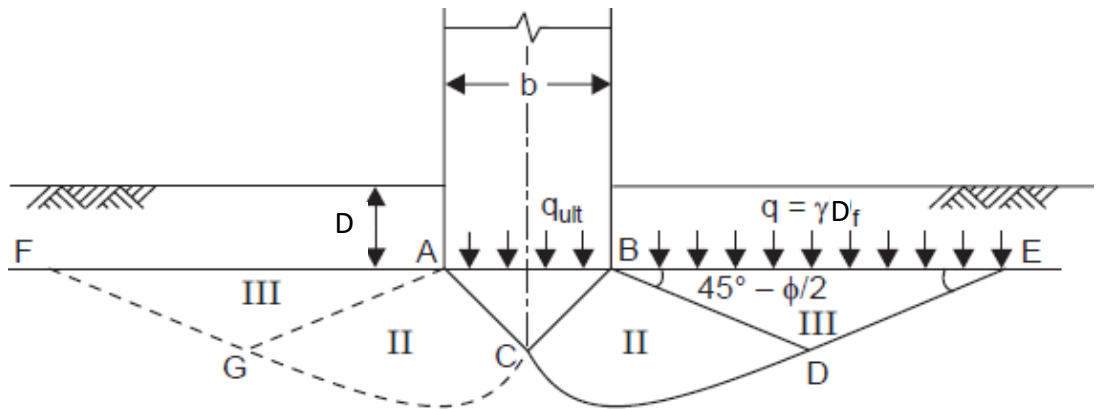


Fig.III. 8. Failure zones.

3. Bearing Capacity:

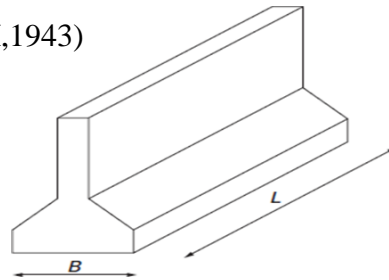
The bearing capacity of soil beneath a strip footing is expressed by:

$$q_u = q_{\gamma} + q_c + q_D \quad (\text{selon TERZAGHI, 1943})$$

$$q_u = 1/2 \gamma B N_{\gamma} + C N_c + \gamma D N_q$$

or : $N_q = \tan^2 (45 + \phi/2) e^{\pi \tan \phi}$

$$N_c = \frac{N_q - 1}{\tan \phi}$$



The values of N_{γ} , N_q , and N_c can be given as a function of ϕ by the table III.1 or chart below (Fig9):

Table III.1. Bearing capacity factors (according to TERZAGHI).

Φ (degré)	N_c	N_q	N_{γ}
0	5,7	1,0	0,0
5	7,3	1,6	0,5
10	9,6	2,7	1,2
15	12,9	4,4	2,5
20	17,7	7,4	5,0
25	25,1	12,7	9,7

30	37,2	22,5	19,7
35	57,8	41,4	42,4
40	95,7	81,3	100,4
45	172,3	173,3	297,5
50	347,5	415,1	1153,2

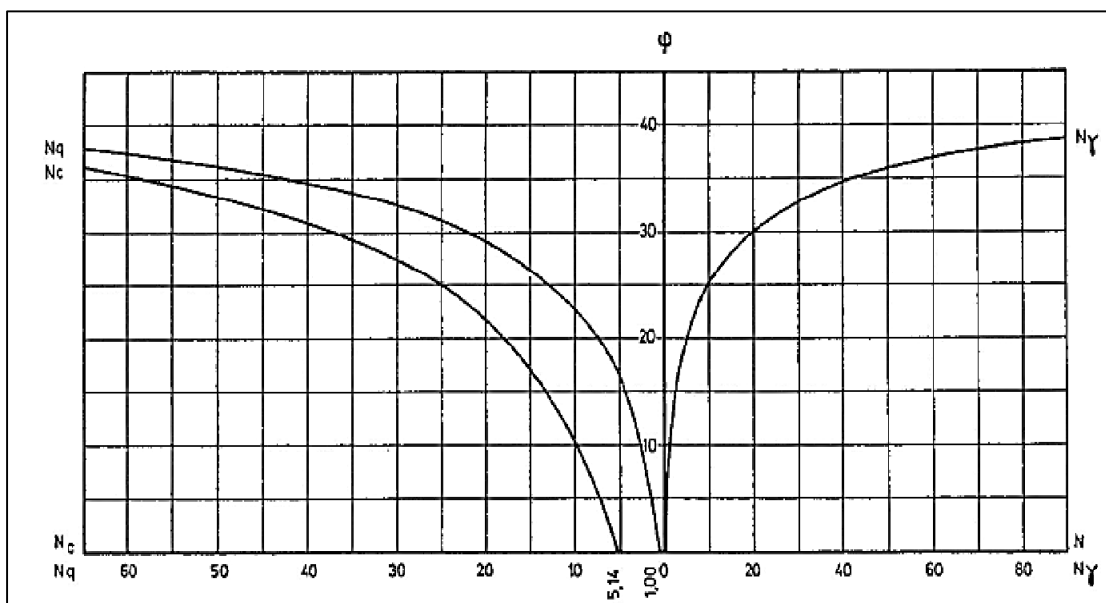


Fig III.9. Coefficients N_q , N_γ , and N_c (bearing capacity factors) as a function of the internal friction angle.

3.1. Rectangular or Circular Foundation

3.1.1. Square foundation with side B

The bearing capacity coefficients of the strip footing must be multiplied by the factors :

$$S_r=0,8 ; S_c=1,2 \text{ et } S_q=1$$

D'où $q_u=0,4 \gamma B N_r + 1,2 C N_c + \gamma D N_q$

3.1.2. Circular foundation with diameter B

$S_\gamma=0,6$ $S_c=1,2$ et $S_q=1$

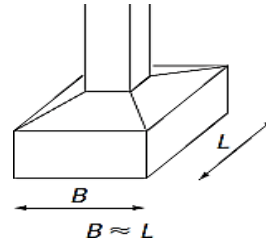
$q_u=0,3 \gamma B N_\gamma + 1,2 C N_c + \gamma D N_q$



3.1.3. Rectangular foundation with width B and length L

$S_\gamma=(1-0,2 \frac{B}{L})$; $S_c=(1+0,2 \frac{B}{L})$; $S_q=1$

$q_u=(1-0,2 \frac{B}{L}) \gamma B N_\gamma + (1+0,2 \frac{B}{L}) C N_c + \gamma D N_q$



3.1.4. Eccentric and Inclined Loads (FigIII. 10)

If the load applied to a foundation of width B is eccentric by “ e ”, MAYRHOF proposed using a reduced width $B' = B - 2e$ in the bearing capacity equation in the term $1/2 \gamma B N_\gamma$

– If the load is inclined, (Meyerhof, 1951). proposed introducing factors to be multiplied by the capacity coefficient

$i_\gamma=(1-\frac{\alpha}{\phi})^2$; $i_c=i_q=(1-\frac{\alpha}{90^\circ})^2$

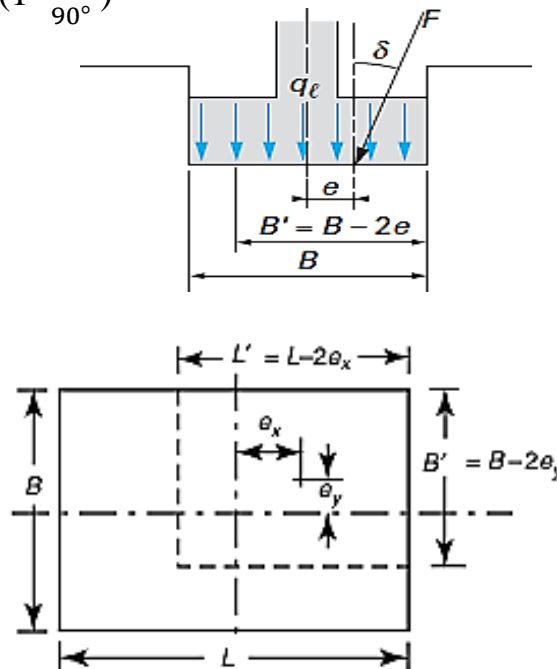


Fig.III. 10. Case of an inclined load.

4. Long-term and Short-term Bearing Capacity

4.1. Cohesive Soils

To determine the short-term bearing capacity of a saturated cohesive soil, undrained test results are used $\varphi = \varphi_u = 0$ $C = C_u$

Pour $\varphi = 0$, $N_\gamma = 0$, $N_q = 1$, $N_c = \pi + 2$

D'où $q_u = \gamma D + C N_c = \gamma D + (\pi + 2) C_u$ (Strip footing)

For the calculation of long-term bearing capacity, drained test results are used

$C = C'$ $\varphi = \varphi'$ For a strip footing

$q_u = 1/2 \gamma' B N_\gamma + C' N_c + \gamma' D N_q$

γ' : Effective unit weight of the soil

4.2. Granular Soils

The calculation, whether short-term or long-term, involves the results of a drained test.

The calculation is done in terms of effective stresses.

$\left\{ \begin{array}{l} C = C' = 0 \\ \varphi = \varphi' \end{array} \right.$ d'où $q_u = 1/2 \gamma' B N_\gamma + \gamma' D N_q$

5. Foundations on Two-layer Soils

In the case where there are two soil layers, the soil bearing capacity can be calculated as follows:

We have:

$$H = B/2 \tan (45 + \phi/2)$$

- If the second layer (soil 2) intersects the rigid corner ABC: an average cohesion and friction angle are calculated, that is to say:

$$C_{moy} = (C_1 h_1 + C_2 h_2) / H \text{ et } \phi_{moy} = (\phi_1 h_1 + \phi_2 h_2) / H$$

So we have: $q_{ult} = C_{moy} N_c + q N_q + 1/2 \gamma' B N_\gamma +$

6. Calculation of the Allowable Bearing Capacity

The net pressure is the difference between the pressure due to the self-weight of the structure q and the pressure from the removed soil γD . That is to say : $q_n = q - \gamma D$

The factor of safety concerning shear failure is defined by: $F = \frac{qu - \gamma D}{q - \gamma D}$

In the case of allowable stress:

$$F = \frac{qu - \gamma D}{q_{adm} - \gamma D} \quad \text{d'où} \quad q_{adm} = \gamma D + \frac{qu - \gamma D}{F} \quad (q_{adm} : \text{Allowable Stress})$$

F : factor of safety between 2 and 4, with the value 3 most commonly adopted.

7. Conclusion

This chapter has provided an in-depth analysis of the fundamental principles governing surface foundations, with particular emphasis on the calculation of the bearing capacity of soils. We examined the calculation methods, using the results of drained tests to evaluate the long-term bearing capacity, while taking into account essential parameters such as the effective unit weight of the soil, the cohesion and the angle of friction, the geometry of the foundation and the loading mode. The formulas presented, especially for strip foundations, illustrate how these factors interact to determine the strength of the foundations. In addition, the importance of granular soils was emphasized, due to their distinct behavior and their influence on the stability of structures.

By integrating these concepts into engineering practice, professionals can design safer and more efficient foundations, adapted to the specific conditions of the site and the requirements of construction projects. This in-depth understanding is essential to minimize the risks of foundation failure and guarantee the durability of the structures. The design and verification of shallow foundations are commonly carried out according to modern geotechnical standards such as Eurocode 7 (EN 1997), the Fascicule 62 – Titre V, and the DTU provisions for building foundations.

*Applications**Application 01*

A purely cohesive soil was tested using the unconfined compression test, and the following compressive strength values were obtained: (39,3 – 43,4 – 37,2 – 44,8 – 48,3 – 42,7 – 40,7) KN/m².

Estimate the bearing capacity of this soil loaded by a strip footing on its surface.

Solution Application 01

$$N_{\gamma}=0 \quad N_q=1 \quad N_c=5,7 \quad (\text{Tableau}) \quad \varphi=0$$

$$C = \frac{\sum F/Z}{2} = 21,17 \text{KN/m}^2$$

$$D=0$$

$$q_u = CN_c \quad q_u = 120,67 \text{KN/m}^2$$

$$q_u = 1/28BN_{\gamma} + CN_c + 8DN_q$$

$$\varphi=0 \quad \{N_{\gamma} = 0; N_q = 1; N_c = 5,7\}$$

$$q_u = 54 * 5,7 + (10 * 1,76) * 2 * 1$$

$$q_u = 343 \text{KN/m}^2$$

$$q_{\text{net}} = q_u - \gamma D$$

$$= 343 - (10 * 1,76) * 2$$

$$q_{\text{net}} = 307,8 \text{KN/m}^2$$

Application 02

A continuous footing 1.5 meters wide rests on the surface of a dry, cohesionless material, $\varphi=17^\circ$; $\gamma=18,4 \text{KN/m}^3$. A flood causes the groundwater table to rise to the surface. By what percentage is the bearing capacity of the foundation reduced?

Solution Application 02

By linear interpolation, we find N_{γ} :

$$15 \rightarrow 2,5 \qquad \frac{15-17}{2,5-N_{\gamma}} = \frac{15-20}{2,5-5}$$

$$17 \rightarrow N_{\gamma}$$

$$20 \rightarrow 5 \qquad N_{\gamma} = 3,5$$

$$q_{\text{usec}} = 1/2 \gamma B N_{\gamma} + C N_c + \gamma D N_q$$

$$= 1/2 * 18,4 * 1,5 * 3,5$$

$$q_{\text{usec}} = 48,3 \text{ KN/m}^2$$

$$q_{\text{usat}} = 1/2 (\gamma' - \gamma_w) B N_{\gamma}$$

$$q_{\text{usat}} = 1/2 (18,4 - 10) * 1,5 * 3,5$$

$$q_{\text{usat}} = 22,05 \text{ KN/m}^2$$

$$q_u = q_{\text{sec}} - q_{\text{sat}} = 48,3 - 22,05$$

$$q_u = 26,25 \text{ KN/m}^2$$

$$\begin{cases} 48,3 \rightarrow 100 \\ 26,25 \rightarrow x \end{cases} \quad x = 54,3\%$$

The settlement after flooding is 54,3%.

Exercise problems

Exercise 1

In a mass-housing complex scheme over a vast area, two types of soils were encountered. One of which is a partially saturated silty clay with $c_u = 5.8 \text{ kN/m}^2$, $\phi_u = 25^\circ$, and $\gamma = 18.5 \text{ kN/m}^3$ and extends over most of the area. The other, predominantly clay having $c_u = 55 \text{ kN/m}^2$ spreads to a lesser extent. The water table is at a greater depth. As per the design, strip footings of the building have to be placed at 1 m depth. Compute the width of the footing required in each type of soil if the load intensity is 150 kN/m run. Adopt a factor of safety of 2.5 in both the soils, and only shear failure need to be considered. For $\phi = 25^\circ$, take $N_c = 20.7$, $N_q = 10.7$, $N_{\gamma} = 10.8$.

If there is a possibility of the water table rising to the ground surface, what should be the change in the width of footing in both areas. The submerged unit weight of the silty clay is 11.2 kN/m^3 .

Exercise 2

A 1 m wide long footing is located at a depth of 1.5 m from the ground surface. The supporting soil is compressible and has shear strength parameters, $c = 30 \text{ kN/m}^2$ and $\phi = 25^\circ$. The total unit weight of the soil, $\gamma = 18.3 \text{ kN/m}^3$. The water table is at a greater depth. Compute the safe load that can be carried by the long footing per metre length of the wall. Adopt a factor of safety of 3.0

Exercise 3

In a warehouse building, two unequally loaded columns are combined by a rectangular combined footing. It is proposed to place the footings at a depth of 1.5 m on a saturated clay with the following soil properties: $c_u = 72 \text{ kN/m}^2$, $\phi_u = 0^\circ$, $\gamma = 17.8 \text{ kN/m}^3$. The loads on the columns are 720 and 1,170 kN, with a spacing of 5 m, and the centre of the 720 kN column is placed at a distance of 0.8 m from the property line (Fig. 14.24). Neglecting the weight of the footing, estimate the dimension of the footing. Adopt a factor of safety of 3.

Exercise 4

A circular concrete pier of 3 m diameter carries a gross load of 3,500 kN. The supporting soil is a clayey sand having the following properties: $c = 5 \text{ kN/m}^2$, $\phi = 30^\circ$, and $\gamma = 18.5 \text{ kN/m}^3$. Find the depth at which the pier is to be located such that a factor of safety of 3.0 is assured. The bearing capacity factors for $\phi = 30^\circ$ are $N_c = 30.1$, $N_q = 18.4$, and $N_\gamma = 22.4$

Exercise 5

The weight of a heavy machinery is 7,600 kN and the base dimensions are $5.5 \text{ m} \times 3.5 \text{ m}$. The machinery has to be installed on a stiff clay soil with a cohesion of 150 kN/m^2 , at a depth of 0.8 m below the ground surface. The total unit weight of the soil is 19.2 kN/m^3 . Determine the size of the foundation required if the minimum factor of safety is 3.0. Assume the load to be rapidly applied so that undrained condition prevails ($\phi = 0$). Neglect the weight of the foundation

IV

- **Slope**
Stability

Chapter IV: Slope Stability

1. Introduction

Slope instability and landslides represent one of the most significant geotechnical hazards affecting both natural terrain and engineered earth structures. Landslides correspond to the movement of soil or rock masses down a slope under the influence of gravity when the shear stresses along a potential failure surface exceed the shear strength of the soil or rock mass (Abramson, Lee, Sharma & Boyce, 2002). Such movements may involve natural slopes composed of rock and soil formations, artificial fills, or combinations of both.

Landslides may occur due to natural causes such as intense rainfall, erosion, earthquakes, or weathering processes. They may also be triggered by human activities including excavation, construction of embankments, deforestation, or changes in groundwater conditions (Duncan, Wright & Brandon, 2014). These events can lead to severe damage to infrastructure, buildings, transportation networks, and sometimes cause significant loss of life.

Potential landslides in natural slopes can often be identified through geological investigations, aerial photographs, satellite imagery, or field reconnaissance surveys. In addition to natural slopes, slope failures may also occur in man-made earth structures, such as embankments, earth dams, highway cuts, and landfills (Chowdhury, Flentje & Bhattacharya, 2010). For this reason, careful selection of construction materials and appropriate construction techniques are essential to ensure slope stability during and after construction.

Slope instability may also affect the performance of foundations and earth-retaining structures, especially when the soil supporting these structures experiences shear failure. One possible cause of ground rupture is insufficient embedment depth of foundations or retaining walls combined with soils having low shear strength (Craig, 2004).

In geotechnical engineering practice, slope stability analysis generally consists of determining the factor of safety against shear failure along a potential slip surface. This analysis allows engineers to evaluate the stability of natural slopes, excavations, embankments, earth dams, and other geotechnical structures. Various analytical methods have been developed to perform such analyses, including limit equilibrium methods, which remain the most widely used in engineering practice (Duncan et al., 2014).

2. Causes of Landslides

The causes of failure of slopes may be external or internal. External causes are those which produce an increase in the stress at unaltered shearing resistance of the material. They include steepening of the slope, deposition of material along the edge of slopes, and earthquake forces.

Internal causes are those that lead to a slide without any change in surface conditions which involve unaltered shearing stresses in the slope material. Some of these conditions are the decrease in shearing resistance brought about by excess pore water pressure, leaching of salts, softening, breakage of cementation bonds, and ion exchange. Intermediate between landslides due to external and internal causes are those due to rapid draw-down, to surface erosion, and to spontaneous liquefaction. (Terzaghi, 1950) reviewed the processes which cause landslides by several modes of action of agents and represented them in a lucid form.

Landslides are often related to the action of water: particularly intense rainfall can carry away large amounts of material deposited in riverbeds, erode slopes, and trigger landslides.

Landslides caused by changes in hydraulic conditions (draining of a reservoir...).

Landslides triggered by an earthquake (Fig IV.1.).

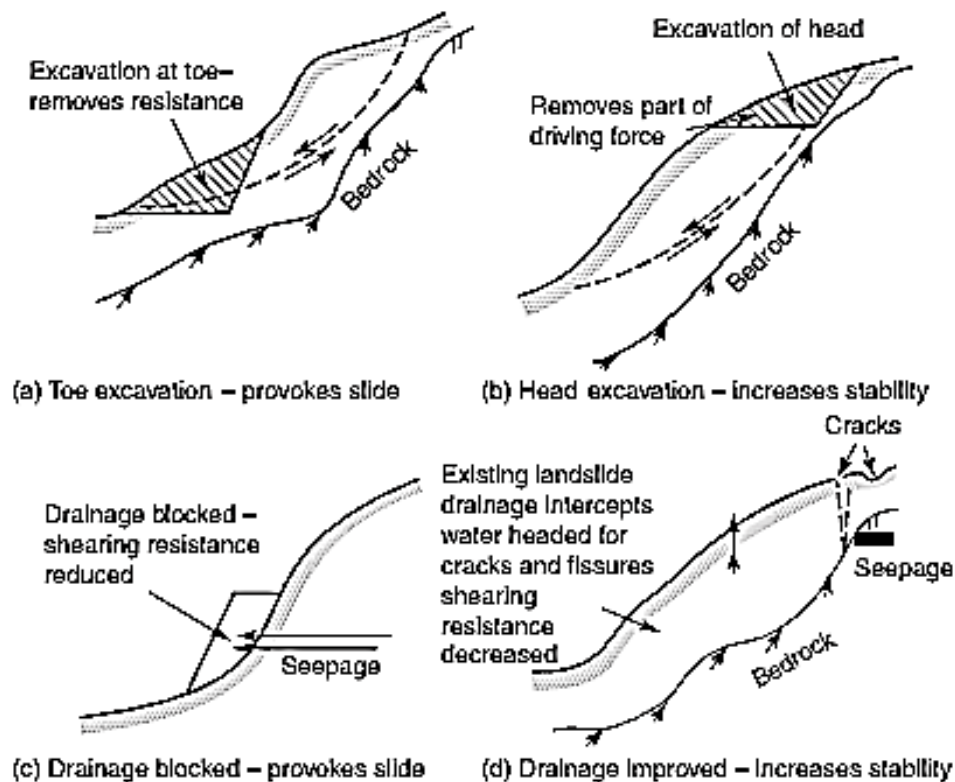


Fig.IV.1. Activities that decrease or increase the probabilities of slides.

3. Classification of Earth Movements

3.1. Slides

A systematic classification of slides in clay and other mass movements was proposed by Skempton and Hutchinson (1969). This includes five basic types and six complex forms of movements (Fig.IV. 2).

3.1.1 Basic Types of Landslides

Falls. The removal of earth support causes bulging at the toe and tension crack at the top. The development of cracks induces additional stresses on the separating mass and leads to an ultimate failure. Clay falls occur in steep slopes and are typical short-term failures. Such failures are found mostly in over-consolidated fissured clays.

Rotational Slides (Slips, Slumps). These types of slides are common in fairly uniform clays or shales. The curved surface of failure, being concave upwards, imparts a backward tilt to the slipping mass, resulting in sinking at the rear and heaving at the toe. Such slides are deep-seated, and the failure surfaces may be circular or non-circular.

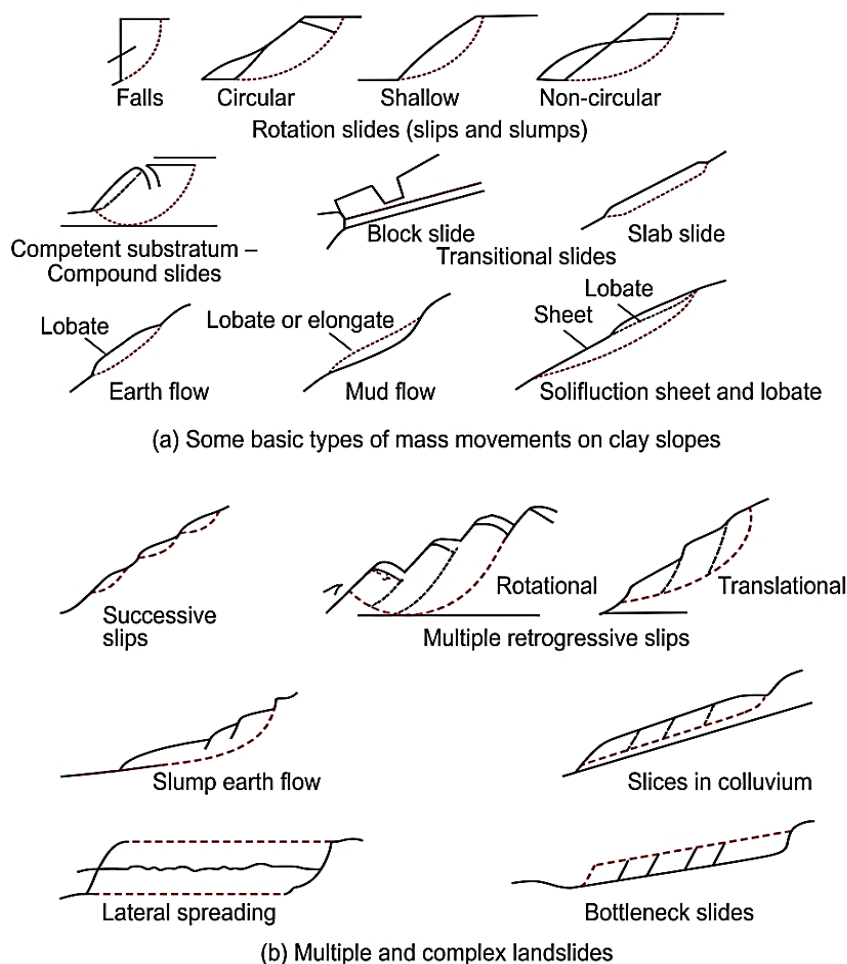


Fig.IV.2. Types of mass movements on clay slopes.

3.1.2. Compound Slides

The surface of failure is predetermined by the presence of heterogeneity within the slope material. Heterogeneity usually consists of a weak soil layer or a structural feature or a boundary between two materials, for example, clay and rock or weathered and unweathered material. Such heterogeneity prevents simple rotational slides but introduces a translational element in the movement in combination with or without rotational slide. Compound slides usually occur in soils with heterogeneity at moderate depth.

3.1.3. Translational Slides

These are planar and most commonly occur in a mantle of weathered material, the heterogeneity being at a shallow depth. Moreover, such slides occur as block or slab slides. Block slides are found in marls and sandstones, whereas slab slides are a type of translational failure in more weathered clay slopes.

3.1.4. Flows

These are mass movements which may be of either earth flow or mud flow. While earth flows are slow movements of softened weathered debris, mud flows are glacier-like in form and are often well developed below the bar in fissured clays (Fig 3).



Fig .IV .3. Rotational Slides.

4. Problems Posed

To study stability under various circumstances, the geotechnician must:

- Establish the geological structure of the site (identify the nature of the soils/grounds).
- Know the groundwater level.
- Specify the depth and shape of the slip surface in order to determine the strength parameters.

- Evaluate the speed of soil movements (set up a monitoring station).

4.1 Factor of safety

In any stability analysis, some measure of the degree of safety has to be provided. Such a measure of safety may be a factor like a limiting stress or strain or a comparative ratio of resistance. Working stresses in any earth structure are much less than the shear strength of the soil so as to ensure the safety of the structure. The working stress is the actual stress at a point or along a continuous surface and may be defined as developed or mobilized strength. In slope stability problems, shear strength is the governing factor for stability; hence, the mobilized or developed shear strength (τ) is also important. If this mobilized strength is less than the available strength (τ_f) of the soil, then the slope is said to be stable. Thus, the factor of safety may be defined, in a form most convenient and acceptable to practical engineers, as the ratio of the shearing resistance available along a slip surface to the total mobilized shear ing resistance; that is, $F = \tau_f / \tau$

4.2. Analysis

The calculation consists of comparing the shear stresses τ acting along S to the maximum shear strength τ_{max} of the soil (Fig IV.4).

It is assumed that failure occurs simultaneously at all points; and the safety factor F is defined by: $F = \tau_{max} / \tau$

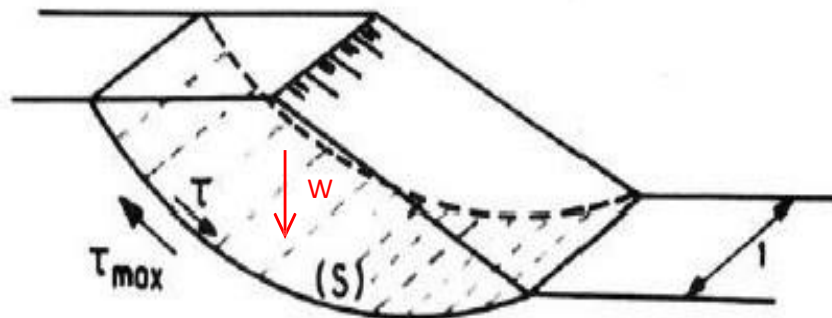


Fig .IV.4. The shear stresses in Rotational Slides.

if $F > 1$: There is no sliding

if $F \leq 1$:There is a possibility of failure by sliding

In the general case, the shear strength τ_{max} is equal to:

$$\tau_{max} = c' + \sigma' tg \phi'$$

And the shear stress τ that can develop along the slip surface is the cause of the forces that move the weight W .

$$\tau_{max} = c' / F + \sigma' tg \phi' / F$$

Goal: We seek the most critical slip surface corresponding to the minimum safety factor;

For this purpose, the most commonly used classical methods are:

Planar Failure Analysis

Circular Failure Analysis (Slice Method)

Non-Circular Failure (Disturbance Method)

5. Analysis of an infinite slope

5.1. Infinite slope without seepage

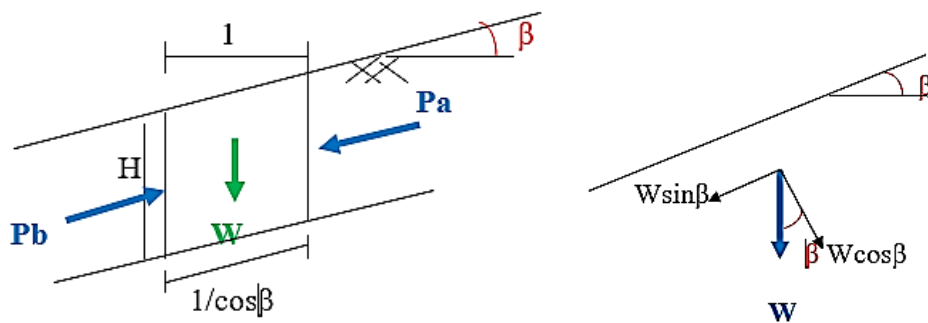


Fig .IV.5. Forces acting on a slice without seepage.

Stability can be analyzed in considering the forces acting on a typical vertical element of unit width (Fig IV.5), this element is subjected to:

The vertical reactions P_a and P_b , which are equal and opposite

To Weight $W = \gamma * H * 1 * 1 = \gamma H$

The reactions at the base

The driving force that causes failure is the component of the weight parallel to the slope

$$F_c = W \sin \beta = \gamma H \sin \beta$$

The force opposing sliding is due to the shear resistance of the soil along the base of the element.

$$F_R = (C + \sigma_n \tan \phi) \cdot \frac{1}{\cos \beta} \quad / \sigma_n = \frac{W \cos \beta}{\frac{1}{\cos \beta} \cdot 1} = \gamma H \cos^2 \beta$$

$$F_R = \frac{C}{\cos \beta} + \frac{\gamma H \cos^2 \beta}{\cos \beta} \tan \phi$$

$$F_R = \frac{C}{\cos \beta} + \gamma H \cos \beta \tan \phi$$

The safety factor of the slope is given by:

$$F = \frac{\frac{C}{\cos \beta} + \gamma H \cos \beta \tan \phi}{\gamma H \sin \beta} = \frac{C}{\gamma H \sin \beta \cos \beta} + \frac{\tan \phi}{\tan \beta}$$

- { granular soil (Cohesionless soil): $C=0$ d'ou $F = \tan \phi / \tan \beta$
- { Short-term soil condition: $C=C_u$; $\phi = \phi_u = 0$ $F = C_u / \gamma H \sin \beta \cos \beta$

5.2. Infinite slope with seepage

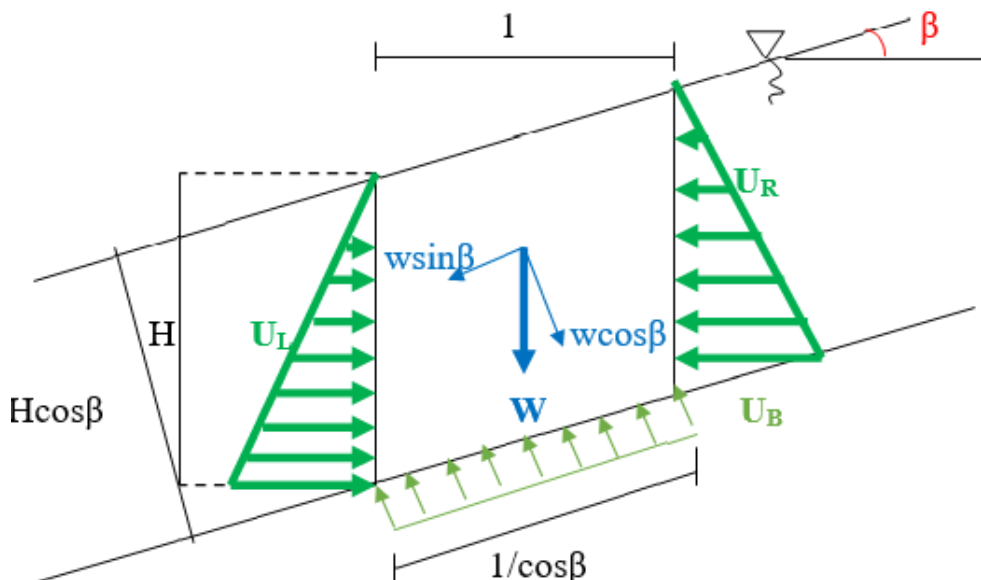


Fig.IV.6. Forces acting on a slice with seepage.

The safety factor $F = \frac{F_r}{F_c}$

$$F_c = \gamma_{\text{sat}} H \sin \beta$$

The components UL and UR have the same magnitude but opposite directions
 The resisting force against failure

$$F_R = (C' + \sigma'_n \tan \phi') \frac{1}{\cos \beta}$$

$$\sigma'_n = (w \cos \beta - u \cos \beta) / 1 / \cos \beta = (\gamma_{\text{sat}} - \gamma_w) H \cos^2 \beta = \gamma' H \cos^2 \beta$$

$$F_R = \frac{C'}{\cos \beta} + \frac{\gamma' H \cos^2 \beta \tan \phi'}{\cos \beta}$$

Therefore $F = \frac{F_R}{F_c} = \frac{\frac{C'}{\cos \beta} + \gamma' H \cos \beta \tan \phi'}{\gamma_{\text{sat}} H \sin \beta}$

In the case of a cohesionless soil $C' = 0$ $F = \frac{\gamma' \tan \phi'}{\gamma_{\text{sat}} \tan \beta}$

6. Analysis of a finite slope

There are two methods: the global method and the method of slices (Fellenius; Bishop)

6.1. Global method

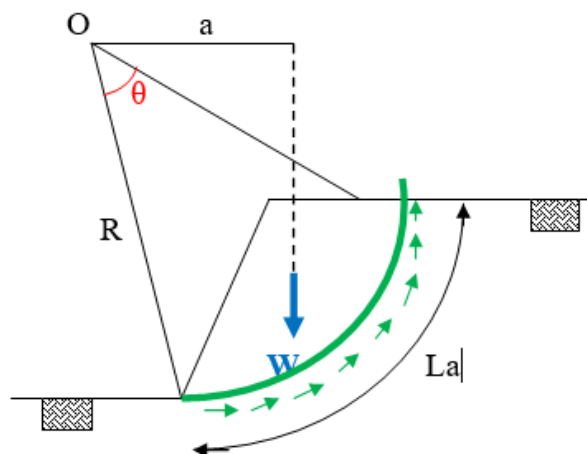


Fig .IV .7. Failure line of a finite slope.

The failure line is approximated by a circular arc with center O and radius R . Let us consider a cohesive soil under short-term behavior (Fig IV.7).

$$C=C_u ; \varphi=\varphi_u=0 ;$$

The safety factor can be defined in terms of moment with respect to the center O. “ M_m ” is the driving moment that promotes failure. “ M_s ” is the stabilizing moment that resists failure.

$$F = \frac{M_s}{M_m}$$

$$M_m = W \cdot a$$

$$M_s = C_u \cdot L_a \cdot R$$

$$\text{Avec } L_a = \frac{\theta}{360} (2\pi R)$$

The moments of all additional forces must be taken into account. It is necessary to analyze the slope for different failure surfaces in order to determine the minimum safety factor.

6.2. Method of slices

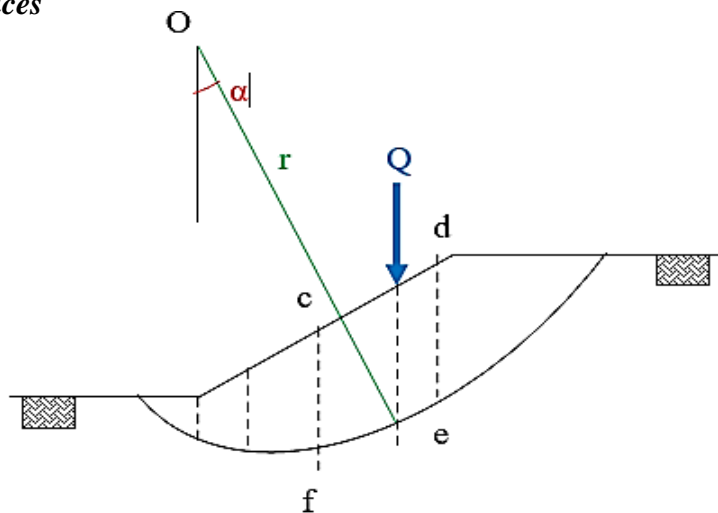


Fig .IV.8. Division of a slope into slices.

In this method, the soil is divided into a certain number of vertical slices (Fig IV.8). The forces acting on each vertical slice are evaluated using limit equilibrium. The equilibrium of the entire mass is determined by summing all the forces acting on the slices. Forces acting on a slice c– d – e – f:

- The weight of the slice W
- The load Q
- The normal and shear forces N and T acting on the failure surface
- The normal and shear forces $E1, X1$ and $E2, X2$ acting on the vertical sides $c-e$ and $d-f$

These forces are shown in the figure IV.9 below.

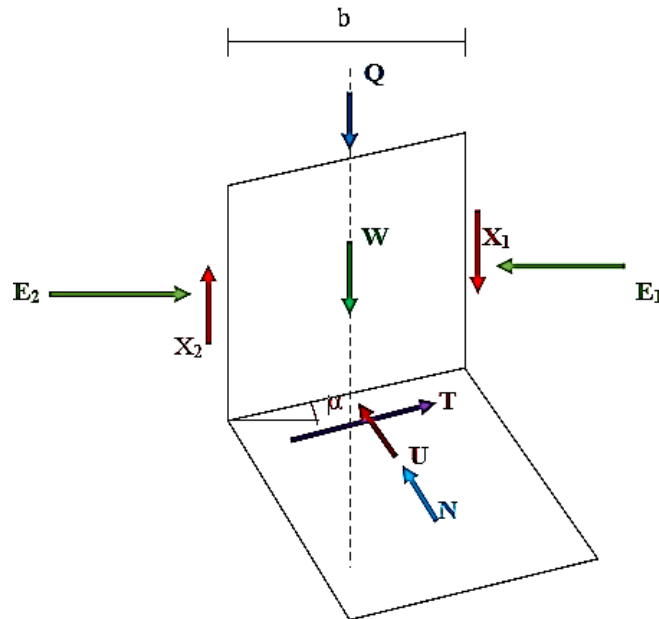


Fig .IV .9. Graphical representation of the forces.

The system is statically indeterminate; to obtain a solution, it is necessary to introduce simplifications regarding $E1$, $X1$ and $E2$, $X2$.

6.2.1.Fellenius method

The summation of forces gives:

$$T = (W+Q)\sin\alpha + (E_1 - E_2)$$

Si $E_1 = E_2 \rightarrow T = (W+Q)\sin\alpha$ Driving force causing sliding

$$\frac{N}{\cos\alpha} = W+Q - \frac{Ul}{\cos\alpha} + (X_1 - X_2)$$

$$N = (W+Q)\cos\alpha - Ul + (X_1 - X_2)\cos\alpha$$

To solve the problem, it is assumed that $X_1 = X_2$

$$N = (W+Q)\cos\alpha - Ul$$

T is the driving force that causes sliding

The resisting force: $\tau \cdot l$

where τ is the shear strength along the slip surface.

$$\tau = C' + \sigma'_n \tan \phi'$$

$$\tau^* l = C' l + \sigma'_n \tan \phi' l$$

$$\sigma'_n = \frac{N}{l}$$

$$F = \frac{\tau r}{\tau} = \frac{\Sigma(C' l + [(W+Q) \cos \alpha - Ul] \tan \phi')}{\Sigma(W+Q) \sin \alpha}$$

The resisting force along the entire slip surface is the sum of the forces from each slice. The tangential force is the sum of the shear forces acting on each slice.

6.2.2. Bishop's method

$$N' \cos \alpha = (W+Q) + (X_1 - X_2) - Ul \cos \alpha - T \sin \alpha$$

If the slope is not about to fail ($F > 1$), the shear force T is equal to the shear strength divided by F .

$$\tau = C' + \sigma'_n \tan \phi'$$

$$\tau^* l = C' l + \sigma'_n \tan \phi' l$$

$$\sigma'_n = \frac{N}{l}$$

$$F = \frac{C' l + N' \tan \phi'}{T} \text{ For a slice.}$$

$$\rightarrow T = \frac{C' l}{F} + \frac{N' \tan \phi'}{F}$$

By substituting T into the first equation, we obtain:

$$N' = [(W+Q) + (X_1 - X_2) - Ul \cos \alpha - \frac{C' l}{F} \sin \alpha] \frac{1}{\cos \alpha + \frac{\tan \phi' \sin \alpha}{F}}$$

The safety factor is:

$$F = \frac{\Sigma[C' l \cos \alpha + [(W+Q) - Ul \cos \alpha + (X_1 - X_2)] \tan \phi'] [\cos \alpha + (\tan \phi' \sin \alpha) / F]^{-1}}{\Sigma(W+Q) \sin \alpha}$$

According to Bishop, the calculations can be simplified if we assume $X_1 = X_2 = 0$, hence:

$$F = \frac{\Sigma[C' \cos \alpha + [(W+Q) - U \cos \alpha] \tan \varphi'] [\cos \alpha + (\tan \varphi' \sin \alpha) / F]^{-1}}{\Sigma(W+Q) \sin \alpha}$$

Since F appears on both sides of the equation, an iterative analysis is required to determine it. Computer programming allows for a fast solution after only a few cycles (generally 2 to 3), by assuming F = 1 on the right-hand side and then calculating the value on the left-hand side. This value is compared with the assumed one. If it is not sufficiently close, the computed safety factor F is used in the next iteration, and so on.

7.Reinforcement methods

Four types of actions can be undertaken to improve the overall stability of an excavation from a given initial state. These actions concern (Fig IV.10 to IV.14):

- Geometry: modification of the profile,
- Hydraulic regime: drainage
- Mechanical forces: support (shoring), rock bolting
- Soil nature: grouting, substratum treatment.

Moreover, it is necessary to ensure the long-term stability of the slope surface and protect it from erosion by covering it with appropriate vegetation

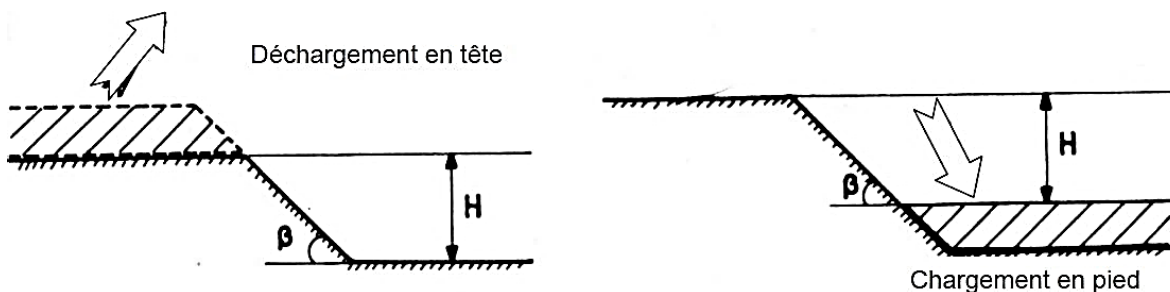


Fig .IV .10. Reduction of the height of a slope.

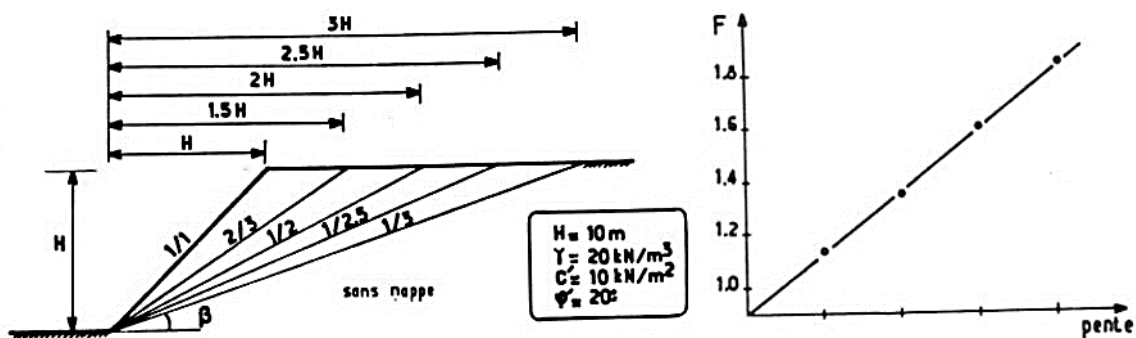


Fig .IV .11. Reduction of the slope.

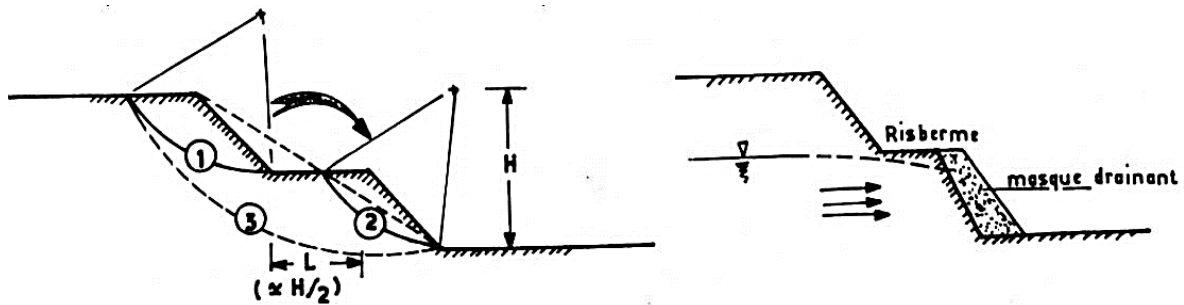


Fig.IV. 12. Construct berms.

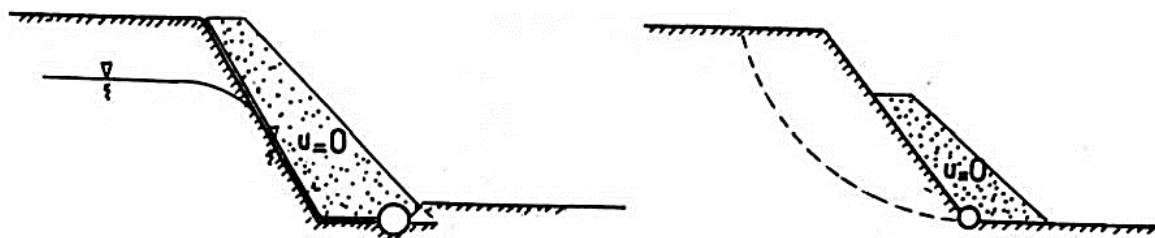


Fig .IV .13. Install a drainage layer.

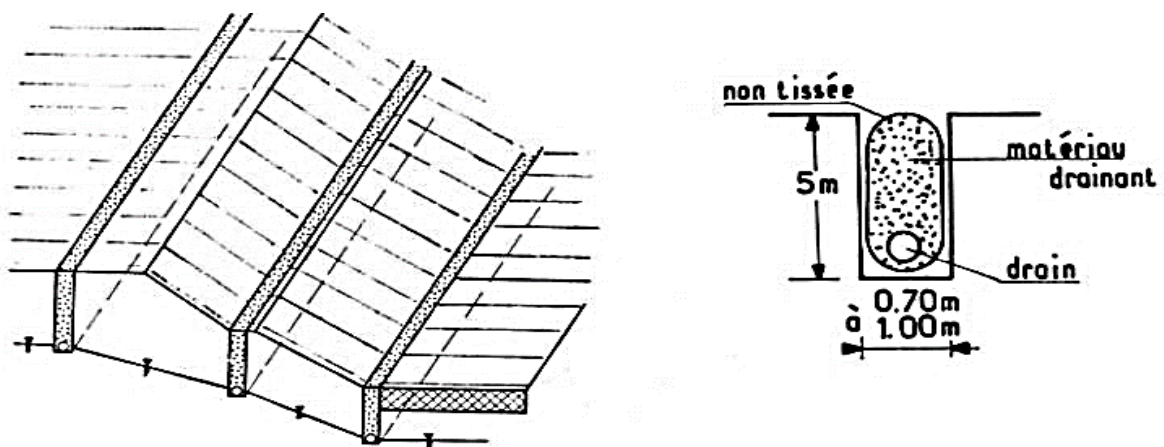


Fig .IV .14. Create drainage trenches.

8. Conclusion

Chapter IV provided a rigorous conceptual and methodological framework for the analysis and management of slope stability. He highlighted the complexity of the phenomena of rupture, distinguishing between intrinsic and extrinsic causes, and presented a taxonomy of mass movements. The safety factor (F), the ratio between the available shear strength and the mobilized stress, is the key parameter for evaluating stability ($F > 1$ for stable, $F \leq 1$ for risk of rupture). The chapter has detailed the analysis methods for infinite and finite slopes, including

global and slice methods (Fellenius, Bishop). Finally, he reviewed various reinforcement and stabilization techniques, such as geometric modifications, drainage, mechanical supports and vegetation, highlighting the need for a combined approach to guarantee the safety and sustainability of developments in geologically constrained environments.

Applications

Application 01

The fissured and overconsolidated clay slope has a long natural inclination of 12° with respect to the horizontal. The groundwater table is at the surface, and the seepage is practically parallel to the slope, extending to a depth of 5 m; $\gamma_{\text{sat}} = 20 \text{ kN/m}^3$. The effective shear strength parameters are $C' = 10 \text{ kN/m}^2$ and $\varphi' = 26^\circ$.

At the residual state, the shear strength parameters are $C_r = 0$ and $\varphi_r = 18^\circ$.

Determine the factor of safety (F_s):

- in terms of C' and φ'
- in terms of C_r and φ_r .

Solution Application 01

Infinite slope with seepage:
$$F = \frac{\frac{C'}{\cos\beta} + \gamma' H \cos\beta \tan\varphi'}{\gamma_{\text{sat}} H \sin\beta}$$

Term of C', φ' :

$$C' = 10 \quad \varphi' = 26^\circ$$

$$F = \frac{\frac{C'}{\cos\beta} + \gamma' H \cos\beta \tan\varphi'}{\gamma_{\text{sat}} H \sin\beta}$$

$$F = \frac{\frac{10}{\cos 12} + 10 * 5 * \cos 12 \tan 26}{20 * 5 * \sin 12}$$

$F = 1,64 > 1,5 \rightarrow$ The slope is stable.

Term of C_r, φ_r :

$$C_r = 0 \quad \varphi_r = 18^\circ$$

$$F = \frac{\gamma' H \cos\beta \tan\varphi_r}{\gamma_{\text{sat}} H \sin\beta} = \frac{\gamma' \tan\varphi_r}{\gamma_{\text{sat}} \tan\beta}$$

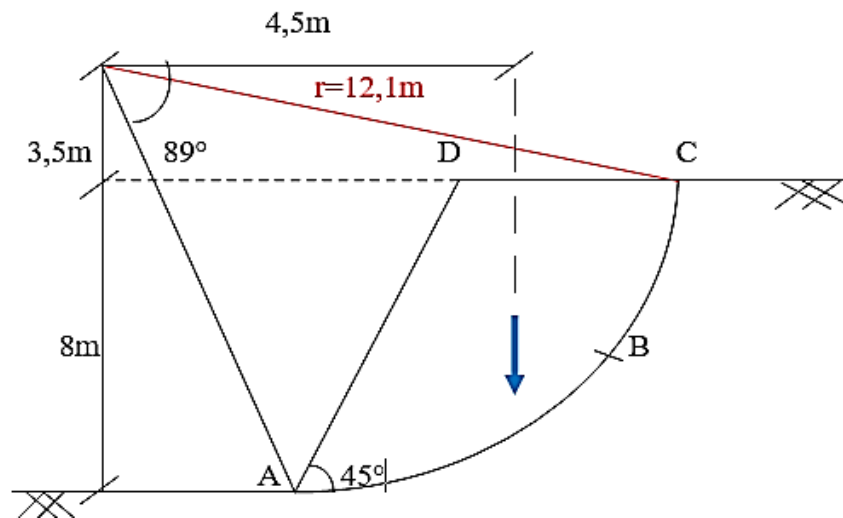
$$F = \frac{10 \tan 18}{20 \tan 12}$$

$F=0,76 < 1,5 \rightarrow$ The slope is unstable.

Application 02

A 45° slope is excavated in a layer of saturated clay with $\gamma = 19 \text{ kN/m}^3$. The shear strength parameters are $C_u = 65 \text{ kN/m}^2$ and $\phi_u = 0$.

Determine the factor of safety (F_s) for the specified slip surface, $S = 70 \text{ m}^2$.



Solution Application 02

$$\phi_u = 0 \quad C_u = 65 \text{ kN/m}^2$$

Global method:

$$F_s = \frac{C_u L_a R}{W \cdot a}$$

$$L_a = \frac{\theta 2 \pi R}{360} = \frac{89,5(2\pi \cdot 12,1)}{360}$$

$$L_a = 18,89 \text{ m}$$

$$W = 8S = 19 \cdot 70$$

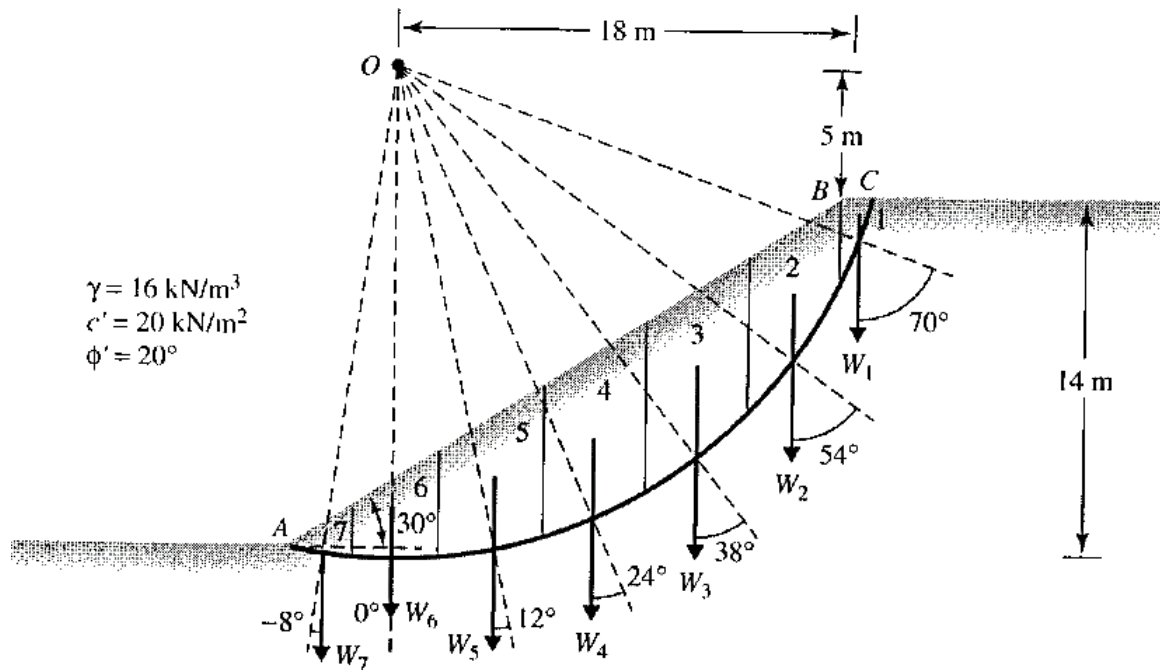
$$W = 1330 \text{ kN}$$

$$F_s = \frac{65 \cdot 18,89 \cdot 12,1}{1330 \cdot 4,5}$$

$F_s = 2,48 > 1,5 \rightarrow$ The slope is stable.

Application 03

Determination of the factor of safety using the method of slices for this figure below.



Solution Application 03

Slice no.	W (kN/m)	α_n (deg)	$\sin \alpha_n$	$\cos \alpha_n$	ΔL_n (m)	$W_n \sin \alpha_n$ (kN/m)	$W_n \cos \alpha_n$ (kN/m)
1	22.4	70	0.94	0.342	2.924	21.1	7.66
2	294.4	54	0.81	0.588	6.803	238.5	173.1
3	435.2	38	0.616	0.788	5.076	268.1	342.94
4	435.2	24	0.407	0.914	4.376	177.1	398.4
5	390.4	12	0.208	0.978	4.09	81.2	381.8
6	268.8	0	0	1	3.076	0	268.8
7	66.58	-8	-0.139	0.99	3.232	-9.25	65.9
Σ					Σ Col. 630.501	Σ Col. 7776.75 kN/m	Σ Col. 81638 kN/m

$$F_s = [(\Sigma \text{ Col. 6})(c') + (\Sigma \text{ Col. 8}) \cdot \tan \phi'] / (\Sigma \text{ Col. 7})$$

Numerical Application:

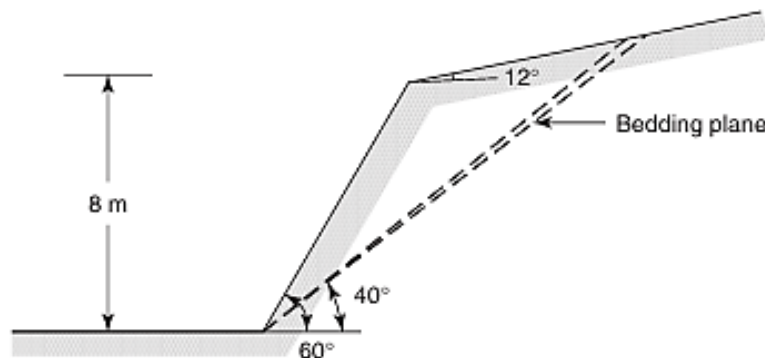
$$F_s = [(30.501)(20) + (1638)(\tan 20)] / 776.75 = 1.55.$$

Exercise problems**Exercise 1**

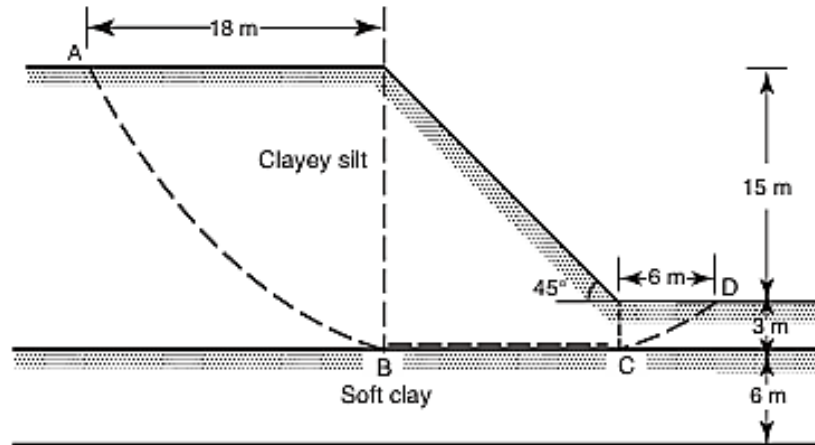
A 21° infinite slope consists of a uniform 5 m thick layer of sandy clay. At 5 m depth, a shale ledge runs parallel to the surface. A laboratory investigation on the sandy clay revealed the following properties: $c = 20$ kPa, $\phi = 15^\circ$, $\gamma = 18$ kN/m³. Compute the factor of safety against sliding on the shale and ledge if (i) no water exists at the top of the shale and (ii) the water level is at the surface of the slope.

Exercise 2

A sub-surface investigation on a 12° natural slope revealed the presence of bedding planes dipping toward the slope at an angle of 40° . A 60° cut slope is to be excavated to a depth of 8 m as shown in Fig below. Estimate the factor of safety of the slope. The shear strength parameters of the soil in the bedding plane are, $c = 15$ kN/m² and $\phi = 28^\circ$. The average unit weight of the soil, on the bedding plane and above, is 18.5 kN/m³.

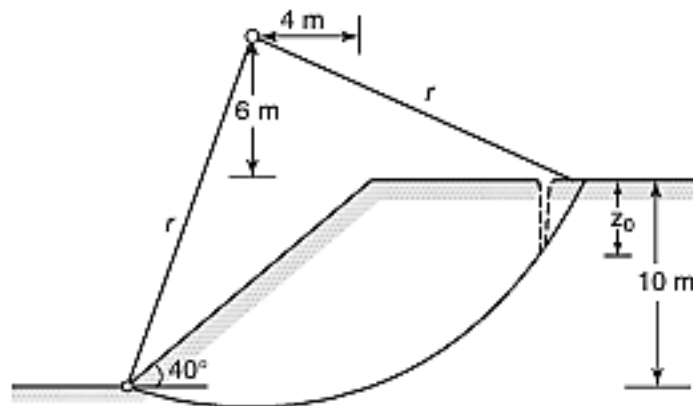
**Exercise 3**

40° A 45° cut was made in a clayey silt soil with $c' = 12$ kPa, $\phi = 30^\circ$, and $\gamma = 19.5$ kN/m³. A sub-surface exploration revealed the presence of a thin soft clay with $c = 13$ kPa and $\phi = 0^\circ$, at a depth of 18 m from the ground surface. Estimate the factor of safety of the slope against sliding along the composite slip surface, as shown in below.



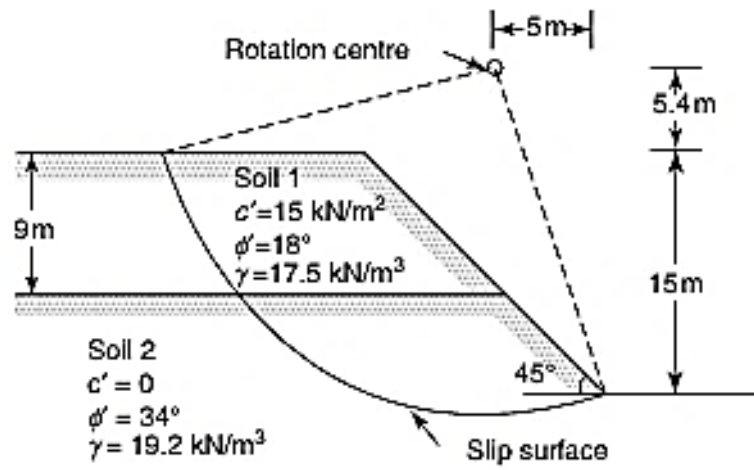
Exercise 4

A cutting in clayey soil is shown in Fig below. The undrained shear strength parameters are $c_u = 48 \text{ kN/m}^2$ and $\phi_u = 0^\circ$. The unit weight of the soil is 20 kN/m^3 . Compute the factor of safety against the slip surface shown when (i) no tension crack is formed, (ii) a tension crack exists with no water in the crack, and (iii) the tension crack is completely filled with water.



Exercise 5

For the soil slope and trial slip surface shown in Fig below, estimate the factor of safety adopting Bishop's simplified method. A preliminary approximate calculation for the slip surface, based on the Fellenius method, gave a factor of safety of 2.



Conclusion

The course Foundations and Geotechnical Structures (FGS) is designed to provide students with a clear and practical understanding of the fundamental principles of geotechnical engineering as applied to civil engineering structures. Through a progressive approach, the course enables students to develop the analytical tools and technical judgment necessary for the safe design of soil-supported structures. It introduces the concept of limit states, where students learn to evaluate soil failure conditions using equilibrium concepts and classical theories such as those of Rankine, Boussinesq, while also determining failure planes through Mohr's circle. The course then explores retaining structures, allowing students to understand their classification, the forces exerted by soils, and the procedures for verifying their global and local stability. In addition, students learn the principles governing the design of shallow foundations, applying classical bearing capacity theories such as those of Terzaghi and Meyerhof, while considering the effects of load eccentricity, foundation geometry, and soil conditions. The course also addresses slope stability, where students analyze natural and artificial slopes and determine safety factors using analytical approaches such as the global method and the method of slices, while accounting for the influence of groundwater and soil parameters. Through these topics, students strengthen their ability to analyze geotechnical problems, integrate soil-structure interaction into design, and apply stability and safety principles in engineering practice. Overall, this course provides a strong foundation for advanced studies in geotechnical engineering and prepares students to address real-world challenges related to the design and construction of safe, reliable, and durable infrastructure.

“Strong structures begin with strong foundations.”

Bibliography

Abramson, L. W., Lee, T. S., Sharma, S., & Boyce, G. M. (2002). Slope stability and stabilization methods. Wiley.

Bieth Emmanuel, M. Murs de soutènement : Cours de Mécanique des sols appliqués. ENTPE année 2009/2010.

Bouafia, A. (2009). Conception et calcul des ouvrages géotechniques. SAB Editions.

Bowles, J. E. (2006). Foundation analysis and design. McGraw-Hill.

Budhu, M. (2000). Soil mechanics and foundations. Wiley.

Cernica, J. N. (1985). Geotechnical engineering: Soil mechanics. Wiley.

Chatzigogos, C. (2007). Comportement sismique des fondations superficielles : Vers la prise en compte d'un critère de performance dans la conception (Doctoral dissertation). Laboratoire de Mécanique des Solides.

Chowdhury, R., Flentje, P., & Bhattacharya, G. (2010). Geotechnical slope analysis. CRC Press.

Coduto, D. P. (2004). Geotechnical engineering: Principles and practices. Prentice Hall.

Coduto, D. P. (2007). Foundation design: Principles and practices. Pearson.

Costet, J., & Sanglerat, G. (1969). Cours pratique de mécanique des sols (Vol. 2). Dunod.

Craig, R. F. (2004). Craig's soil mechanics. Taylor & Francis.

Craig, R. F. (2007). Craig's soil mechanics (7th ed.). Taylor & Francis.

Das, B. M. (1999). Shallow foundations: Bearing capacity and settlement. CRC Press.

Das, B. M. (2007). Principles of foundation engineering. Cengage Learning.

Das, B. M. (2008). *Advanced soil mechanics* (3rd ed.). Taylor & Francis.

Didier, D., Le Brazidec, M., Nataf, P., & Simon, G. (1998). *Précis structures de génie civil: Projets, dimensionnement, normalisation*. Nathan.

D.T.U 13-12 : Document Technique Unifié, Règles pour le calcul des fondations superficielles, Cahiers du Centre Scientifique et Technique de Bâtiment (CSTB) 1988.

Duncan, J. M., & Wright, S. G. (2005). *Soil strength and slope stability*. Wiley.

Eurocode 7. (2004). *Geotechnical design – General rules*.

Fang, H. Y. (2017). *Foundation engineering handbook*. Springer.

Fascicule 62 – Titre V : Règles techniques de conception et de calcul des fondations des ouvrages de génie civil. Paris, France.

Favre, J.-L. (2004). *Géotechnique : Sécurité des ouvrages et risques*. Ellipses.

Frank, R. (1996). *Fondations superficielles*. Presses de l'École Nationale des Ponts et Chaussées.

Frank, R. (2003). *Calcul des fondations superficielles et profondes*. Presses de l'École Nationale des Ponts et Chaussées.

Habib, P. (1997). *Génie géotechnique : Application de la mécanique des sols et des roches*. Ellipses.

Hatzor, Y., Lisjak, A., & Shi, G. (2017). *Discontinuous deformation analysis in rock mechanics practice*. CRC Press.

Holtz, R. D., & Kovacs, W. D. (2015). *An introduction to geotechnical engineering*. Pearson.

Lambe, T. W., & Whitman, R. V. (1969). *Soil mechanics*. Wiley.

Magnan, J. P. (1997). Capacité portante des fondations superficielles : Pressiomètre et essais de laboratoire. *Bulletin des Laboratoires des Ponts et Chaussées*, 211, 53–72.

Magnan, J. P., & Baghery, S. (1982). Statistiques et probabilités en mécanique des sols. Laboratoire Central des Ponts et Chaussées.

Meyerhof, G.G. (1951). The ultimate bearing capacity of foundations. Géotechnique.

Philipponnat, G. (1987). Fondations et ouvrages en terre. Eyrolles.

Philipponnat, G., & Hubert, B. (2002). Fondations et ouvrages en terre. Eyrolles.

Rankine, W.J.M. (1857). On the stability of loose earth. Philosophical Transactions of the Royal Society.

Rao, N. S. V. K. (2011). Foundation design: Theory and practice. Wiley.

Salençon, J. (1983). Calcul à la rupture et analyse limite. Presses de l'ENPC.

Schlosser, F. Techniques de l'Ingénieur : Murs de soutènement. Traité construction Volume C 244-2. P23, Paris.

Terzaghi : Terzaghi, K. (1943). Theoretical Soil Mechanics. Wiley.

Vesic, A. S. (1975). Bearing capacity of shallow foundations. In H. Y. Fang (Ed.), Foundation engineering handbook. Van Nostrand Reinhold.
