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Department: Electrical Engineering

Educational Handout

Title:

Fundamental Electrical Engineering 01

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This document is primarily intended for students in the second year of the Electrical Engineering degree. The curriculum aligns with the objectives of the fundamental electrical engineering module I, which encompasses the comprehensive examination of the electrical system, starting from dipoles and electrical regimes to the conversion of electrical energy through transformers and electrical machines. The purpose of teaching this module is to familiarize students with the concepts specific to electrical engineering in order to allow them to continue their university training in electrical engineering, electronics and automation…..

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List of Symbols

z	Complex number
z*	Complex conjugate
ρ	Modulus (length)
Ø	Angle (phase)
R	Resistance
Ω	Ohm
W	Watt
s	Second
F	Farad
H	Henry
Wb	Weber
J	Joule
C	coulomb
kg	Kilogram
m	Meter
Y_{eq}	Equivalent admittance[Ω^{-1}]
R_{eq}	Equivalent resistance [Ω]
L	Inductors [H]
C	Capacitors [F]
i	Curent [A]
v(t)	Voltage[V]
P	Active power[W]
Q	Reactive power [VAR]
S	Apparent power [V .A]
cos(φ)	Power factor
H	Intensity of the magnetic field [A. m−1]
N	Number of turns [T]
Fmm	Magnetomotive force [AT: ampere-turn]
μ	Permeability [$H.m^{-1}$]
μ_0	Vacuum permeability= $4\pi.10^{-7}\ Wb/\ A.m$
μ_r	Relative permeability of the material
ℜ	Rulctance [AT/Wb]
Ø	Flux [m.Wb]
H	Strength of the magnetizing field [AT/m]

Introduction

Introduction

This document is primarily intended for students in the second year of the Electrical Engineering degree. The curriculum aligns with the objectives of the fundamental electrical engineering module I, which encompasses the comprehensive examination of the electrical system, starting from dipoles and electrical regimes to the conversion of electrical energy through transformers and electrical machines. The purpose of teaching this module is to familiarize students with the concepts specific to electrical engineering in order to allow them to continue their university training in electrical engineering, electronics and automation.

This document is divided into six distinct sections. In the initial section, we will provide an overview of intricate numerical concepts. We will examine the role of complex numbers in electricity and how an electrical quantity can be represented by a complex number. The second section, entitled "Reminder on the fundamental laws of electricity," is devoted to the identification of electric dipoles (electrical components) and the investigation of circuits in the continuous, variable, and transient regimes. We will focus on the study of electrical energy in the form of single-phase (2-wire) or three-phase (3 or 4-wire) alternating voltages and currents. We will emphasize the general laws linking the various quantities: powers, intensities, voltages, impedances, etc.

The fourth section, Magnetic Circuits, is devoted to the study of the main concepts of magnetic circuits. We will present the organization of an electric cable and a ferromagnetic material in a similar magneto-electric circuit, in order to allow an understanding of the operation of electrical machines. The last part, Transformer, is devoted to the study of a static machine (transformer). This machine, based on a magnetic circuit, which allows to modify the voltage level, is widely used in electronics. The last part will be devoted to the study of direct current electrical machines. These (motors or generators) are made up of electrical circuits (conductors) closely linked in a magnetic circuit, etc.

Chapter 1:

Reminder About Complex Numbers

Chapter 1:

Reminders about Complex Numbers

1.1 Introduction

Complex numbers can be understood as a specific type of two-dimensional vector, comprising a real component and an imaginary component. They serve as an extension of the real number system into the complex plane, thereby offering a broader and more versatile mathematical framework. In both mathematics and physics, complex numbers play a pivotal role due to their capacity to encode two-dimensional information in a compact algebraic form. This added dimensionality often facilitates more efficient analytical approaches and can lead to the derivation of solutions that are otherwise inaccessible using only real numbers. For instance, complex analysis provides powerful techniques for evaluating definite integrals, solving differential equations, and modeling oscillatory behavior, such as in quantum mechanics, signal processing, and electrical engineering. Their utility stems not only from their algebraic properties but also from their rich geometric interpretation, which provides deeper insights into the structure and dynamics of a wide range of physical and mathematical systems.

1.2 Form of complex numbers

Complex numbers can be introduced in the following component form:

$$z = u + iv \qquad (1.1)$$

Where *u* belongs to the real numbers and *v* also belongs to the real numbers. A special number is the one called the imaginary number. Where:

$$i^2 = -1 \qquad (1.2)$$

The real and imaginary parts are:

$$u = Re[z] \quad \text{and} \quad v = Im[z] \qquad (1.3)$$

The modulus or absolute value of a complex number is defined by:

$$|z| = \sqrt{u^2 + v^2} \qquad (1.4)$$

Complex conjugate z^* of a complex number $z = u + iv$ is defined by:

$z^* = u - iv$ (1.5)

1.3 Addition and subtraction of complex numbers

Addition and subtraction of complex numbers are defined component-by:

$z_1 \pm z_2 = u_1 \pm u_2 + i(v_1 \pm v_2)$ (1.6)

And the commutation and association properties are fulfilled,

$z_1 + z_2 = z_2 + z_1$ (1.7)

$(z_1 + z_2) + z_3 = z_2 + (z_1 + z_3)$ (1.8)

1.4 Product of a complex number

If we have two complex numbers, $z_1 = u_1 + iv_1$, and $z_2 = u_2 + iv_2$ then multiplication of z_1 and z_2 is written as:

$z_1 z_2 = (u_1 + iv_1)(u_2 + iv_2) = u_1u_2 - v_1v_2 + i(u_1v_2 + u_2v_1)$ (1.9)

$Re[z_1 z_2] = u_1u_2 - v_1v_2$, $Im[z_1 z_2] = u_1v_2 + u_2v_1$ (1.10)

The product of a complex number and its complex conjugate is a real number:

$z z^* = (u + iv)(u - iv) = u^2 - i^2v^2 = u^2 + v^2$ (1.11)

I.5 Division of complex numbers

To divide two complex numbers, we start by multiplying the numerator and denominator by the conjugate of the denominator. This step eliminates the imaginary part of the denominator.

$\frac{z_1}{z_2} = \frac{z_1 z_2^*}{z_2 z_2^*} = \frac{z_1 z_2^*}{[z]^2} = \frac{u_1u_2 + v_1v_2 + i(v_1u_2 - u_1v_2)}{u^2 + v^2}$ (1.12)

I.6 Trigonometric & exponential form

Just like vectors in *2D*, complex numbers can be described by their modulus (length) ρ and their angle (phase) $\emptyset$ as follows :

$z = \rho\left(Cos(\emptyset) + i\ Sin(\emptyset)\right)$ (1.13)

Where

$\rho = [z]$ (1.14)

$\text{Cos}\ (\emptyset) = \frac{a}{|z|}$ (1.15)

$\text{Sin}\ (\emptyset) = \frac{b}{|z|}$ (1.16)

This formula can be brought into a more compact and elegant shape;

$z = \rho e^{i\emptyset}$ (1.17)

Where

$e^{i\emptyset} = Cos(\emptyset) + i\ Sin(\emptyset)$ (1.18)

So if you have two complex numbers $z_1 = \rho_1 e^{i\emptyset_1}$ and $z_2 = \rho_2 e^{i\emptyset_2}$ multiplication rule:

$z_1 z_2 = \rho_1 e^{i\emptyset_1} \rho_2 e^{i\emptyset_2} = \rho_1\, \rho_2 e^{i(\emptyset_1+\emptyset_2)} = \rho_1\, \rho_2 (Cos(\emptyset_1 + \emptyset_2) + i.Sin(\emptyset_1 + \emptyset_2))$ (1.19)

In particular,

$Re[z_1 z_2] = \rho_1\, \rho_2 Cos(\emptyset_1 + \emptyset_2)$ (1.20)

$Im[z_1 z_2] = \rho_1\, \rho_2 Sin(\emptyset_1 + \emptyset_2)$ (1.21)

For division rule:

$\frac{z_1}{z_2} = \frac{\rho_1}{\rho_2} e^{i(\emptyset_1 - \emptyset_2)}$ (1.22)

$z^2 = \rho^2 (Cos(\emptyset) + i\ Sin(\emptyset))^2 = \rho^2 (Cos(\emptyset)^2 - \ Sin(\emptyset)^2 + 2\ i\ Cos(\emptyset)\ Sin(\emptyset))$ (1.23)

$z^2 = \ \rho^2 e^{2i\emptyset} = \ \rho^2 \big(\ Cos(2\emptyset) + i\ Sin(2\emptyset)\big)$ (1.24)

By putting the real and imaginary parts of these two formulas on the same plane, we obtain the following trigonometric identities:

$Cos(2\emptyset) = Cos(\emptyset)^2 + Sin(\emptyset)^2$ (1.25)

$Sin(2\emptyset) = 2Sin(\emptyset).Cos(\emptyset)$ (1.26)

One can derive formulas for *Sin* and *Cos* of any multiple arguments with this method.

I.7 Geothermal representation of complex numbers

Just like vectors in *2D*, complex numbers can be represented by fig 1.1 .

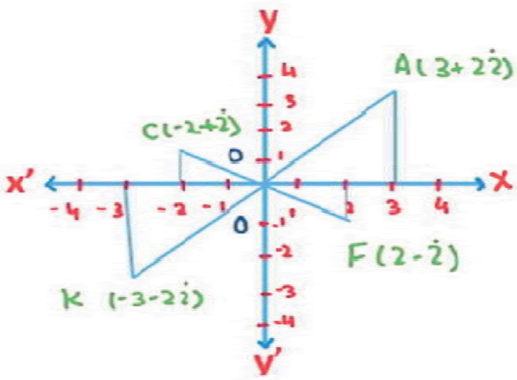

Fig 1.1 Geothermal representations of complex numbers

In this illustration, the x-axis is referred to as the real axis , while the *y* axis is referred to as the imaginary axis . The illustration that represents more complex numbers on the complex plane is known as an Argand diagram . The points on the *x* axis symbolize real numbers, while those on the *y* axis correspond to imaginary numbers. *x* and *y* illustrate the complex number $x + iy$. And $\sqrt{(a^2 + b^2)}$ is the modulus of the complex number $a + ib$.

In the right-angled triangle *OMA*, we have, by Pythagoras theorem.

$$|\overline{OA}|^2 = |\overline{OM}|^2 + |\overline{OA}|^2 \; and \;\; |OA| = \sqrt{x^2 + y^2} \tag{1.26}$$

$$\overline{MA} \perp \overline{OX} \;\; \text{and} \; \overline{OM} = x \, , \overline{MA} = y \tag{1.27}$$

The polar form of a complex number consider adjoining representing the complex number $z = x + iy$

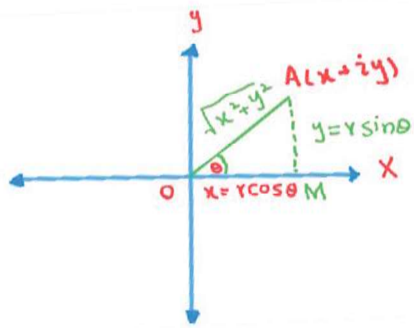

Fig 1.2 Polar form of a complex number

From the diagram, we see that $x = \rho.cos\theta$ and $y = \rho.sin\theta$ where $\rho = |z|$ and θ is called arguments of z.

I.8 Moivre formula

In mathematics, de Moivre's formula states that for any real number x and integer n it is the case that:

$$\rho e^{i\theta} = \rho(cos\theta + isin\theta) \quad (1.28)$$

$$\rho^n e^{in\theta} = \rho^n(cosn\theta + isinn\theta) \quad (1.29)$$

I.9 Euler's formula

$$Cos(x) = \frac{e^{ix}+e^{-ix}}{2} \quad (1.30)$$

$$Sin(x) = \frac{e^{ix}-e^{-ix}}{2i} \quad (1.31)$$

TD N°1 : Exercise in Complex Numbers (CN)

Exercise 1.1

Write in the form x + iy:

a- $\frac{1}{3+3i}$;

b- $\frac{1}{i^3}$;

c- $i(1+i)(1-i)^2$.

Exercise 1.2

Write in the polar and the exponential polar form:

a- $\frac{1}{2+2i}$;

b- $-1+i\sqrt{3}$

c- $\sqrt{1+i}$

Exercise 1.3

Consider the complex number z with modulus ρ, argument θ and complex conjugate $\overline{z}$

1- Calculate the inverse of z for $z = 2 + i5$

2- If $z = a+jb$; What is the solution to $2z + \overline{z} = 6 + i2$

3- Put the following complex numbers in algebraic form :

a- $z_1 = \frac{1-i2}{3+i}$;

b- $z_2 = \frac{(3+i5)^2}{1-i2}$;

c- $z_3 = \left(\frac{1+i}{2-i}\right)^2 + \frac{3+i6}{3-i4}$

4- Perform the following operations; *(3 + 2i) (1 − 3i)*.

Exercise 1.4

Deduce the module and the argument of the following complex numbers and then put in trigonometric form :

a- $z_1 = 1 + \sqrt{3}i$;

b- $z_2 = \sqrt{3} + i$;

c- $z_1 + z_2$;

d- $z_1{}^{27}$.

Exercise 1.5

Solve $z^4 + 16 = 0$ for complex z,

Give all roots (solutions) of $z^2 + Z + 1 = 0$.

Exorcise 1.5

Use Euler's formulas to transform the following expression into a sum;

$f(x) = sin(2x).sin(x)$

$F(x) = sin^2(x)$

Exercise 1.6

A dipole carries a current $\mathrm{i(t)} = 2\sqrt{2}sin(314t + 6\pi)$ when subjected to voltage

$u(t) = 220sin(314t)$

- Determine the impedance of this dipole.

Chapter 2:

Reminders on the Fundamental Laws of Electricity

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2.1 Introduction

A solid grasp of the basic laws of electricity is essential for any student beginning studies in electrical or electronic engineering. These fundamental principles provide the foundation for understanding how electric circuits function and are crucial for more advanced topics that will be encountered throughout the curriculum. This chapter offers a clear and concise review of the fundamental laws of electricity, including Ohm’s Law and Kirchhoff’s Laws, as well as key electrical quantities such as current, voltage, resistance, and power. By revisiting these core concepts, students will strengthen their analytical skills and develop the necessary tools to successfully approach more. In this chapter, we will discuss the main electric dipoles and the fundamental laws that govern them.

2.2 Continuous regime

In continuous mode, the current and voltage quantities are constant over time.

2.2.1 Electric dipole

An electric dipole is a single component or a set of components, connected to two (02) terminals (see figure 2.1).We place a meaning for the Koran.

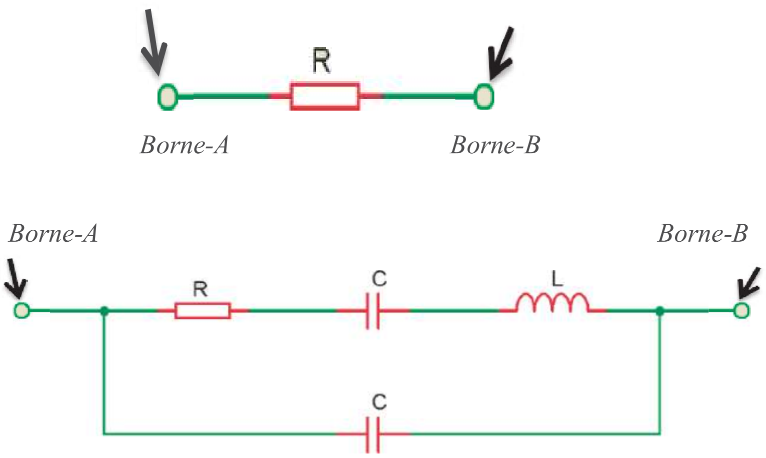


Fig 2.1 Electric dipoles

Receiver convention: current i and voltage u are oriented in opposite directions.
Generator convention: current i and voltage u are oriented in the same direction.

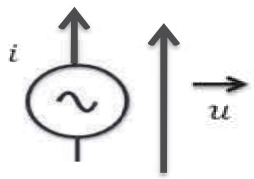


Fig. 2.2 Generator conventions

Passive dipole: It is a dipole which consumes electrical energy and does not contain any energy source. Examples include: resistance, inductance, bulb…..

Fig 2.3 Passive dipole

Active dipole: It is a dipole which contains a source of energy. For example, we can cite battery, or direct current electric motor.

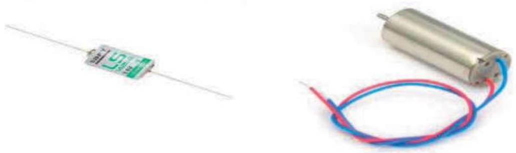


Fig 2.4 Active dipole

2.2.2 Properties of dipoles

2.2.2.1 Polarity of dipoles

A dipole is polarized when its terminals cannot be swapped, for example: chemical capacitor, direct current generator, diode, etc. If the terminals are reversed, operation can be disrupted of the circuit. For a non-polarized dipole, the permutation of their terminals does not influence the operation of the circuit. The resistor is a non-polarized dipole.

2.2.2.2 Linearity of dipoles

A dipole is linear when it meets the mathematical criteria of linearity. The current/voltage characterization is a straight line. A pure resistor is a linear dipole, on the other hand the diode is a non-linear dipole.

2.2.3 Association of dipoles

In an electrical circuit, dipoles can be associated in series or in parallel.

A. Dipoles in series : Dipoles are associated in series when they are connected one after the other. The current i is common to all dipoles. The voltage u is the sum of the voltages across each dipole.

B. Dipoles in parallel : The voltage u is common to all dipoles. The total current i is the sum of the currents across each dipole.

2.2.4 Association of elementary dipoles R, *L* and *C*

2.2.4.1 Association of resistances (*R*) in series

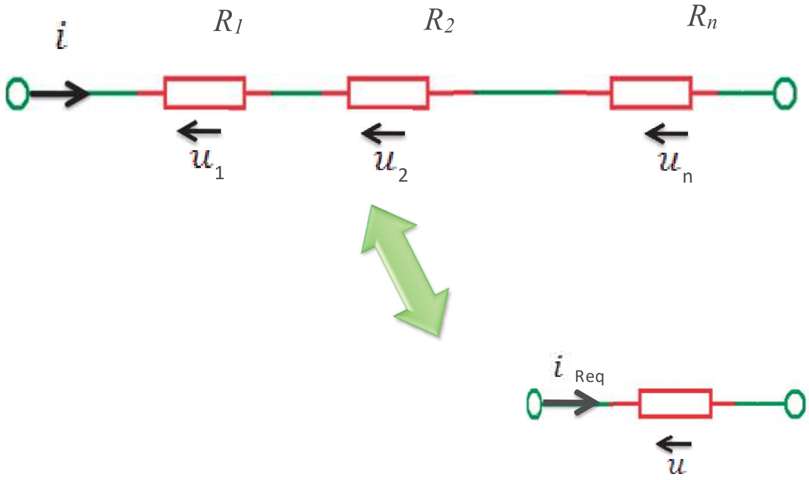


Fig.2.5 Association of resistances (R) in series

$$u = u_1 + u_2 + u_3 + \cdots + u_n = (R_1 + R_2 + R_3 + \cdots + R_n).i = R_{eq}.i \quad (2.1)$$

The equivalent resistance is then equal to the sum of the resistances placed in series. Its unit is *Ω*.

$$R_1 + R_2 + R_3 + \cdots + R_n = \sum_1^n R_n \quad (2.2)$$

2.2.4.2 Association of resistances (*R*) in parallel

In parallel, the voltage is common to all resistors. The current which enters the whole is given, according to the law of nodes, by:

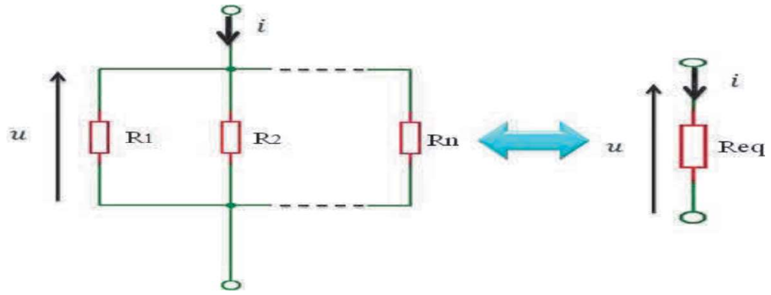


Fig.2.6 Association of resistances (R) in parallel

$$i = i_1 + i_2 + i_3 + \cdots + i_n = \frac{u}{R_1} + \frac{u}{R_2} + \frac{u}{R_3} + \cdots + \frac{u}{R_n} \qquad (2.3)$$

$$i = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}\right) u \qquad (2.4)$$

$$i = \frac{1}{R_{eq}} . u \qquad (2.5)$$

The equivalent admittance is equal to the sum of the reciprocals of the resistances placed in parallel. Its unit is Ω^{-1}.

$$Y_{eq} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n} = \sum_1^n \frac{1}{R_n} \qquad (2.6)$$

- Case of 2 resistors placed in parallel

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \qquad (2.7)$$

2.2.4.3 Association of inductors (L) in series

Associating inductors in series means increasing the total number of turns. The voltage across an inductor crossed by a current of variable intensity as a function of time is given by:

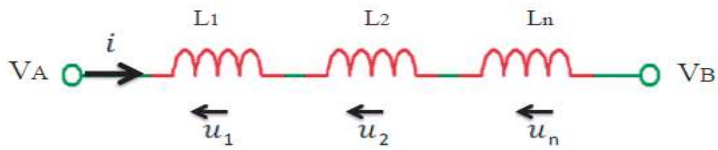


Fig.2.7 Association of inductors (L) in series

$$u_L = L \frac{di}{dt} \qquad (2.8)$$

$$V_A - V_B = u_1 + u_2 + u_3 + \cdots + u_n \qquad (2.9)$$

$$V_A - V_B = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + \cdots + L_n \frac{di}{dt} = (L_1 + L_2 + \cdots + L_n) \frac{di}{dt} = L_{eq} \frac{di}{dt} \qquad (2.10)$$

The equivalent inductance is then equal to the sum of the inductances placed in series. (It is assumed that the current has the same direction of flow in the coils).

$L_1 + L_2 + \cdots + L_n = \sum_1^n L_n$ (2.11)

2.2.4.4 Association of inductors (L) in parallel

In parallel, the voltage is common to all inductors. The current that enters the whole is (law of knots):

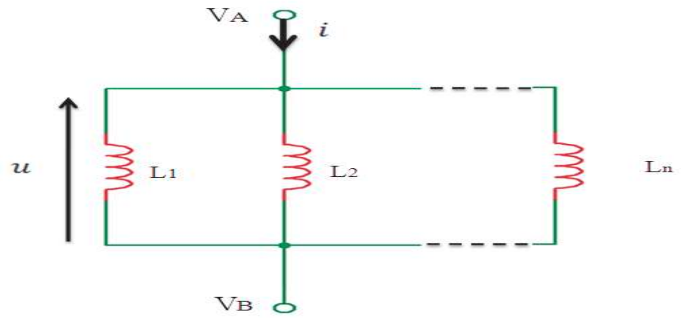


Fig.2.8 Association of inductors (L) in parallel

$i = i_1 + i_2 + i_3 + \cdots + i_n$ (2 .12)

$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt} + \cdots + \frac{di_n}{dt} = \frac{V_A - V_B}{L_1} + \frac{V_A - V_B}{L_2} + \frac{V_A - V_B}{L_3} + \cdots + \frac{V_A - V_B}{L_n}$ (2 .13)

$\frac{di}{dt} = (V_A - V_B)\left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_n}\right) = (V_A - V_B)\left(\frac{1}{L_{eq}}\right)$ (2 .14)

The equivalent admittance is equal to the sum of the inductances placed in parallel:

$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_n} = \sum_1^n \frac{1}{L_n}$ (2 .15)

2.2.4.5 Association of capacitors (C) in series

A capacitor is characterized by its capacitance, denoted C and expressed in Farads (symbol F).The voltage across a capacitor crossed by a current of variable intensity as a function of time is:

$u_C = \frac{1}{C} \int i.dt$ (2 .16)

In the figure 2.9 , we see a series association fed by a voltage source V. The sum of the voltage drops in the capacitors shall be equal to the voltage V of the source.

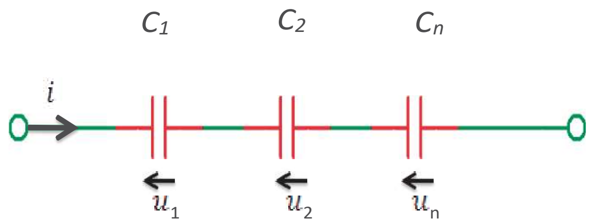


Fig.2.9 Association of capacitors (C) in series

$$u = u_1 + u_2 + u_3 + \cdots + u_n \quad (2.17)$$

$$u = \frac{1}{C_1}\int i.dt + \frac{1}{C_2}\int i.dt + \frac{1}{C_3}\int i.dt + \cdots + \frac{1}{C_n}\int i.dt \quad (2.18)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n} = \sum_1^n \frac{1}{C_n} \quad (2.19)$$

As you will no doubt notice, this is exactly the opposite of the phenomenon exhibited by resistors.

2.2.4.6 Association of capacitors (C) in parallel

In parallel, the voltage is common to all capacitors. The current (see figure 2.10) which enters the whole is (law of knots):

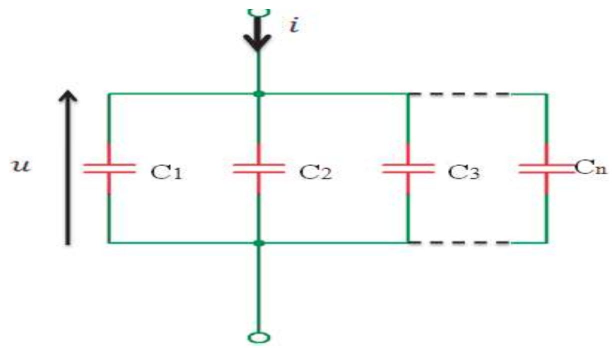


Fig. 2.10 Association of capacitors (C) in parallel

Here the current is common to all capacitors. The voltage across the assembly is:

$$i = i_1 + i_2 + i_3 + \cdots + i_n \quad (2.20)$$

$$i = C_1\frac{du}{dt} + C_2\frac{du}{dt} + C_3\frac{du}{dt} + \cdots + C_n\frac{du}{dt} \quad (2.21)$$

$$i = (C_1 + C_2 + C_3 + \cdots + C_n)\frac{du}{dt} \quad (2.22)$$

$$C_1 + C_2 + C_3 + \cdots + C_n = \sum_1^n C_n \quad (2.23)$$

2.3 Transitional regime

A transient regime is the evolution regime of a system that has not yet reached its permanent regime. It is characterized by a characteristic duration τ, called relaxation time (also called time constant).

2.3.1 RL Transitional regime

Consider the circuit in the figure. At *t=0*, we close the switch *K*. For $t < 0$: $i(t)=0$. For $t > 0$, the mesh law is written: which make up the circuit $\tau = L/R$.

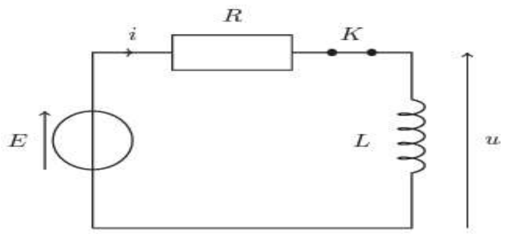


Fig.2.11 Circuit *RL*

$$E = Ri + L\frac{di}{dt} \qquad (2.24)$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L} \qquad (2.25)$$

The solution to this equation is written:

$$i(t) = Ae^{\frac{-t}{\tau}} + \frac{E}{R} \qquad (2.26)$$

With $i(t=0)=0$; so : $A = -\frac{E}{R}$

$$i(t) = \frac{E}{R}(1 - e^{\frac{-t}{\tau}}) \qquad (2.27)$$

and $u(t) = L\frac{di}{dt} = Ee^{\frac{-t}{\tau}}$ (2.28)

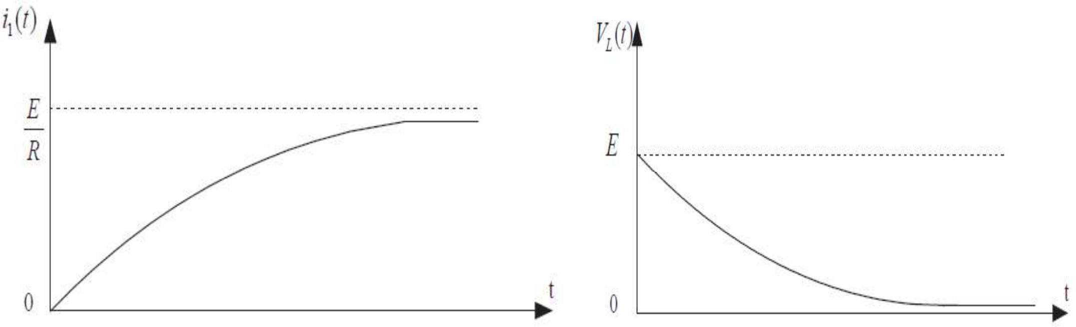


Fig.2.12 Current and voltage variations in the inductance

2.3.2 RC Transitional regime

The RC circuit is made up of a generator, a resistor and a capacitor. In their series configuration, *RC* circuits make it possible to produce low-pass electronic filters or high pass. The time constant τ of an *RC* circuit is given by the product of the value of these two elements which make up the circuit $\tau = RC$.

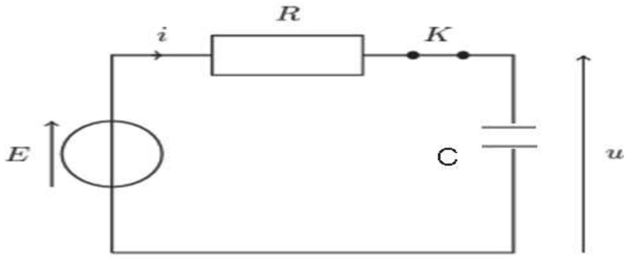


Fig.2.13 RC Circuit

We flip the switch to position *1* thus, we apply a voltage *E* across the capacitor.

$E = RI + U_C$ and $I = C\frac{du_c}{dt}$ (2 .29)

$E = RC\frac{du_c}{dt} + U_C \longrightarrow \frac{du_c}{dt} + \frac{1}{RC}U_C = \frac{E}{RC}$ (2 .30)

The solution to this equation is:

$U_C(t) = Ae^{\frac{-t}{\tau}} + E$ (2 .31)

If $t=0$ $U_c = 0$ *So* $A = -E$

$U_C(t) = E(1 - e^{\frac{-t}{\tau}})$ (2 .32)

$i_1(t) = C\frac{du_c}{dt} = C\frac{d}{dt}\left[E\left(1 - e^{\frac{-t}{RC}}\right)u(t)\right]$ (2 .33)

$i_1(t) = \frac{E}{R}\, e^{\frac{-t}{RC}}\, u(t)$ (2 .34)

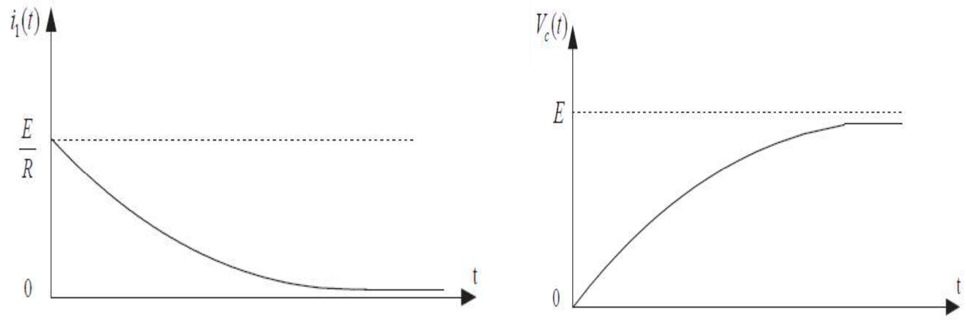


Fig.2.14 Current and voltage variations in the capacitor

2.3.3 RLC transitional regime

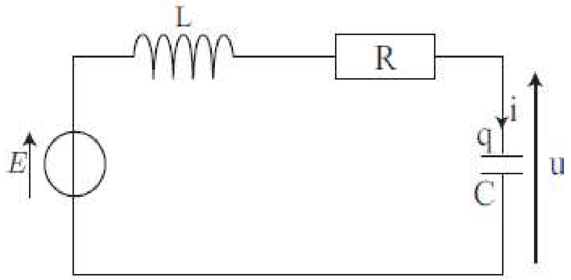


Fig.2.15 RLC Circuit

According to the law of meshes we have:

$-E + Ri + L\frac{di}{dt} + \frac{1}{C}\int idt = 0$ (2.35)

Let's look for the differential equation, consider the voltage and current across the capacitor:

$i = \frac{dq}{dt}$; $q = CU_c$ Where $i = C\frac{dU_C}{dt}$ (2.36)

$E = RC\frac{dU_C}{dt} + LC\frac{d^2U_C}{dt^2} + U_C$ (2.37)

$\frac{d^2U_C}{dt^2} + \frac{R}{L}\frac{dU_C}{dt} + \frac{1}{LC}U_C = \frac{E}{LC}$ (2.38)

$\frac{d^2U_C}{dt^2} + 2\lambda\omega_0\frac{dU_C}{dt} + \omega_0^2U_C = \omega_0^2E$ (2.39)

Where $\omega_0^2 = \frac{1}{LC}$ proper pulsation as the frequency at which this system oscillates when it is in free evolution so : $2\lambda\omega_0 = \frac{R}{L}$ and $\omega_0^2 = \frac{1}{LC}$.

$\Delta = 4\omega_0^2(\lambda^2 - 1)$ (2.40)

1- If $\Delta < 0$ ⟶ $\lambda < 1$ Pseudo-periodic regime

2- If $\Delta > 0$ ⟶ $\lambda > 1$ A-periodic regime

3- If $\Delta = 0$ ⟶ $\lambda = 1$ critical regime

Figure 2.16 shows the response of the *RLC* circuit to a voltage step for the 3 different cases

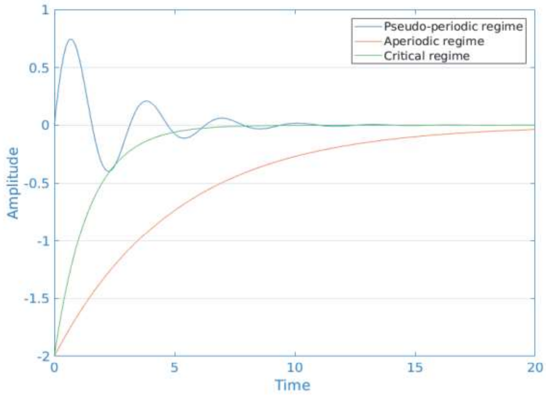


Fig.2.16 Pseudo-periodic, a-periodic and critical voltage for the transient response

2.4 Harmonic regime (Sinusoidal)

We call sinusoidal regime (or harmonic regime) the state of a system for which the variation over time of the quantities characterizing it is sinusoidal. The electrical circuit, in this case, is powered by a sinusoidal alternating voltage; *V(t)* and traversed by a sinusoidal alternating current *i(t)*.

2.4.1 Alternating current

A sinusoidal alternating current is a periodic bidirectional current. The same is true for a sinusoidal alternating voltage.

Voltage: $u(t) = U_M \sin(wt + \Phi_u)$ and Current, $i(t) = I_M \sin(wt + \Phi_i)$

With:

u(t): Instantaneous value, U_M **:** Maximum value (*V*);

($wt + \Phi_u$) :Instantaneous phase *(rd)*; *(w)*:Pulsation. **(**Φ_u) and (Φ_i) :Phase shift relative to the phase origin; $\Delta\phi = \Phi_u - \Phi_i$:is the phase shift between current and voltage.

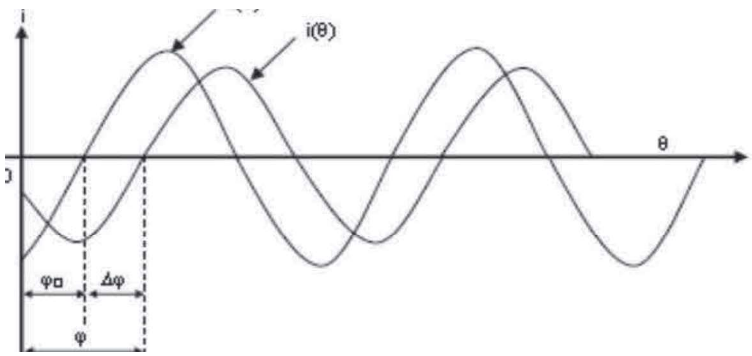


Fig.2.17 Alternating current

2.4.2Average values of sinusoidal current

We have: $i(t) = I_M \sin(\omega t + \emptyset_i)$ (2 .41)

$I_{moy} = \frac{1}{T}\int_0^T i(t)dt = \frac{1}{T}\int_0^T I_M \sin(\omega t)\, dt = \frac{I_M}{T}\left[\frac{-\cos(\omega t)}{\omega}\right]_0^1 = \frac{I_M}{T}\left[\frac{-\cos(\omega t)}{\omega}\right]_0^1$ (2 .42)

$I_{moy} = \frac{I_M}{T}[\cos(\omega t) - \cos(0)\,]$ (2 .43)

$I_{moy} = \frac{I_M}{2\pi}[1 - 1\,] = 0$ (2 .44)

2.4.3 RMS values of sinusoidal current

$I_{eff}^2 = \frac{1}{T}\int_0^T I^2(t)dt = \frac{2}{T}\int_0^{T/2} I^2(t)dt = \frac{2}{T}\int_0^{T/2} I_M^2 sin^2(\omega t)dt$ (2 .45)

$I_{eff}^2 = \frac{2I_M^2}{T}\int_0^{T/2}\frac{1-cos2\omega t}{2}dt = \frac{2I_M^2}{2T}\left[t - \frac{sin\omega t}{2\omega}\right]_0^{\frac{T}{2}} = \frac{I_M^2}{2}$ (2 .46)

$I_{eff} = \frac{I_M}{\sqrt{2}}$; $U_{eff} = \frac{U_M}{\sqrt{2}}$ (2 .47)

2.4.4 Fresnel representation

Or a sinusoidal quantity $S_{eff}\sqrt{2}\sin(wt + \varphi)$.This quantity can be represented at each moment by a vector $\bar{G}$ called Fresnel vector associated with the sinusoidal quantity *g(t)*.We choose an axis of origin of the phases and we represent the vector. The vector rotates with a constant speed ω in the trigonometric direction, the interest of the Fresnel representation is to separate the temporal part (wt) from the part phase (φ). Let the signal be:

$S(t) = S_M sin(wt+\varphi) = \sqrt{2}\, S_{eff} sin(wt+\varphi)$

Which can be a voltage or a current. This signal can be represented by a vector $\overrightarrow{OM}$ the module S_{eff} placed relative to the axis (*OX*) origin of the phases, such that φ = angle between axis $(OX)$and the vector $\overrightarrow{OM}$

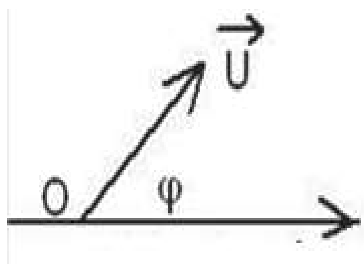


Fig.2.18 Fresnel vector

In electricity, this representation will easily make it possible to find the sum vector of two other.

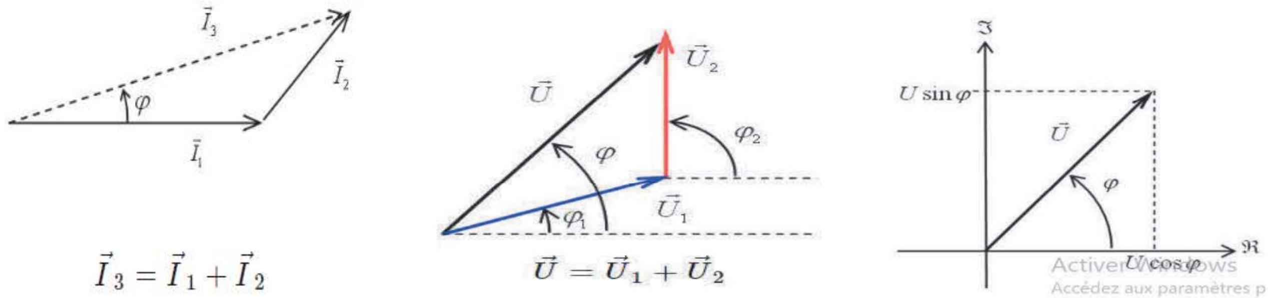


Fig.2.19 Fresnel representation

2.4.5 Complex notation

Let's be the voltage: $u(t) = U_M \sin(wt+\varphi_u)$: and the curent, $i(t) = I_M \sin(wt+\varphi_i)$ **.** We can associate complex numbers with them in the form

$\underline{U} = Ue^{j\varphi_u}$ and $\underline{I} = Ie^{j\varphi_i}$ (2 .48)

And

$\underline{U} = Ue^{j\varphi_u}$; $\underline{U} = \underline{Z} \cdot \underline{I}$; $\underline{I} = \underline{Y} \cdot \underline{U}$ (2 .49)

2.4.6 Determination of elementary dipole impedances (*RLC*)

2.4.6.1 Case of an Ohmic resistance

Ohm's law

$u(t)=R\,i(t)$ So $i(t) = \frac{u(t)}{R}$ (2 .50)

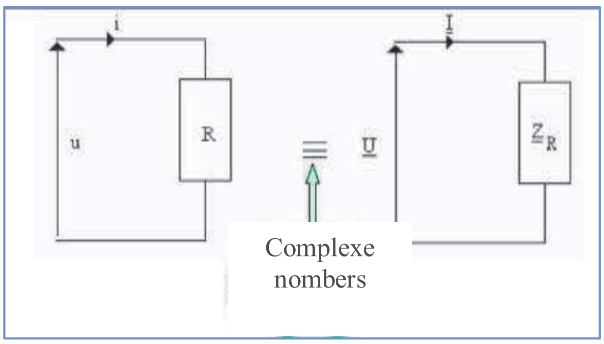


Fig.2.20 Ohmic resistance

$$i(t) = \frac{U\sqrt{2}\,sin(\omega t+\varphi_u)}{R} \qquad (2.51)$$

$$i(t) = \frac{U}{R}\sqrt{2}sin(\omega t+\varphi_u) = \frac{U}{R}e^{j\varphi_u} \qquad (2.52)$$

$$\underline{Z} = \frac{U}{\underline{I}} = \frac{Ue^{j\varphi_u}}{\frac{U}{R}e^{j\varphi_u}} = Re^{j0} \qquad (2.53)$$

Where $\underline{Z_R} = R$ and $arg(Z_R) = 0$

Resistive impedance is purely real. Voltage and current are in phase.

Fig.2.21 Ohmic resistance ;voltage and current vectors

2.4.6.2 Case of a capacitor

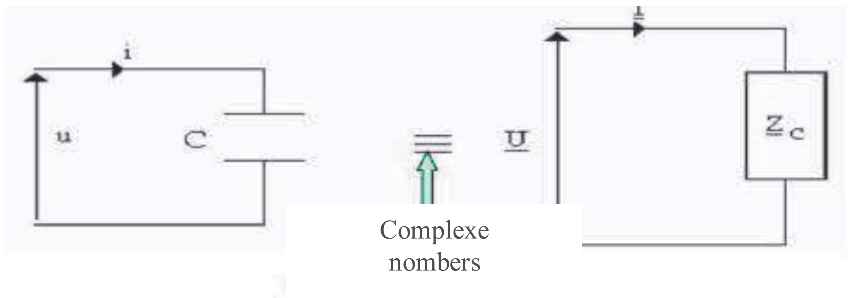


Fig.2.22 Capacitor in complex number

$$u(t) = \frac{1}{C}\int i(t)dt \qquad (2.54)$$

And

$$i(t) = C\frac{du(t)}{dt} \qquad (2.55)$$

$$i(t) = C\omega U\sqrt{2}\,cos(\omega t+\varphi_u) = C\omega U\sqrt{2}\,sin\left(\omega t+\varphi_u+\frac{\pi}{2}\right) \qquad (2.56)$$

$$\underline{I} = C\omega Ue^{j\left(\varphi_u+\frac{\pi}{2}\right)} \qquad (2.57)$$

$$\underline{Z} = \frac{U}{\underline{I}} = \frac{U\,e^{j\varphi_u}}{C\omega Ue^{j(\varphi_u+\frac{\pi}{2})}} = \frac{e^{-j\frac{\pi}{2}}}{C\omega} \qquad (2.58)$$

$$\underline{Z_C} = -J\frac{1}{C\omega} \qquad (2.59)$$

And

$$\text{Arg}\,(\underline{Z_C}) = -\frac{\pi}{2} = \varphi_C \qquad (2.60)$$

Capacitive impedance is pure imaginary capacitive reactance $X_C = \frac{-1}{C\omega}$

Current is quadrature ahead (in advance) of voltage $\underline{U} = -j\frac{1}{C\omega}\underline{I}$

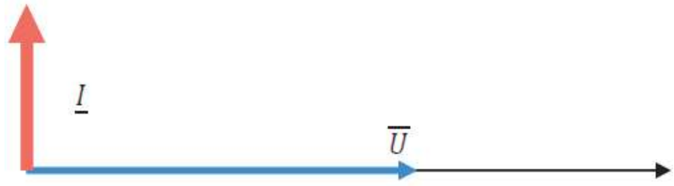


Fig. 2.23 Capacitor ; voltage and current vectors

2.4.6.3 Case of a Coil

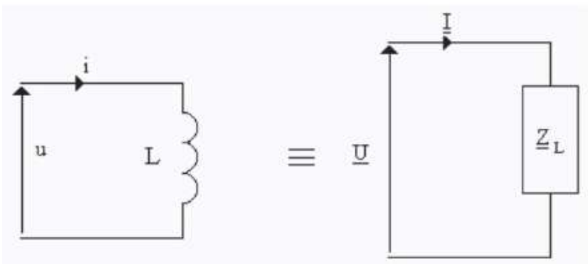


Fig.2.24 Coil

$U(t) = L\frac{di(t)}{dt}$ so , $i(t) = \frac{1}{L}\int u(t)dt$ (2 .61)

$i(t) = -\frac{1}{L\omega}U\sqrt{2}\,cos(\omega t + \varphi_u) = \frac{1}{L\omega}U\sqrt{2}\,sin\left(\omega t + \varphi_u - \frac{\pi}{2}\right)$ (2 .62)

And

$\underline{I} = \frac{U}{L\omega}e^{j(\varphi_u - \frac{\pi}{2})}$ (2 .63)

The impedance of a coil is purely inductive of inductive reactance and the current is in quadrature lagging behind the voltage .

$X_L = L\omega(\Omega)$ and $\varphi = \frac{\pi}{2}$; $\underline{U} = jL\omega\underline{I}$ (2 .64)

Fig.2.25 Coil ; voltage and current vectors

Noticed : $\underline{Z} = |Z|e^{j\varphi} = R + jX$ (2 .65)

- If $X = 0$ Impedance is resistive and $\varphi = 0$

-If $R = 0$ and $X > 0$ Impedance is purely inductive and $\varphi = \frac{\pi}{2}$

-If $R = 0$ and $X < 0$ Impedance is purely capacitive and $\varphi = -\frac{\pi}{2}$

Tab.2.1 The law of cosines

$\sin(-\alpha) = -\sin(\alpha)$	$\cos(-\alpha) = \cos(\alpha)$
$\sin(\alpha + 2n\pi) = \sin(\alpha)$	$\cos(\alpha + 2n\pi) = \cos(\alpha)$
$\sin(\alpha + \pi) = -\sin(\alpha)$	$\cos(\alpha + \pi) = -\cos(\alpha)$
$\sin(\pi - \alpha) = \sin(\alpha)$	$\cos(\pi - \alpha) = -\cos(\alpha)$
$\sin(\alpha + \frac{\pi}{2}) = \cos(\alpha)$	$\cos(\alpha + \frac{\pi}{2}) = -\sin(\alpha)$
$\sin(\frac{\pi}{2} - \alpha) = \cos(\alpha)$	$\cos(\frac{\pi}{2} - \alpha) = \sin(\alpha)$

2.4.6.4 RL Circuit

$$U_T = U_R + U_L \; ; \quad U_T = RI + jX_L I = RI + jL\omega I = ZI \; ; \quad Z = R + jL\omega \qquad (2.66)$$

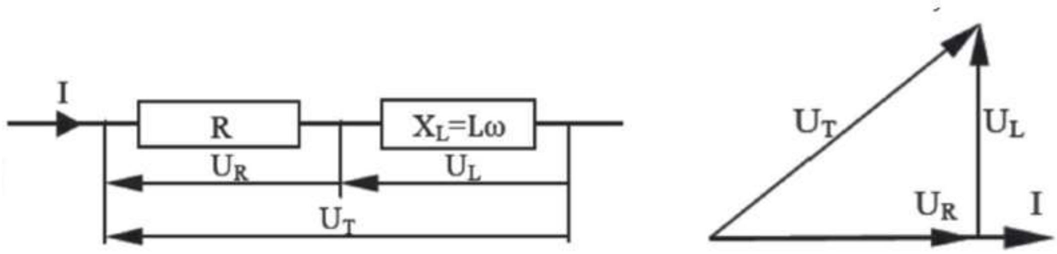


Fig.2.26 RL Circuit

2.4.6.5 RC Circuit

$$U_T = U_R + U_C \quad , \quad U_T = R.I + jX_C I = R.I - \frac{j}{C\omega} I = Z.I \qquad (2.67)$$

And

$$Z = R - \frac{j}{C\omega} \qquad (2.68)$$

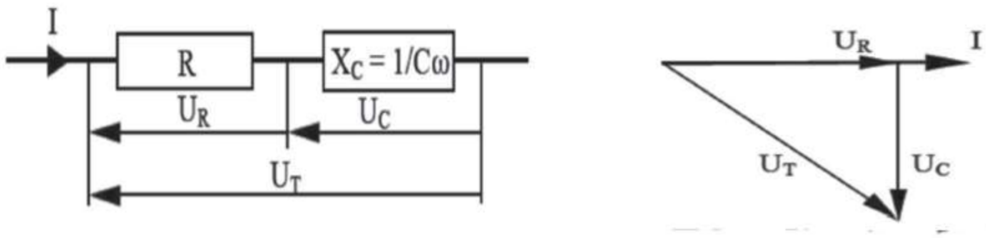


Fig.2.27 ***RC*** Circuit

2.4.6.6 RLC Circuit

$$U_T = U_R + U_R + U_C = (R + jL\omega - \frac{j}{C\omega})I \qquad (2\ .69)$$

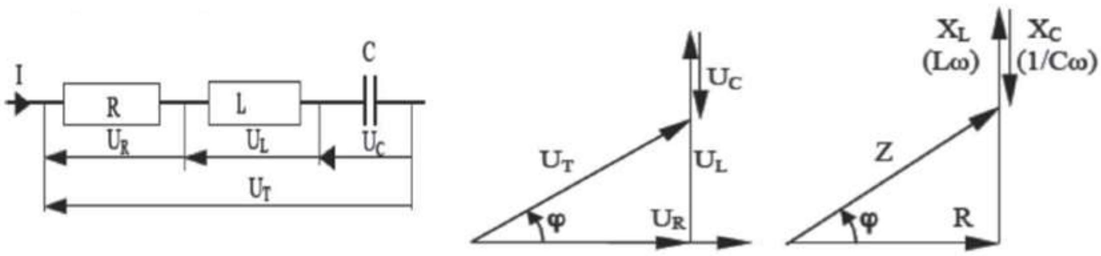


Fig.2.28 *RLC* Circuit

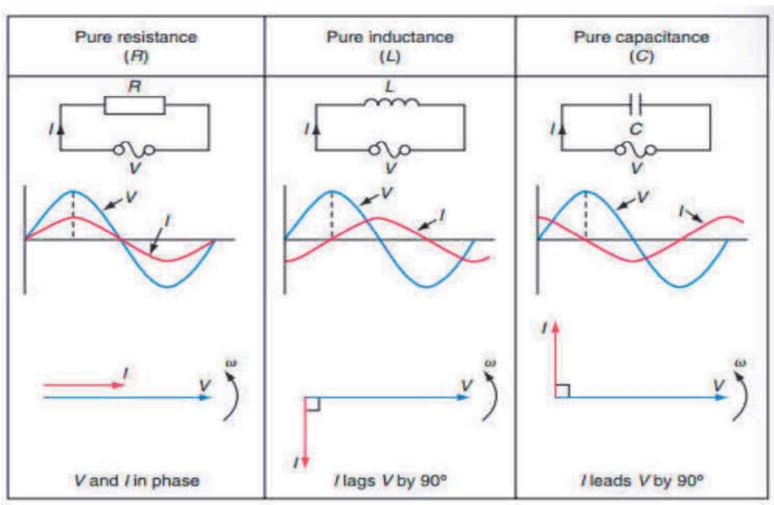


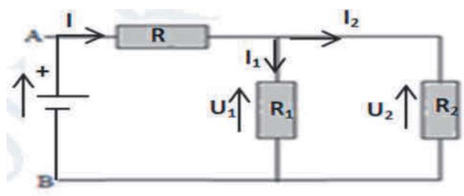
Fig.2.29 *V* and *I* for Resistance; Inductance and capacitance

TD N°2 : Exercise in Fundamental Laws of Electricity

Exercice 2.1

Consider the electrical circuit opposite: $U=15V$, $R=10\Omega$,$_1=5\Omega$ and $R_2=10\Omega$,
Calculate:

- The equivalent resistance R_{eq}
- Current intensity I_2
- The voltage U_1 across R_1



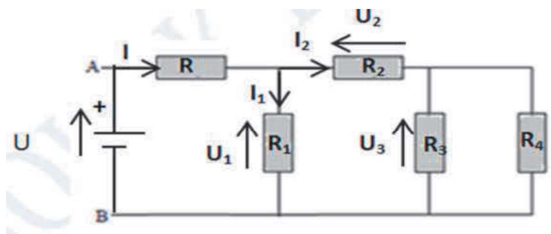
Exercice 2.2

Consider the electrical circuit opposite:

$U=15\ V$; $R=10\Omega$, $R_1=5$, $R_2=10\Omega$, $R_3=10\Omega$, $R_4=10\Omega$

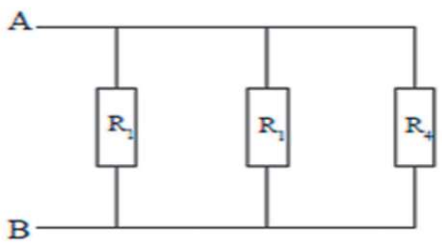
Calculate:

- The equivalent resistance R_{eq}
- The intensity of the current I_2
- The voltage U_1 across R_1
- The voltage U_3 across R_3.

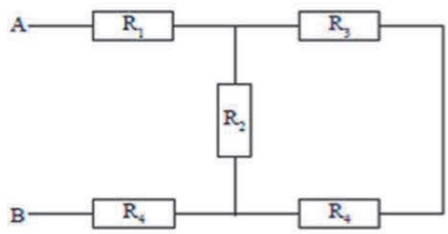


Exercice 2.3

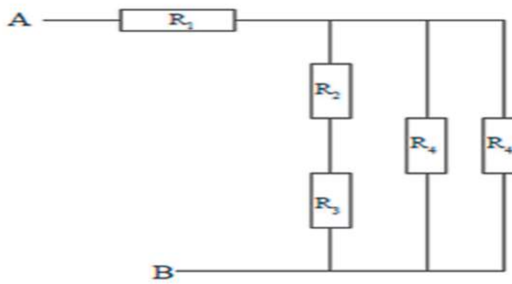
Consider : $R_1=10\Omega$, $R_2=15\Omega$, $R_3=10\Omega$, $R_4=5\Omega$ .Calculate the equivalent resistance seen from points A and B for the different assemblies.



(1)



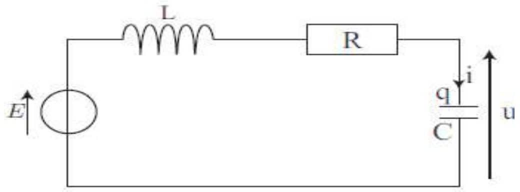
(2)



Exercice 2.4

(3)

Consider the following circuit: The generator is considered to be perfect with emf E. Initially, no current flow through the coil and capacitor C is discharged. At $t = 0$, switch K is closed.



1- Write the differential equation that satisfies $q(t)$ the charge of capacitor C.

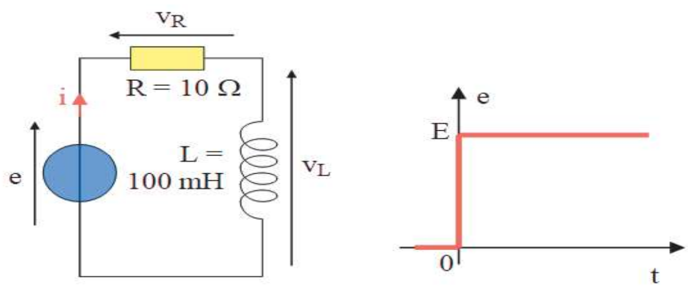
2- Write the differential equation that satisfies $i(t)$ the intensity of the current flowing through L

3- Give a relationship between R, L and C so that the circuit regime is "critical".

4- What must be the condition between R, L and C so that the regime is pseudo periodic? In this case, give the expressions for the pseudo-pulsation ω, the pseudo-frequency f and the pseudo-period T.

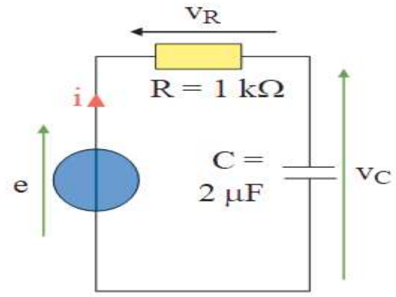
Exercice 2.5

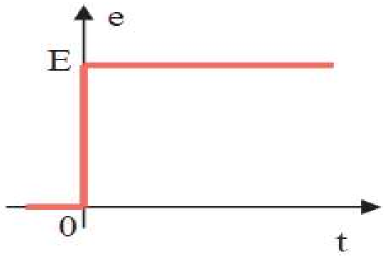
Let the $R.L$ series dipole opposite be supplied by a voltage source e(t) producing a voltage step E from t = 0. For t < 0: i(t) = 0 and e(t) = 0. Give the expression and represent v_L (t) for t > 0.



Exercice 2.6

Let the R.C series dipole opposite be supplied by a voltage source e(t) producing a voltage step E from t = 0. For t < 0: i(t) = 0 and e(t) = 0. Give the expression and represent v_C (t) for t > 0.





Exercice N° 2.7

Consider an *RL* circuit whose effective current is $I=1A$. R=100Ω, $L=38.6mH$, $f=50Hz$.

1- Determine the effective values U_R , U_L and U_t and the corresponding phase shift.

Consider an *RC* circuit whose effective current is $I= 1A$. $R=100\Omega$, $C=35\mu F$, $f=50Hz$.

2- Determine the effective values de U_R , U_C et U_t and the corresponding phase shift.

Eercice N° 2.8

A $25\ \Omega$ resistor, a $10\ \mu F$ capacitor and a $0.1H$ inductor which has an internal resistance of $12\ \Omega$ are connected in series. Determine for a frequency of $50\ Hz$:

1- The coil impedance and the capacitor impedance.

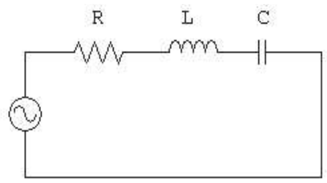
2- The module of the overall circuit impedance.

3- What is the nature of the load ?

4- Calculate the effective current in the circuit for a sinusoidal voltage of maximum value *300 V*.

Exercice N° 2.9

We consider the circuit represented in the figure where is the complex representation of a sinusoidal voltage with effective value $V_{eff}=100\ V$ and frequency $50\ Hz$. The components of this circuit are directly characterized by the value of their complex impedance. White $R=20\Omega$; $X_L=j10\Omega$; $X_C=-j5\Omega$.



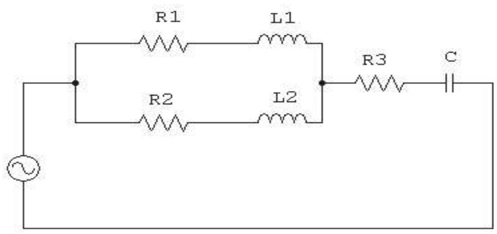
1- Calculate the effective value *I* of the current.

2- Calculate the phase of the current if we consider the voltage at the origin of the phases. Then write the time expression for the voltage *v(t)* and the current *i(t)*.

3- Represent all the complexes forming this mesh law on a vector diagram in the complex plane (Fresnel diagram).

Exercice N° 2.10

Calculate the impedances of each branch and the equivalent impedance and the current *i(t),* Knowing that $V(t)= 100\sqrt{2}\sin 314t$.Whit : $R_1=8\Omega$; $X_{L1}=j10\Omega$; $R_2=7\Omega$; $X_{L2}=j9\Omega$;$R_3=5\Omega$; $X_C=-j2\ \Omega$.



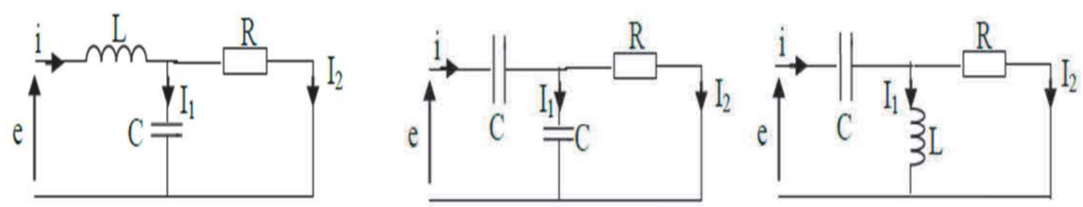
Exercice N° 2.11

For the following three circuits, determine:

1- Their complex impedance;

2- Intensities *i* and I_1;

A. N: For a sinusoidal voltage, $e(t)= 220\sqrt{2}\sin 314t$ and 25Ω for resistor, a $10\mu F$ capacitor and a 0.1H inductor .



-A **-B** **-C**

Chapter 3:

Circuits and Electrical Power

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Circuits and Electrical Power

3.1 Introduction

The study of electricity forms one of the most essential pillars in the education of engineers and scientists. Its fundamental laws provide the foundation upon which modern electrical engineering, electronics, telecommunications, and many other related disciplines are constructed. Understanding these laws is not merely an academic exercise but a practical necessity, as they enable us to analyze, design, and optimize a broad range

This chapter aims to revisit the core principles governing electric phenomena. Emphasis will be placed on their proper mathematical formulation, physical interpretation, and their role as building blocks for more complex electrical concepts.

By the end of this chapter, readers will have refreshed their knowledge of these fundamental laws, allowing them to confidently tackle advanced electrical engineering problems and appreciation.

Indeed, in 1884, engineers Lucien Gaulard and John Gibbs developed a high-power transformer using three-phase current and electricity has been produced since the end of the 19th century from different primary energy sources. The first power plants ran on wood. Today, production can be done from fossil energy (coal, natural gas or oil), nuclear energy, hydroelectric energy, solar energy, wind energy and biomass.

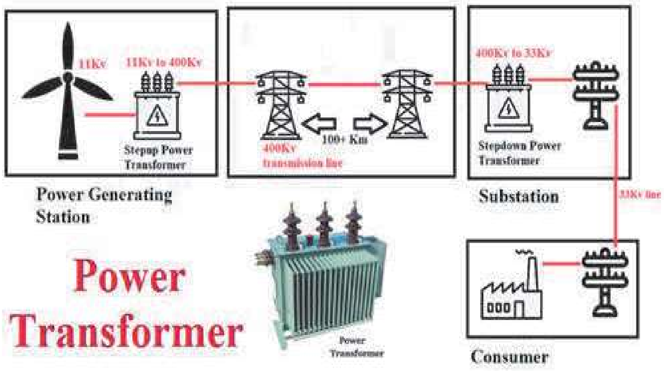


Fig.3.1 Power transformer

The most convenient solution for industrially producing electrical energy is to drive an alternator by a turbine, all rotating around an axis. Naturally, these installations produce sinusoidal voltages.

3.2 Sinusoidal power

3.2.1 Active power

This is the energy actually recoverable by the load. It is called active power because it is what is really useful (for example, in a motor, it is the active power which is transformed into mechanical power). The active power is the average value of the instantaneous power expressed in Watt [w].

$$P = \frac{1}{T}\int_0^T u(t)i(t)dt = UIcos\varphi \qquad (3.1)$$

3.2.2 Reactive power

Reactive power appears when the installation contains inductive or capacitive receivers. Its unit is: Volt-Ampere-Reactive [VAR].

$$Q = UIsin\varphi \qquad (3.2)$$

-If the reactive power is positive then the receiver is inductive.

-If the reactive power is negative then the receiver is capacitive.

3.2.3 Apparent power

It is equal to the vector sum of the two active and reactive powers and it makes it possible to determine the value of the current absorbed by the load. Expressed in .

$$S = UI \qquad (3.3)$$

Relationship between power (Triangle of powers)

$$S^2 = P^2 + Q^2 \qquad (3.4)$$

$$S = \sqrt{P^2 + Q^2} \qquad (3.5)$$

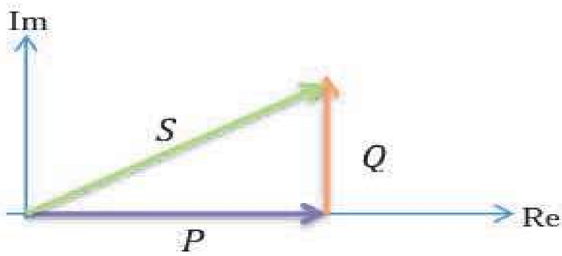


Fig.3.2 Triangle of powers

Noticed :

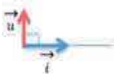
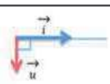
- The power supplied by the generator is equal to the power absorbed by the receiver.
- A resistor does not consume reactive power.
- The coil does not consume active power. It consumes reactive power.
- The capacitor does not consume active power. It is a reactive power generator.

3.2.4 Power factor

Power factor is a factor (without units) measuring the active power production efficiency of the system.

$$cos(\varphi) = \frac{p}{s} \qquad (3.6)$$

Tab.3.1 Different dipoles

Dipoles	Temporal	Module	Impedance Complexes	Fresnel representation	Active power	Reactive power
Resistance in Ohm R(Ω)	$u(t) = R.i(t)$	$U = R.I$	$Z_R = R$; $\varphi = 0$		$P = R.I^2$	$Q = 0$
Inductance In Henry L(H)	$u(t) = L.\frac{di}{dt}(t)$	$U = L.\omega.I$	$Z_R = jL\omega$; $\varphi = \frac{\pi}{2}$		$P = 0$	$Q = L\omega I^2$
Capacity In Farad (F)	$u(t) = R.i(t)$	$U = \frac{1}{c\omega}.I$	$Z_R = \frac{-j}{c\omega}$; $\varphi = -\frac{\pi}{2}$		$P = 0$	$Q = \frac{I^2}{c\omega}$

3.3 Boucherot's Theorem

The total active power consumed by a system is the sum of the active powers consumed by each element;

$$P_T = \sum_{i=1}^{n} P_i = P_1 + P_2 + P_3 + \cdots + P_n \qquad (3.7)$$

The total reactive power consumed by a system is the sum of the reactive powers consumed by each element. The apparent power consumed by a system is calculated from the relationship:

$$Q_T = \sum_{i=1}^{n} Q_i = Q_1 + Q_2 + Q_3 + \cdots + Q_n \qquad (3.8)$$

$$S = \sqrt{P^2 + Q^2} \qquad (3.9)$$

Tab.3.2 Power of dipoles

Impedance	Ohm's Law	Phase shift	Power factor	Active power	Reactive power	Apparent power
$Z = R$	$U = ZI$	$\varphi = 0$ $\varphi = 0\ rads$	$Cos\varphi = 1$ $Sin\varphi = 0$	$P = UI$ $P = RI^2$ $P = U^2/R$	$Q = 0\ VAR$	$S = P$ $S = UI$
$Z = L\omega$	$U = L\omega I$	$\varphi = 90$ $\varphi = \frac{\pi}{2} rads$	$Cos\varphi = 0$ $Sin\varphi = 1$	$P = 0\ W$	$Q = UIsin\varphi$ $Q = UI$ $Q = L\omega I^2$	$S = Q$ $S = L\omega I^2$
$Z = \frac{1}{C\omega}$	$U = \frac{1}{C\omega}$	$\varphi = -90$ $\varphi = -\frac{\pi}{2} rads$	$Cos\varphi = 0$ $Sin\varphi = -1$	$P = 0\ W$	$Q = UIsin\varphi$ $Q = -UI$ $Q = -U^2C\omega$	$S = Q$ $S = -U^2C\omega$

3.4 Power measurement

$$P = U.I.cos\varphi \qquad [W] \qquad (3.10)$$

$$Q = U.I.sin\varphi \qquad [VAR] \qquad (3.11)$$

$$S = \sqrt{P^2 + Q^2} = U.I \qquad [VA] \qquad (3.12)$$

3.4.1 Measure active power P

To measure *P*, simply connect a wattmeter according to the downstream assembly. It has at least four terminals: two terminals for measuring voltage and two terminals for measuring current. There are therefore two connections to make: a parallel connection, like a voltmeter, to measure the voltage, and a series connection, like an ammeter, to measure the current.

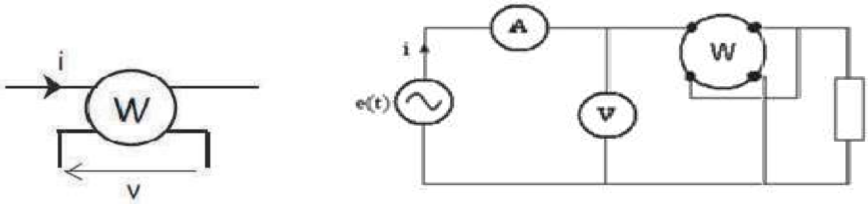


Fig.3.3 Measure active power P

3.4.2 Measuring apparent power *S*

To measure *S*, you must use an ammeter and a voltmeter in order to determine the effective values of the current and the voltage according to the diagram of the following assembly:

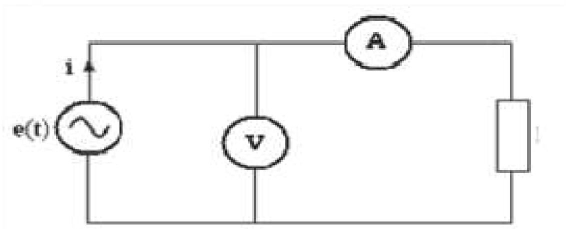


Fig.3.4 Measuring apparent power S

3.4.3 Measuring reactive power *Q*

To measure reactive power *Q*, simply connect an ammeter, a voltmeter and a wattmeter. Then calculate taking into account the type of receiver:

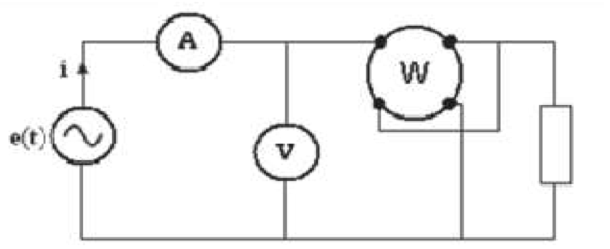


Fig.3.5 Measuring reactive power Q

- For a resistive receiver $Q = 0$
- For an inductive receiver $Q > 0$
- For a capacitive receiver $Q < 0$

3.5 Three-phase alternating current power

Three-phase receivers: These are receivers made up of three identical dipoles, Z impedance.

Balanced: because the three elements are identical.

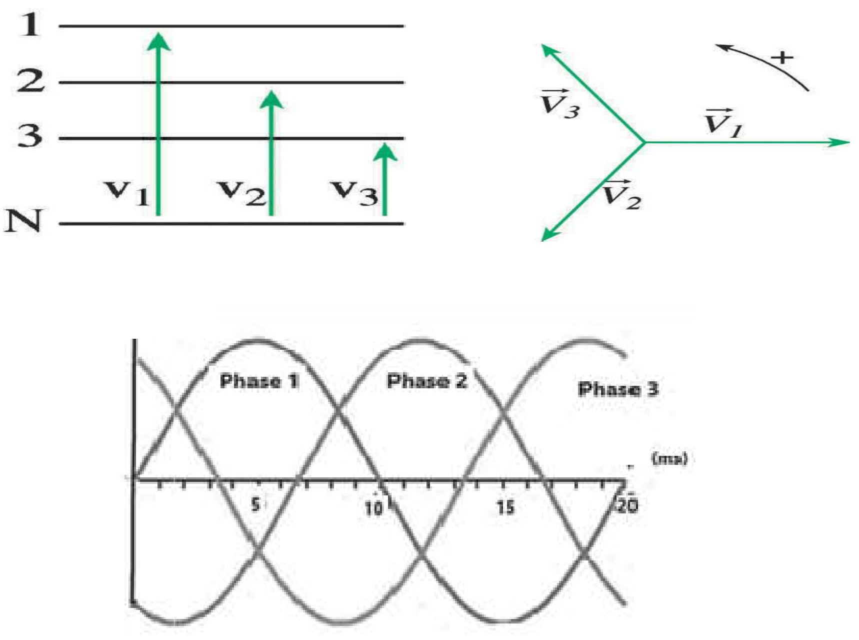


Fig.3.6 Three-phase alternating simple voltages

Composite voltages between phases u_{12} , u_{23} , u_{31}

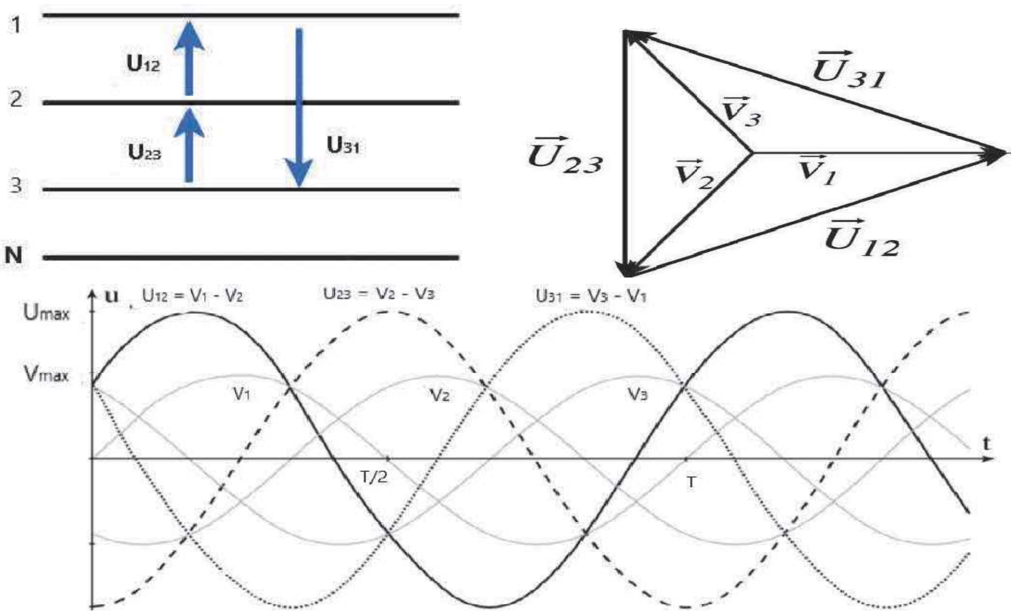


Fig.3.7 Three-phase alternating composite voltages

$$\vec{U}_{12} = \vec{V}_1 - \vec{V}_2 \ ; \vec{U}_{23} = \vec{V}_2 - \vec{V}_3 \ ; \vec{U}_{31} = \vec{V}_3 - \vec{V}_1 \qquad (3.13)$$

$$u_{12}(t) = U\sqrt{2}\sin\left(wt + \frac{\pi}{6}\right) ; u_{23}(t) = U\sqrt{2}\sin\left(wt - \frac{\pi}{2}\right) ; u_{31}(t) = U\sqrt{2}\sin\left(wt + \frac{7\pi}{6}\right) \quad (3.14)$$

3.5.1 Relationship between *U* and *V*

$$U = 2V\cos(30°) \qquad SO \qquad U = 2V\frac{\sqrt{3}}{2} \qquad (3.15)$$

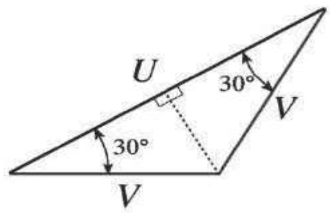


Fig.3.8 Relationship between *U* and *V*

Currents per phase: these are the currents which pass through the *Z* elements of the receiver three-phase. Symbol *J*.

Line currents: these are the currents which pass through the wires of the three-phase network. Symbol: *I*.

The network and the receiver can be connected in two different ways: in a star or in a triangle.

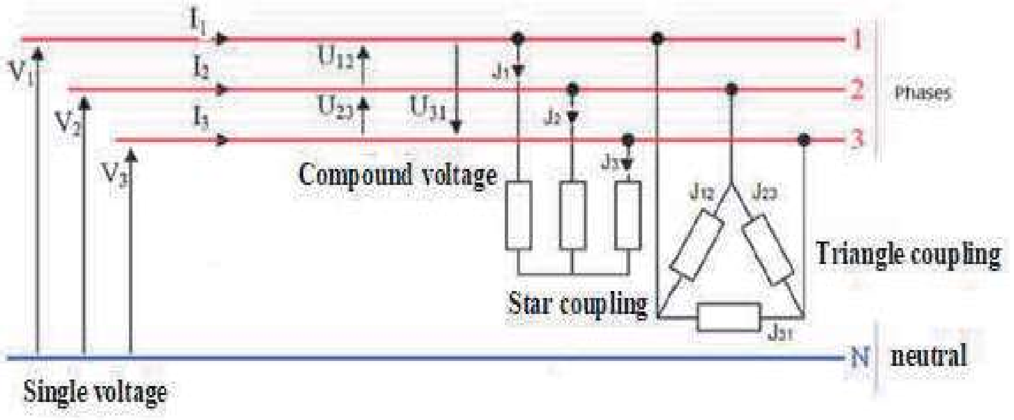


Fig.3.9 Star and triangle coupling

3.5.2 Star coupling

As they are the same impedances, therefore $I_1 + I_2 + I_3 = 0$. The current in the neutral wire is zero. The neutral wire is therefore not necessary.

Noticed :

For a balanced three-phase system, the neutral wire is useless.

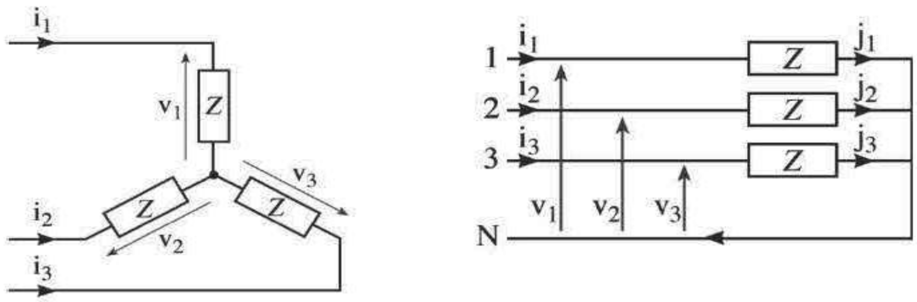


Fig.3.10 Star coupling

Line currents are equal to currents per phase.

$i_1 = j_1$; $i_2 = j_2$; $i_3 = j_3$

In addition, the load and the network are balanced, so

$I_1 + I_2 + I_3 = I = J$

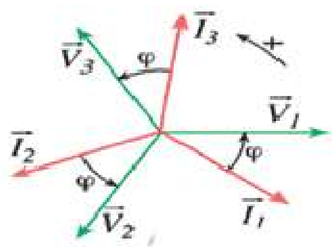


Fig.3.11 Line and voltage per phase

For a receiver phase

$P_1 = VIcos\varphi \quad and \quad \varphi(\vec{I},\vec{V})$ (3.16)

Finally for the star coupling

$P = 3VIcos\varphi$ (3.17)

$V = \frac{U}{\sqrt{3}}$ (3.18)

$P = \sqrt{3}UIcos\varphi$ (3.19)

$Q = \sqrt{3}UIsins\varphi$ (3.20)

$S = UI$ (3.21)

Power factor: $K = cos\varphi$ (3.22)

Losses by Joule effect: Let us consider that the resistive part of the receiver.

For a receiver phase:

$P_{J1} = rI^2$ (3.23)

For a receiver couplet:

$P = 3P_{J1} = 3rI^2$ (3.24)

Resistance seen between two terminals:

$R = 2r$ (3.25)

Finally for the star coupling:

$P = \frac{3}{2}RI^2$ (3.26)

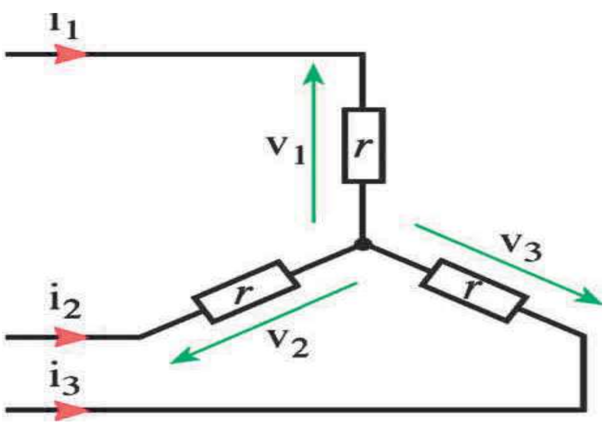


Fig.3.12 Star coupling

3.5.3 Triangle coupling 

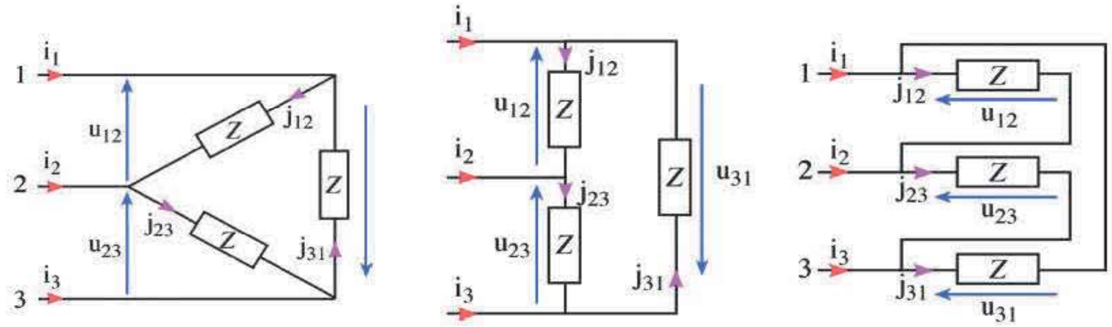


Fig.3.13 Triangle coupling

As they are the same impedances, here in no case is the neutral wire necessary.

Relationships between currents :

$i_1 + i_2 + i_3 = 0 \, , And \;\; J_{12} + J_{23} + J_{23} = 0$ (3.27)

$i_1 = J_{12} - J_{31} \rightarrow \vec{I_1} = \overrightarrow{J_{12}} - \overrightarrow{J_{31}}$ (3.28)

$i_2 = J_{23} - J_{12} \rightarrow \vec{I_2} = \overrightarrow{J_{23}} - \overrightarrow{J_{12}}$ (3.29)

$i_3 = J_{31} - J_{23} \rightarrow \vec{I_3} = \overrightarrow{J_{31}} - \overrightarrow{J_{23}}$ (3.30)

The three-phase system is balanced: $I_1 = I_2 = I_3 = I$ and $J_{12} = J_{23} = J_{31} = J$

For triangle coupling, the relationship between *I* and *J* is the same as the relationship between *V* and *U*.

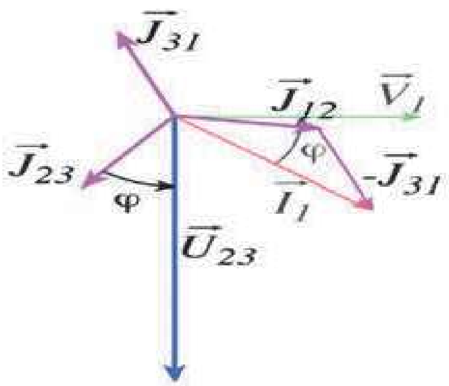


Fig.3.14 Relationships between currents

For triangle coupling:

Puissances

For a receiver phase: $P_1 = UJcos\varphi \quad Whit \quad \varphi(\vec{U},\vec{J})$ (3.31)

For the complete receiver: $P = 3\ P_1 = UJcos\varphi \quad and \quad J = \frac{I}{\sqrt{3}}$ (3.32)

Finally for the star coupling: $P_1 = \sqrt{3}UJcos\varphi$ (3.33)

In the same way**:** $Q = \sqrt{3}UJsin\varphi$ (3.34)

And: $S = UI$ (3.35)

Power factor: $K = cos\varphi$ (3.36)

Losses due to Joule effect

Consider that the resistive part of the receiver.

For a receiver phase:

$P_{J1} = rI^2$ (3.37)

For the complete receiver :

$P = 3P_{J1} = 3rj^2$ (3.38)

Resistance seen between two terminals:

$R = \frac{2rr}{2r+r} = \frac{2}{3}r$ (3.39)

Finally for the star coupling :

$P = \frac{3}{2}RI^2$ (3.40)

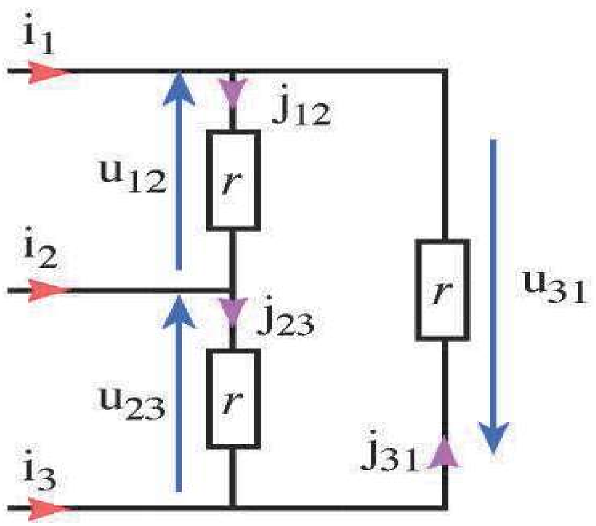


Fig.3.15 Triangle coupling

Tab.3.3 Relation between star coupling and triangle coupling

	Star coupling	**Triangle coupling**
Relation between *U* and *V*	$U = \sqrt{3}V$	$U = \sqrt{3}V$
Relation between *I* and *J*	$I = J$	$I = \sqrt{3}J$
Phase shift	$\varphi(\vec{I},\vec{V})$	$\varphi(\vec{J},\vec{U})$
Power active	$P = 3P_1 = 3VIcos\varphi$ $P = \sqrt{3}UJcos\varphi$	$P = 3P_1 = 3UJcos\varphi$ $P = \sqrt{3}UJcos\varphi$
Joule losses	$P = 3rI^2$ $P = \frac{3}{2}RI^2$	$P = 3rJ^2$ $P = \frac{3}{2}RI^2$
Equivalent resistance	$P = 2r$	$P = \frac{2}{3}r$
Power reactive	$Q = \sqrt{3}UIsins\varphi$	$Q = \sqrt{3}UIsins\varphi$
Apparent power	$S = \sqrt{3}UI$	$S = \sqrt{3}UI$
Power factor	$K = cos\varphi$	$K = cos\varphi$

3.6 Different types of Generator-Receiver coupling

3.6.1 Star – star coupling

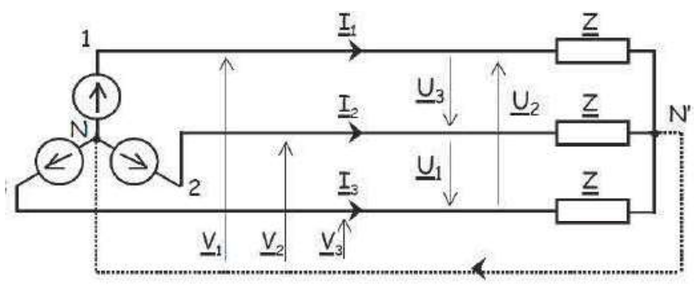
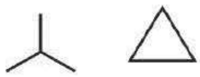


Fig.3.16 Star – star coupling

3.6.2 Star – triangle coupling



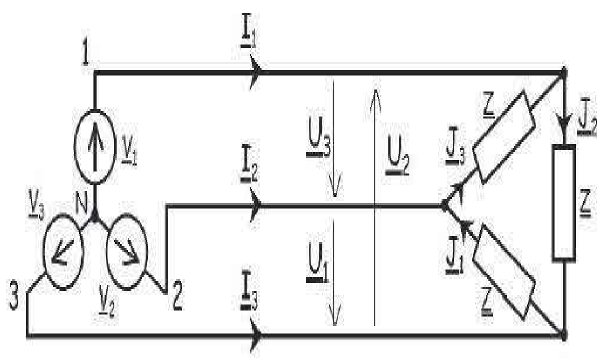


Fig. 3.17 Star – triangle coupling

3.6.3 Triangle – Starc coupling 

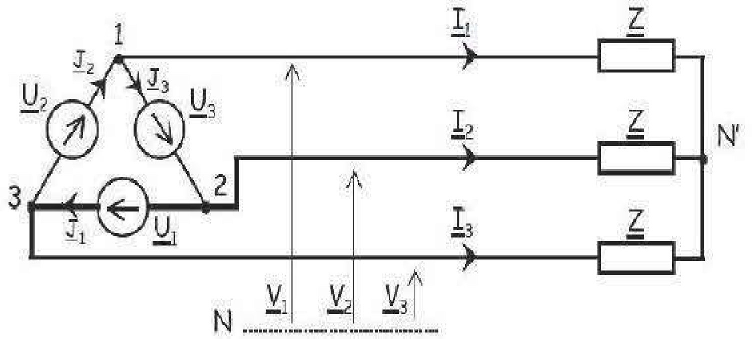
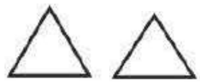


Fig.3.18 Triangle – Starc coupling

3.6.4 Couplage triangle – triangle 

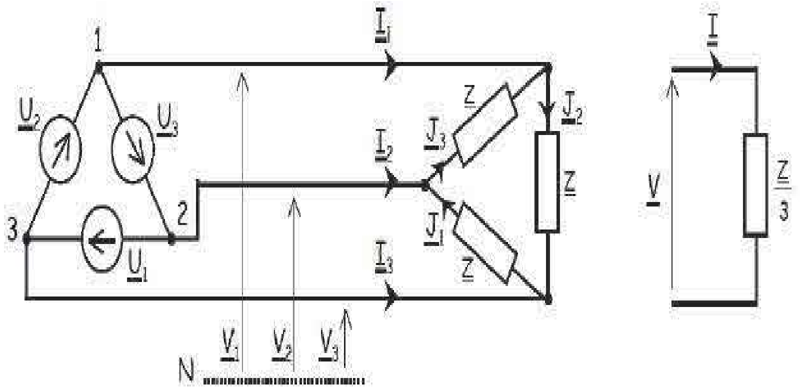
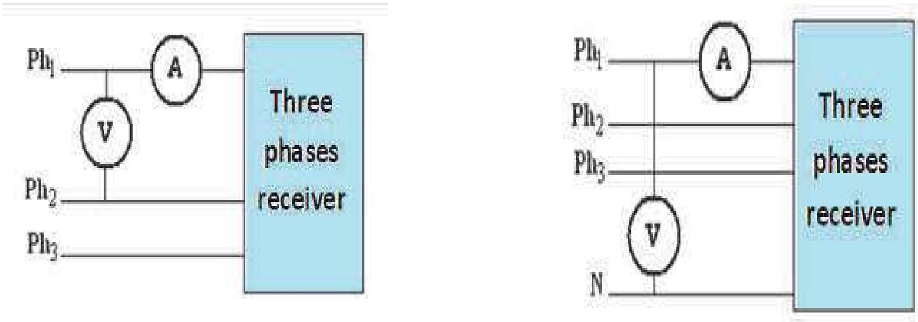


Fig. 3.19 Triangle – triangle

3.7 Power measurement

3.7.1 Measurement of apparent power *S*

To measure *S*, you must use a voltmeter and an ammeter to determine the simple or compound voltage and the current crossing a power line (we assume that the available three-phase system is direct balanced) according to the two arrangements in the following figure:



Line without neutral wire $S = \sqrt{3}.U.I$

Line with neutral wire $S = 3.V.I$

Fig. 3.20 Measurement of apparent power

3.7.2 Measurement of active power *P*

3.7.2.1 Single wattmeter method with neutral wire

When the receiver is balanced, a single wattmeter can measure the active power absorbed. The principle diagram is given by the following figure:

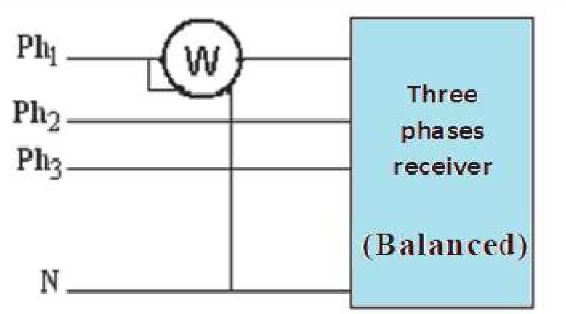


Fig.3.21 Single wattmeter method

The power meter, as plugged in, measures power :

$$P_1 = V.I.cos\varphi \quad (3.33)$$

The power absorbed by the balanced three-phase receiver is:

$$P = 3.P_1 \quad (3.34)$$

Indeed, we can write:

$$P = 3.P_1 = 3.V.I.cos\varphi = \sqrt{3}.U.I.cos\varphi \quad (3.35)$$

This measure requires that the neutral wire be accessible.

3.7.2.2 Two wattmeter method

For an unbalanced system or a balanced system where the neutral is not accessible, the active power is measured using two wattmeters. The assembly diagram is as follows:

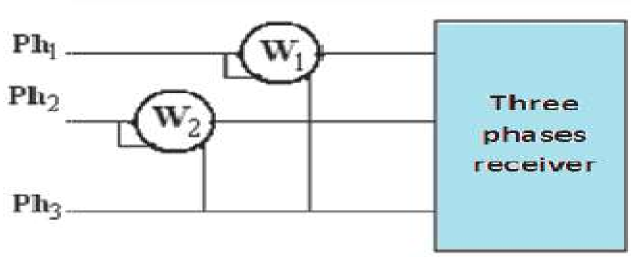


Fig.3.22 Two wattmeter method

If we call P_1 and P_2 the powers measured by the wattmeters W_1 and W_2, we determine the active power absorbed by the load using the relationship:

$$P = P_1 + P_2 \quad (3.36)$$

3.7.3 Reactive power measurement Q
3.7.3.1 Single wattmeter method

To measure reactive power using a single wattmeter, simply mount the voltage circuit between phase 2 and 3 wires as shown in the following figure:

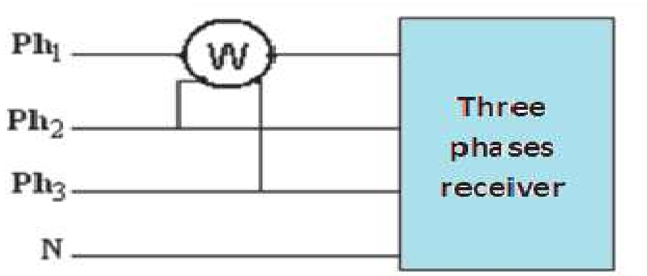


Fig.3.23 Measure reactive power using a single wattmeter

The reactive power is given by the following expression:
With P_1 the power measured by the wattmeter W

$$Q = \sqrt{3}.P_1 \quad (3.37)$$

3.7.3.2 Two wattmeter method

This is the same method used for measuring active power. But we can determine the reactive power by the following relationship:

$$Q = \sqrt{3}.(P_1 - P_2) \quad (3.38)$$

TD N°3 : Exercise in Circuits and Electrical Power

Exercice N° 3.1

A- The single-phase motor of a washing machine consumes 5A at a voltage of 230 V, 50 Hz. Its power factor is $Cos\ \varphi = 0,75$.

1- Calculate the apparent power of the engine.

2- Calculate the active power absorbed by the motor.

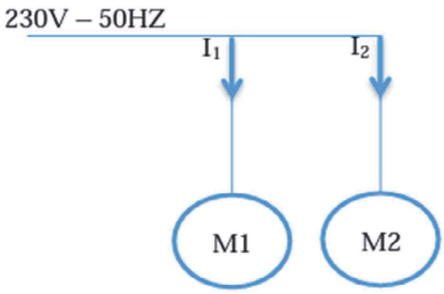
3- Calculate the reactive power absorbed by the motor.

B- Let the group of motors, in the figure opposite, be powered by an effective voltage of 230V. The group is made up of two dipoles:

D_1 is a motor such that $I_1 = 5\ A$; $Cos\varphi_1 = 0,8$ et D_2 is a second motor such that $I_2 = 10A;$ $Cos\ \varphi_2 = 0,7$.

1- Calculate powers (active, reactive and apparent) of each engine, as well as that of the group;

2- Calculate the phase shift between the supply voltage and current.



Exercice N°3.2

A single-phase electrical installation includes: ten (10) bulbs of 75 W each; a 1.875 kW electric heater; three (03) identical electric motors each absorbing a power of 1.5 kW with a power factor of 0.80. These different devices operate simultaneously.

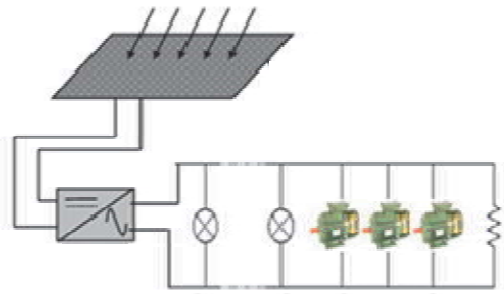
1- What is the active power consumed by the bulbs ?

2- What is the reactive power consumed by a motor ?

3- What are the active and reactive powers consumed by the installation?

4- What is its power factor?

5- What is the effective intensity of the current in the line cable?



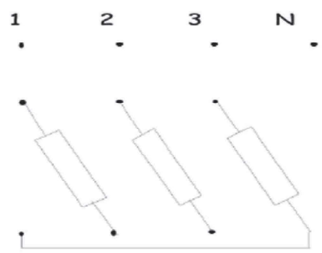
Exercice : 3.3

Three identical inductive single-phase receivers (coils) with impedance Z=50Ω and power factor 0.8 are connected to the *220/380V; 50Hz* network.

1- The impedances are triangle coupled with neutral. Complete the wiring diagram and calculate the line currents and active and reactive powers.

2- The impedances are star-coupled on the network. Complete the wiring diagram and calculate the line currents and active and reactive powers.

3- Calculate the active power ratio: *PΔ/Py* and conclude.



Exercice : 3.4

1- On a network *(230 V / 400 V, 50 Hz*) without neutral, three identical capacitive receivers with resistance *R = 20Ω* are connected in a star pattern with a capacitance *C= 20 µF*. Determine the complex impedance of each receiver. Calculate its module and its argument.

2- Determine the effective value of the line currents, as well as their phase shift with respect to the simple voltages.

3- Calculate the active and reactive powers consumed by the three-phase receiver, as well as the apparent power.

Exercice :3.5

Study of a freight elevator driven by an alternating three-phase motor. The motor is powered by the *220V/380V50Hz* network. The power absorbed is measured using the 2 Wattmeter method: $P_1=4800W$ and $P_2=1500W$.

1- Give the diagram for measuring the powers of the 2 wattmeter method

2- Calculate the active and reactive powers.

3- Deduce the line current and the power factor of the motor.

4- Propose another active power measurement setup.

Exercice :3.6

The nameplate of a three-phase motor displays the following information:

U=400V, Y, 50Hz and Cos$(\varphi)=0.78$.

This motor is connected to a three-phase network $U=400\ V$. Power measurement using a single wattmeter method yielded a power absorbed by one phase $P_1=1100\ W$.

1- Determine the active power absorbed by the motor.

2- Determine the line current. Deduce the current per winding.

3- Determine the impedance "Z" of one motor winding.

4- Determine the reactive power and apparent power absorbed by the motor.

Exercice :3.7

A balanced Y-connected load with a phase impedance of is supplied by a balanced, positive sequence Δ- connected source with a line voltage of 210 V.

Calculate the phase currents. Use as reference.

Exercice :3.8

A three-phase motor can be regarded as a balanced Y-load. A three phase motor draws 5.6 kW when the line voltage is 220 V and the line current is 18.2 A. Determine the power factor of the motor

Chapter 4:

Reminders on Magnetic Circuits

Chapter 04:

Reminders on Magnetic Circuits

4.1 Introduction

In the electronics industry, there are another kind of circuits called 'magnetic circuits'. These circuits like any other circuits have a closed path but the path is followed by magnetic lines of forces creating a field of magnetic flux instead of a flowing current. In this article, we will study what are magnetic circuits, and what components make up the circuit. Magnetic circuits are similar to normal electrical circuits that have a closed path followed by magnetic lines of force. It is important to know that in a magnetic circuit, the magnetic lines of force originate from a point and end at the same point after completing the full path. Despite being a circuit, it is important to note that nothing flows in a magnetic circuit like current that flows in a standard electrical circuit.

As the name suggests, a magnetic circuit consists of magnetic materials which have high permeability, these materials are usually steel or iron. Magnetic circuits also include electric motors, transformers, generators galvanometers, etc.

4.2 Applications of Magnetic Circuits

Magnetic circuits remain indispensable in electrical engineering thereby having various applications in real-day life like.

Magnetic circuits are majorly used in transformers. This is mainly because of their ability to transfer energy efficiently. These circuits result in efficient power transfer by giving direct control over power supply in circuit. The circuits are used in inductors for storage of energy efficiently. The magnetic circuits generate a magnetic field of electrons and the energy of these electrons is stored in inductors. Using magnetic circuits also reduces electrical noise.

Other appliances using magnetic circuits include electric motors and generators. They help to operate motors by converting electrical energy to mechanical energy. The generators use these circuits for performing the exact opposite task i.e. converting

mechanical energy to electrical energy. Some general devices that work on the principles of magnetism involve usage of magnetic circuits. Some of the examples of these devices are magnetic sensors, magnetic amplifiers, magnetic recording devices and magnetic couplers. They are general use items that involve magnetic circuits.
Hospitals make use of magnetic circuits in operating devices and performing surgeries. Medical imaging in machines like MRI (Magnetic Resonance Imaging) involves strong magnets and magnetic circuits which generate detailed images for medical diagnosis.

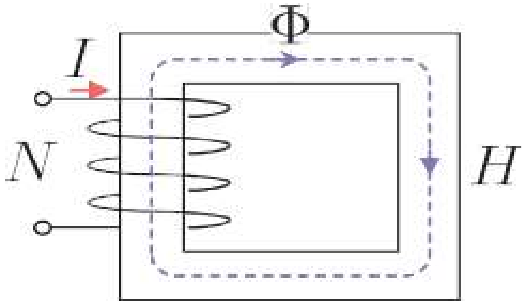


Fig.4.1 Magnetic Circuits

Any electric current flowing in a conductor is surrounded by a magnetic field *H*. Ina broader sense, it can be regarded that both are mutually dependent which is expressed by the following statement: the line integral of the magnetic field *H* over a closed path is equal to the netcurrent *i* enclosed by the path, where *H* is the magnetic field intensity [*unit:* $A\,m^{-1}$].

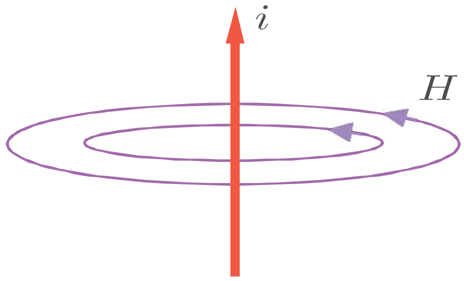


Fig. 4.2 Magnetic field intensity

$$H.dl = i \qquad (4.1)$$

$$H.2\pi r = i \qquad (4.2)$$

4.3 Magneto-motive Force

Similar as in electric circuits the magnetic field represents a tension which causes an equalizing flow of a so-called magnetic flux Φ. Just as in an electric circuit where the current is confined to thecopper conductor a magnetic circuit is built of high-permeability magnetic material the magnetic flux is confined to. Each single turn of the winding in figure 4.2 increases the cause of the electric current at building up the magnetic field. The relation given above for H over a closed path can beexpanded as:

$$H.dl = N.I \quad (4.3)$$

Where

N: number of turns

I : current [A]

The source of magnetic flux is the magneto-motive force mmf or F given by the current flowing in N-turn windings [unit: A, earlier also as At (ampere-turn)]:

$$mmf = F = N.I = H.I \quad (4.4)$$

4.4 Magnetic field

Flux lines strive to be as short as possible and take the path with the highest permeability. The flux density is defined as follows:

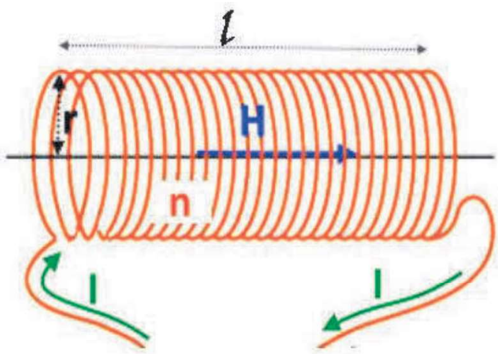


Fig.4.3 Magnetic field

$$B = \frac{\emptyset}{A} \quad (4.4)$$

Wher :

B :Wb /m^2 *=teslas (T) :*

Ø :Webers (Wb)

A : m^2

The "pressure" on the system to establish magnetic lines of force is determined by the applied magneto motive force mmf , which is directly related to the number of turns and current of the magnetizing coil as appearing in the following equation:

$$F = N.I \qquad (4.4)$$

F : ampere-turns (At)
N: turns (t)
I : amperes (A)

The level of magnetic flux established in a ferromagnetic core is a direction function of the permeability of the material. Ferromagnetic materials have a very high level of permeability, while nonmagnetic materials such as air and wood have very low levels. The ratio of the permeability of the material to that of air is called the relative permeability and is defined by the following equation:

$$\mu_r = \frac{\mu}{\mu_0} \qquad (4.5)$$

$$\mu_0 = 4\pi.10^{-7}\ Wb/\ A.m$$

μ : *H.m*$^{-1}$

μ_0 : Vacuum permeability.

μ_r : Relative permeability of the material

The applied magnetizing force has a pronounced effect on the resulting permeability of a magnetic material. As the magnetizing force increases, the permeability rises to a maximum and then drops to a minimum, as shown in figure. The flux density and the magnetizing force are related by the following:

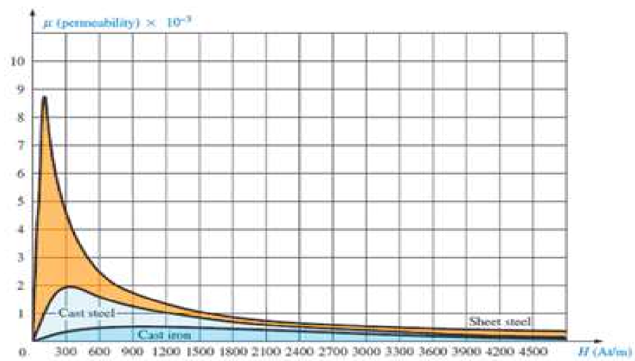


Fig.4.4 Permeability

4.5 Rulctance

The reluctance of a material to the setting up of magnetic flux lines in the material is determined by the following equation:

$$\mathfrak{R} = \frac{L}{\mu A} \qquad (rels; or\ At/Wb) \qquad (4.6)$$

Where $\mathfrak{R}$ is the reluctance, L is the length of the magnetic path, and A is the cross-sectional area.

4.6 Ohm's aw for magnetic circuits

For magnetic circuits, the effect desired is the flux magnetomotive force (mmf) . The cause is the Φ, which is the external force (or "pressure") required to set up the magnetic flux lines within the magnetic material. The opposition to the setting up of the flux Φ is the reluctance $\mathfrak{R}$.Substituting, we have:

$$\emptyset = \frac{F}{\mathfrak{R}} \qquad (4.7)$$

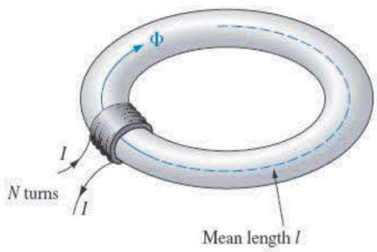


Fig.4.5 Direction of the flux

The magneto motive force per unit length (flux intensity) is called the magnetizing force (H). In equation form,

$$H = \frac{F}{L} \qquad (At/m) \qquad (4.8)$$

Substituting for the magnetomotive force results in

$$H = \frac{NI}{L} \qquad (At/m) \qquad (4.9)$$

Note in figure that the direction of the flux can be determined by placing the fingers of your right hand in the direction of current around the core and noting the direction of the thumb. It is interesting to realize that the magnetizing force is independent of the type of core material it is determined solely by the number of turns, the current, and the length of the core.

$$B = \mu H \qquad (4.10)$$

Tab.4.1 Electric and magnetic circuits

	Electric Circuits	Magnetic Circuits
Cause	*E*	*F*
Effect	*I*	*Φ*
Opposition	*R*	$\mathscr{R}$

4.7 Hystrisis

Magnetic hysteresis refers to the phenomenon of hysteresis observed during the magnetization of a material . Thus, when an external magnetic field is applied to a ferromagnetic material such as iron , the atomic magnetic dipoles align according to the field. When the field is removed, some of the alignment remains within the material. The material has been magnetized .

The relationship between field strength (*H*) and magnetization (*M*) is not linear. Thus, if the material is demagnetized (*H = M = 0*), then the initial magnetization curve increases rapidly at first, then becomes asymptotic upon reaching the magnetic saturation point . If, subsequently, the magnetic field is reduced monotonically, then M follows a different curve, hence the phenomenon of hysteresis. When the field becomes zero, the magnetization is shifted from the origin by an amount equal to the remanence .

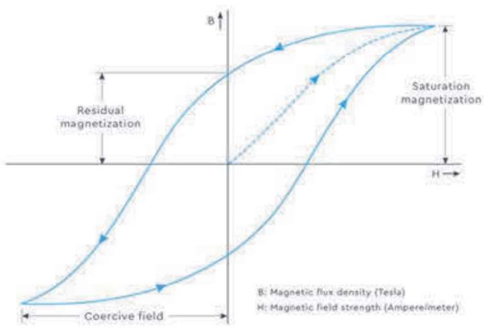


Fig.4.6 Hysteresis curve

EXEMPLE 01 /Linear Homogeneous Circuits

For the circuit shown in figure 4.6 with: $S = 16cm^2$, $l = 40cm$, $N = 350$ *and* $\mu_r=50000$, to obtain a magnetic flux density (magnetic induction) = 1.5T, find:

– The flux;

– The current required through the coil.

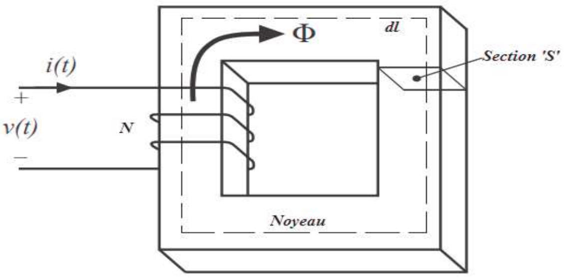


Fig.4.7 Magnetic circuits

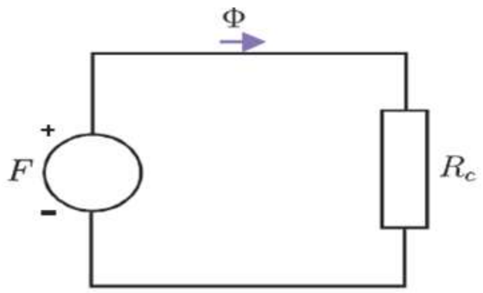


Fig.4.8 Equivalent circuit

Solution:

The flux is :

$$\text{Ø} = B.S = 1.5.10^{-4} = 2.4mWb$$

The current is: $\mathrm{N.I} = \Re.\text{Ø}$

Whit $\Re = \frac{1}{\mu.S} = \frac{1}{\mu_0.\mu_r.S} = \frac{40.10^{-2}}{4.\pi.10^{-7}.50000.16.10^{-4}} = 3979\ A.t/Wb$

$I = \frac{\Re.\text{Ø}}{N} = = 27{,}3\ mA$

EXEMPLE 02 / Linear heterogeneous circuits

A circuit is said to be heterogeneous when it is made up of different materials or geometries with variable sections. The methodology will consist, as in an electrical circuit, in using the known associations of reluctances in order to calculate the different quantities.

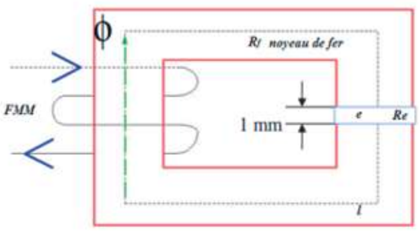


Fig.4.9 Linear heterogeneous circuits

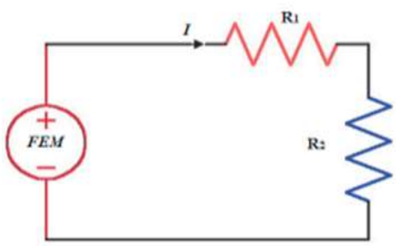


Fig.4.10 Equivalent circuit

The 2 frequent cases are the heterogeneous series and parallel circuits for each circuit, we represent the corresponding electrical analogy:

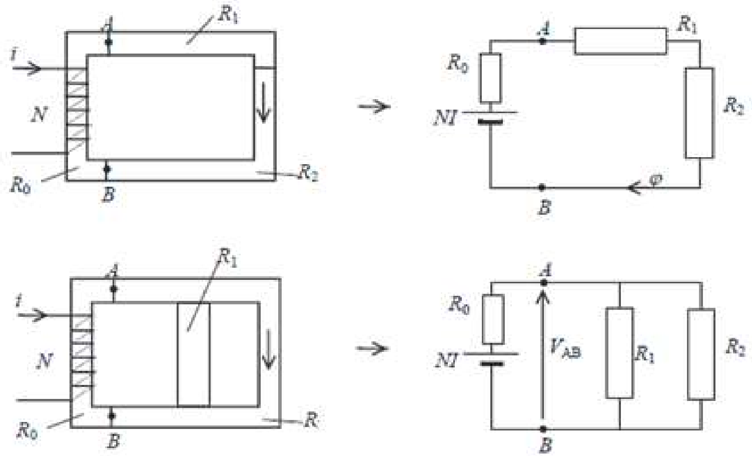
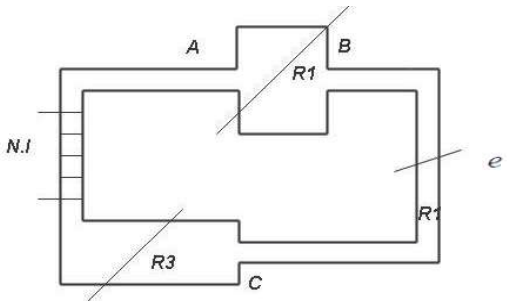


Fig.4.11 Series and parallel circuits

EXEMPLE 3 :

For the following circuit, calculate the equivalent reluctances and deduce the magnetic excitation and inductions B.



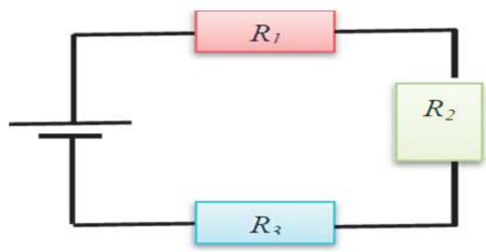


Fig.4.12 Series and parallel circuits

Fig.4.13 Magnetic Series and parallel circuits

$$\oint \vec{H}\,\vec{dl} = N.I = \int_A^B H_1.dl + \int_B^C H_2.dl + \int_C^A H_3.dl = H_1.l_1 + H_2.l_2 + H_3.l_1 \quad (4.11)$$

$$N.I = \Re_1.\emptyset_1 + \Re_2.\emptyset_2 + \Re_3.\emptyset_3 = (\Re_1 + \Re_2 + \Re_3)\emptyset = \Re_{eq}\emptyset \quad (4.12)$$

$$\Re_{eq} = \Re_1 + \Re_2 + \Re_3 \quad (4.13)$$

$$H.l = \Re_{eq}\emptyset \quad (4.14)$$

$$H = \frac{\Re_{eq}\emptyset}{l} \quad (4.15)$$

$$\emptyset = B.S \longrightarrow B = \frac{\emptyset}{S} \quad (4.16)$$

$$B_1 = \frac{\emptyset}{S_1} \;;\; B_2 = \frac{\emptyset}{S_2} ; \quad B_3 = \frac{\emptyset}{S_3} \quad (4.17)$$

TD N°4 : Exercise in Magnetic Circuits

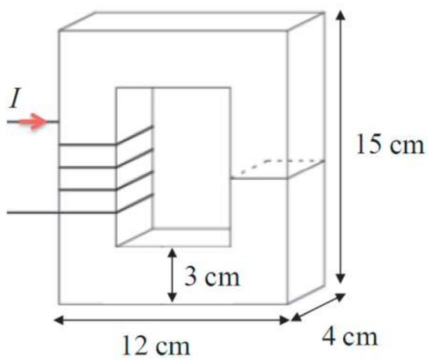
Exercice 4.1 :

Consider the following magnetic circuit, the relative permeability of the material is μ_r=3000, the number of turns is N=300 turns. This magnetic circuit carries a current of 1.2 A.

1- Calculate the geometric parameters of the circuit (average length and section).

2- Calculate the reluctance of this circuit? With $\mu_0=4\pi\ 10^{-7}$ H/m

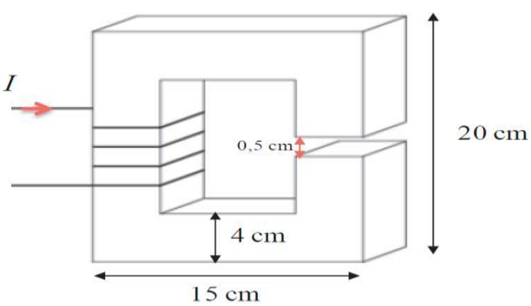
3- Calculate the magnetic flux, then deduce the magnetic induction?



Exercice 4.2 :

Consider the following circuit, the current intensity is 2 A, the relative permeability of the material is μ_r=2500 with an air gap thickness of 0.5 cm, the number of turns is 250.Knowing that the depth is 4 cm, calculate:

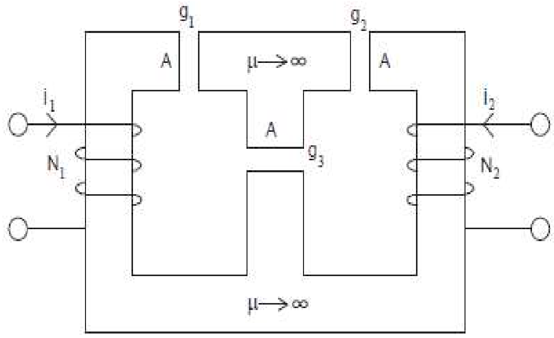
1- Calculate the geometric parameters of the circuit (average length and section).
2- Give the equivalent electrical diagram?
3- Calculate the reluctance of this circuit (material and air gap)?
4- Calculate the magnetic flux, then deduce the magnetic induction?



Exercice 4.3 :

Fig shows a magnetic circuit with air gaps $g_1 = g_2 = g_3 = 1$ mm and coils $N_1 = 100$ turns and $N_2 = 200$ turns. The cross sectional area A of the circuit is $200\ mm^2$. Assume the permeability of the core material approaches infinity and the fringing effect is negligible, calculate:

the self and mutual inductances; (b) the total magnetic energy stored in the system, if the currents in the coils are $i_1 = i_2 = 1\ A$; (c) the mutual inductance between N_1 and N_2 , if the air gap g_3 is closed.



Exercice 4.4 :

A torus-shaped magnetic circuit, $\mu_r=700$ (unsaturated) with interior radius $R_i=10\ cm$ and exterior radius $R_e=15\ cm$, carries a winding of *300 turns*, knowing that the intensity of the current in the winding is *4 A*, we are asked to determine:

1- The magnetic field H, the magnetic induction B and the magnetic flux Φ, in the case where the magnetic circuit is without air gap;

2- Same question in the case where the magnetic circuit has a 2 mm gap.

Chapter 5:

Electrical Transformers

Chapter 5 :

Electrical Transformers

5.1 Introduction

An electrical transformer is an electromagnetic static converter which allows the values of alternating voltage and current to be transformed into a voltage and current of different values but of the same frequency and shape. Figure 5.1 illustrates a set of three-phase transformers for electrical networks.

Fig. 5.1 Electrical transformers

A transformer allows the use of low voltage electrical energy and allows high voltage transmission or the opposite. In fact, the transport of electrical energy can only be done at high voltage; it will therefore be necessary to raise the voltage provided by the alternator of the power plants (from 2 to 20kv) before being able to transport it and it is necessary to lower the voltage after transport and it is the transformers that carry out these operations most economically. There are different types of transformers, but we will limit ourselves to the study of the power transformer, because it is of greatest interest in the development of network interconnection.

5. 2 Constitution

It consists of two essential parts, the electrical windings placed around a common magnetic circuit. Generally, the construction of transformers can be column or armoured , consisting of a yoke thus closing the ferromagnetic core (magnetic circuit) and one or more windings that will be called according to the destination: primary or secondary, low or high voltage as shown in figure 5.2 below.

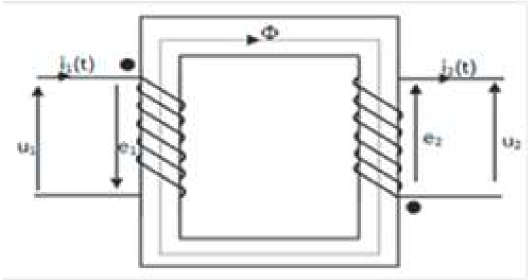


Fig.5.2 Schematic diagram of a single-phase electrical transformer

The magnetic circuit of a transformer is subjected to a magnetic field that varies over time. For transformers connected to the distribution network, this frequency is 50 or 60 Hertz. The magnetic circuit is formed of laminated steel sheets glued together with varnish or special paper to reduce eddy current losses. Paper insulation is much cheaper than varnish, but its conductivity and heat resistance and mechanical strength are lower. The magnetic circuit is crossed by an alternating magnetic flux. For the most common transformers, the stacked sheets have the shape of *E* and *I*, thus allowing a coil to slide inside the windows of the magnetic circuit thus formed. The magnetic circuits of "high-end" transformers have the shape of a torus. The number of turns of the two windings (primary and secondary) are different. The winding that has the most is called (high voltage). It is made of thinner wire than the second, called (low voltage). Transformers can be single-phase or three-phase, column or armored.

Fig.5.3 Three-phase transformer

5.3 Operating principle of a single-phase transformer

The operation of the single-phase transformer is described as follows: the secondary is not electrically connected to a source, but it supplies devices that we want to operate. If the effective value of the voltage applied to the primary is greater than that delivered by the secondary, the transformer is a step-down transformer. It is a step-up transformer in the opposite case. Figure 5.4 below shows what has been explained. When the primary is supplied with an alternating voltage, the variation in current creates a magnetic flux at the level of the column in accordance with the law:

$$d\Phi = -L\frac{dI}{dt} \quad ; \; Weber \qquad (5.1)$$

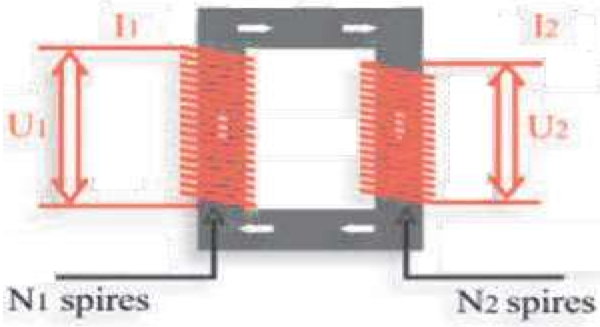


Fig.5.4 Schematic presentation of a single-phase transformer

The magnetic flux that will circulate in the magnetic core is called the main magnetic flux. The remainder at the primary and secondary coils is called the leakage or dispersion flux. The latter constitutes the magnetic losses in the vicinity of the windings. At the secondary level and according to Faraday's Law, an electromotive force (EMF) will be created and induced by the variation of the magnetic flux also considered as alternating and is expressed as follows:

Lois de Faraday : $\mathrm{e} = -N\frac{\mathrm{d}\Phi}{dt}$ (5.2)

This EMF in turn generates an induced current which will be delivered to the circuit connected with the secondary winding.

5.4 Boucherot's formula and its application to the transformer

Boucherot' s formula relates the sinusoidal voltage across a coil wound around a magnetic circuit to the magnetic field within the circuit. It is often used to determine the magnitude of the magnetic field in a transformer's magnetic circuit.

$$E = 4{,}44.N.\Phi.f \qquad (5.3)$$

Where E is the expression of the effective voltage across a winding, Φ represents the variable magnetic flux, N being the number of turns in the winding. This relationship can only be used in sinusoidal voltage conditions. This formula comes directly from Faraday's law applied to a sinusoidal magnetic field. Boucherot's formula then allows us to determine the amplitude of the magnetic field in the magnetic circuit and to verify that the latter is not saturated. Now applying this law to the circuits of the single-phase transformer :

$$E_1 = 4{,}44.N_1.\Phi.f_1 \qquad (5.4)$$

$$E_2 = 4{,}44.N_2.\Phi.f_2 \qquad (5.5)$$

E_1 for the primary circuit and E_2 for the secondary circuit. The term "transformation ratio" means the ratio of the electromagnetic forces induced in the primary and secondary of the transformer, taking into account that the frequencies f_1 and f_2 are equal, and is expressed as follows:

$$K = \frac{E_1}{E_2} = \frac{4{,}44.N_1.\Phi.f_1}{4{,}44.N_2.\Phi.f_2} = \frac{N_1}{N_2} \qquad (5.6)$$

Since the apparent powers of the primary and secondary circuits are also equal according to the law of conservation of energy, we can write the following relation:

$$K = \frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1} \qquad (5.7)$$

5.5 Reduced transformer

In the general case, the number of turns of the primary N_1 is different from N_2 and automatically the currents I_1 and I_2 must be different. The simplest method to simplify the problem is to bring the parameters of the secondary circuit back to the primary.

FEM reduced :

$$E_2' = K.E_2 = E_1 \qquad (5.8)$$

This implies:

$$K = \frac{E_1}{E_2} = \frac{N_1}{N_2} \qquad (5.9)$$

$$E_2'.I_2' = E_2.I_2, \qquad (5.10)$$

Which implies that :

$$I_2' = \frac{E_2.I_2}{E_2'} = \frac{I_2}{K} \qquad (5.11)$$

$$I_2' = \frac{I_2}{K} = I_1 \qquad (5.12)$$

Reduced resistance: according to the law of conservation of energy we have :

$$K^2.R_2 = R_1 \quad (5.13)$$

Reduced reactance:

$$X_2' = K^2.X_2 = X_1 \quad (5.14)$$

Reduced impedances:

$$Z_2' = K^2.Z_2 = Z_1 \quad (5.15)$$

If we consider that F_0 is the *MMF* of the magnetic circuit, F_1 is the *MMF* of the primary circuit and F_2, the *MMF* of the secondary circuit, we can write that:

$$F_0 = F_1 + F_2 \quad (5.16)$$

According to F=N.I, we write:

$$N_1.I_0 = N_1 I_1 + N_1.I_2' \quad (5.17)$$

F_0 is the *MMF* required to create the main magnetic flux which is distributed uniformly along the section of the ferromagnetic core. We deduce that:

$$I_0 = I_1 + I_2' \quad (5.18)$$

The FEM equations are written as follows:

$$\begin{cases} V_1 = -E_1 + I_1.Z_1 \\ V_2' = E_2' - I_2'.Z_2' \\ E_2' = E_1 = V_2' + I_2'.Z_2' \end{cases} \quad (5.19)$$

Starting from the following equations interpreting the electric circuits (Ohm's law) and the magnetic circuit (Faraday's law):

$$\begin{cases} V_1 = R_1.I_1 + N_1.\frac{d\Phi_1}{dt} \\ -V_2 = R_2.I_2 + N_2.\frac{d\Phi_2}{dt} \end{cases} \quad (5.20)$$

Φ_1 and Φ_2 are the fluxes of the primary and secondary windings respectively.

$$\begin{cases} \Phi_1 = \Phi + \Phi_{\delta 1} \\ \Phi_2 = \Phi + \Phi_{\delta 2} \end{cases} \quad (5.21)$$

$\Phi_{\delta 1}$et $\Phi_{\delta 2}$ represent the leakage flows of the primary and secondary circuits respectively.

$$\begin{cases} N_1 .\Phi_{\delta 1} = L_1 .I_1 \\ N_2 .\Phi_{\delta 2} = L_2 .I_2 \end{cases} \quad (5.22)$$

We can deduce that:

$$\begin{cases} N_1 .\Phi_1 = N_1 .\Phi + L_1 .I_1 \\ N_2 .\Phi_2 = N_2 .\Phi + L_2 .I_2 \end{cases} \quad (5.23)$$

The general equations are written in this case as follows:

$$\begin{cases} V_1 = I_1.R_1 + L_1.\frac{dI_1}{dt} + N_1.\frac{d\Phi}{dt} \\ -V_2 = I_2.R_2 + L_2.\frac{dI_2}{dt} + N_2.\frac{d\Phi}{dt} \\ N_1.I_1 + N_2.I_2 = \bar{R}.\Phi \end{cases} \quad (5.24)$$

Wher $\bar{R}$ is the reluctance of the magnetic circuit. We can also write that:

$$e_f = -L.\frac{dI_1}{dt} = -L.\frac{d(I_m .\sin\omega t)}{dt} = -L.\omega.I_m .\cos\omega t \quad (5.25)$$

When *L.ω=X,* then the leakage *EMF* is written:

$$e_f = -X.I_m .\cos\omega t = -j.I.X \quad (5.26)$$

Then we can also write the system of equations on the basis of which we can establish the equivalent diagram and the vector diagram.

$$\begin{cases} V_1 = I_1.R_1 + J.I_1.X_1 - E_1 \\ V_2 = -I_2.R_2 - J.I_2.X_2 + E_2 \\ N_1.I_1 + N_2.I_2 = \bar{R}.\Phi \end{cases} \quad (5.27)$$

5.6 Perfect or ideal transformer

The perfect transformer is virtual without any loss. It is used to model real transformers. In the case where all losses and flux leaks are neglected, the ratio of the number of transformers.

Example: A transformer whose primary has 400 turns supplied by a sinusoidal voltage of 800 V of effective voltage, the secondary which has 100 turns will present at its terminals a sinusoidal voltage whose effective value will be equal to 200 V. The operating equations are written in this case as follows:

$$\begin{cases} V_1 = N_1.\frac{d\Phi}{dt} \\ -V_2 = N_2.\frac{d\Phi}{dt} \\ N_1.I_1 + N_2.I_2 = 0 \end{cases} \quad (5.28)$$

The perfect transformer is ideal for currents, voltages and powers:

$$S_1 = S_2 \rightarrow V_1.I_1 = V_2.I_2 \rightarrow \frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} \quad (5.29)$$

All powers are conserved:

$$P_1 = P_2 V_1 I_1.\cos(\varphi_1) = V_2 I_2.\cos(\varphi_2) \quad (5.30)$$

$$Q_1 = Q_2 V_1 I_1.\sin(\varphi_1) = V_2 I_2.\sin(\varphi_2) \quad (5.31)$$

5.7 Equivalent diagram of a real transformer

The equivalent diagram (see Fig.5.5), must first satisfy the equations of the *FEM* and *FMM* of the transformer. The magnetic circuit is electrically simulated by a resistance and reactance at no load (R_0 and X_0).

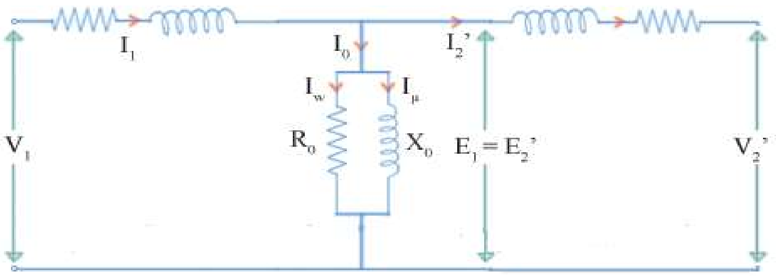


Fig.5.5 Equivalent T-circuit diagram of a transformer

$-E_1 = I_0.Z_0 = I_0.(R_0 + jX_0)$ (5.32)

Z_0 est l'impédance du circuit magnétique et Z_{ch} est l'impédance de charge

$R_0 = \frac{P_f}{I_0^2}$ and $V_2' = I_2'(Z_2' + Z_{ch})$ (5.33)

$I_2'(Z_2' + Z_{ch}) = E_2' = E_1$ then $I_2' = \frac{E_1}{(Z_2' + Z_{ch})}$ (5.34)

$I_1 = I_0 + I_2' = -\frac{E_1}{(Z_2' + Z_{ch})} + \frac{-E_1}{Z_0}$ (5.35)

$-E_1 = I_1 \frac{1}{\left(\frac{1}{Z_0} + \frac{1}{(Z_2' + Z_{ch})}\right)}$ (5.36)

Since : $V_1 = -E_1 + I_1 Z_1$, We will have:

$I_1 = V_1 \frac{1}{Z_1 + \left(\frac{1}{Z_0} + \frac{1}{(Z_2' + Z_{ch})}\right)} = \frac{V_1}{Z_{eq}}$ (5.37)

5.8 Power losses of a transformer

a- **Joule effect losses:** These are due to the flow of current in the windings and are also called "copper losses". They depend on the resistance of the copper, the square of the intensity of the current flowing through them and the time it will last.

b- **Eddy current losses:** The magnetic circuit which is the seat of a sinusoidal induction is traversed by induced currents called eddy currents. They produce a release of heat within the same magnetic circuit by Joule effect. To reduce them, we are led to laminate the sheets then stick them to each other and isolate them with vernier or special paper.

c- **Hysteresis losses:** hysteresis is a phenomenon that leads to energy consumption that appears in the form of heat in the magnetic circuit like eddy currents. The air in the hysteresis cycle corresponds to the energy lost. This is why we choose materials that have a narrow hysteresis cycle (e.g. silicon sheets).

5.9 The different types of transformers

These transformers are differentiated according to their various possible applications. In this sense we distinguish:

5.9.1Auto-transformers

This is a transformer without insulation between the primary and secondary. This structure has the particularity of the secondary being part of the primary winding. The ratio between the input voltage and the output voltage is set by the number of turns of the secondary involved in the transformation (Fig.5.6). With equal efficiency, an autotransformer takes up less space than a conventional transformer; this is due to the fact that there is only one winding and that the common part of the single winding is traversed by the difference in the primary and secondary currents. The autotransformer is interesting when the input and output voltages are close, for example (230V/115V).

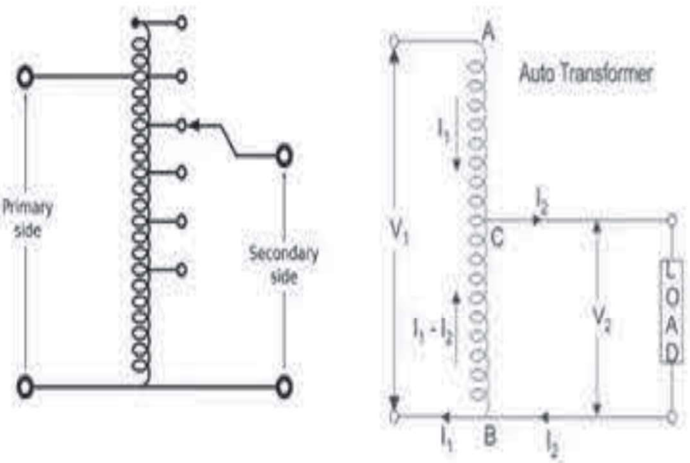


Fig.5.6 Autotransformers

5.9.2 Power transformers

Distribution transformers with a voltage of at least one phase exceeding 1000 V are considered power transformers. Their role is essential in the electrical network to allow electricity to be transported over long distances (see Fig.5.7).

Fig.5.7 Power transformers

5.9.3 Variable transformer - Variac - Alternostat

As shown in Figure 5.8, this is a type of autotransformer, since it has only one winding. The secondary output branch can be moved by means of a sliding contact on the primary turns. A "variac", or variable autotransformer, consists of a toroidal steel core, a single-layer copper coil and a carbon brush. By varying the position of the brush on the coil, the autotransformer ratio is proportionally varied. It has the advantage, compared to a rheostat, of producing much less Joule losses and its secondary voltage depends much less on the load. The presence of a fuse between the secondary and the load is essential to avoid burning the turns in the case where the secondary voltage and the load impedance are low. In fact, in this case, there is almost a short circuit distributed over very few turns.

Fig.5.8 Variable ratio auto transformer

5.9.4 Transformateur d'isolement

The isolation transformer is only intended to create electrical isolation between several circuits for reasons often of safety or resolution of technical problems and the output voltage has the same effective value as that of the input. These transformers have almost the same number of turns in the primary and secondary. They are, for example, widely used in operating theatres : each room in the operating room is equipped with its own isolation transformer, to prevent a fault that appears there from causing malfunctions in another room. Another interest is to change the neutral regime (Case of use of computer equipment and/or sensitive electronic equipment in an IT installation).

5.9.5 Current transformer

This type of transformer, also called an intensity transformer, is dedicated to the adaptation of currents involved in different circuits. Such a transformer allows the measurement of high alternating currents. It has a primary turn, and several secondary turns, the transformation ratio allows the use of a conventional ammeter to measure the intensity at the secondary. The intensity at the primary can reach several kilo-amperes (*kA*) (see Fig.5.9).

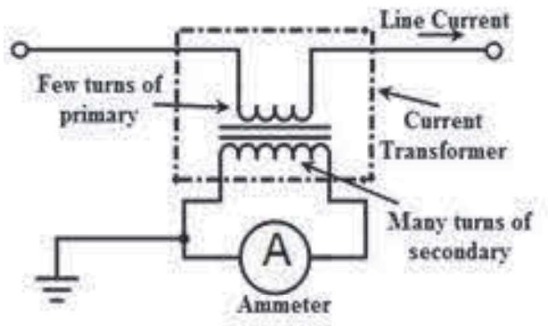


Fig.5.9 Current transformer

5.9.6 Voltage transformer

This transformer is one of the means of measuring high alternating voltages. It is a transformer that has the particularity of having a precisely calibrated transformation ratio, but designed to deliver only a very low load to the secondary, corresponding to a voltmeter. The transformation ratio makes it possible to measure primary voltages in kilovolts (*kV*) in *HTA* and *HTB* networks (see Fig.5.10).

Fig.5.10 Voltage transformer

5.10 Nameplate

The voltages indicated on the nameplate have the nominal value V_1n of the primary voltage and the effective value of the no-load voltage V_{2o} of the secondary voltage. The nominal apparent power S_n and the nominal frequency f of use of the transformer, power factor $Cos\varphi_2$ are also indicated. The nameplate allows you to quickly calculate the quantities not listed using the relationships seen previously. Figure 5.11 shows us an example of a nameplate for a *400KVA* three-phase transformer.

Fig.5.11 Nameplate of a 400 KVA three-phase transformer

5.11 Utility

The three-phase transformer plays a fundamental role in the transport and distribution of electrical energy. The transport of the latter can only be economical in high voltage (e.g. *400Kv*). The power plants for the production of electrical energy, whether hydroelectric, thermal or nuclear, have a relatively low output voltage. The increase in voltage is ensured by three-phase step-up transformers. In three-phase electrical networks, it would be perfectly possible to consider using 3 transformers, one per phase. On each of the columns are arranged a primary winding and a secondary winding. The three secondary windings can be coupled in a triangle, star or zig-zag.

Each coupling mode is symbolized by a letter:

Star: *Y* or *y*;

Triangle: *D* or *d*;

Zigzag: *Z* or *z*.

The three-phase voltage systems are: "delta" (*D or d*) and "star" (*Y or y*). The first letter of the coupling index is always in capital letters and indicates the three-phase system with the highest voltage; the second letter is in lower case and indicates the system with the lowest voltage. In the "star" system, the "neutral" (central point of the star) can be brought out to the transformer terminal block: this is indicated by the presence of the letter *N* (or n) in the coupling index. There is also the zig-zag coupling (*z*), used mainly on the secondary; it has a neutral. This coupling allows, when a phase is lost on the primary, to have a practically identical voltage on the secondary on the three phases. This results in six possible combinations of coupling group: *Y-y* or *Y-d* or *Y-z* or *D-y* or *D-d* or *D-z*, for the case where the transformer is step-down; *d-D* or *d-Y* or *d-Z* or *y-D* or *y-Y* or *y-Z*, for the case where the transformer is a step-up transformer.

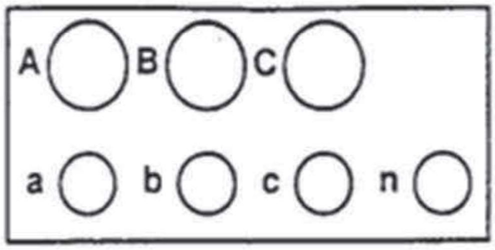


Fig.5.12 Terminal plate of a three-phase transformer

The coupling index "hour index" is the phase shift between the primary voltage and the secondary voltage which gives, by a shift of 30°, the hourly phase shift in 12^ths^ between the primary and the secondary of the transformer (e.g.: 11 = 11x30° = 330° clockwise or 30° counter clockwise). For example, a coupling index "*Dyn11*" therefore defines a transformer whose: the high voltage three-phase system is in "delta"; the low voltage three-phase system is in "star" with neutral brought out (indicated by the "n") and the shift between the two systems is *330° (= - 30° or 11 x 30°).* The hour index is a system for recognizing the temporary and spatial position of the primary windings in relation to those of the secondary (see Fig.5.13).

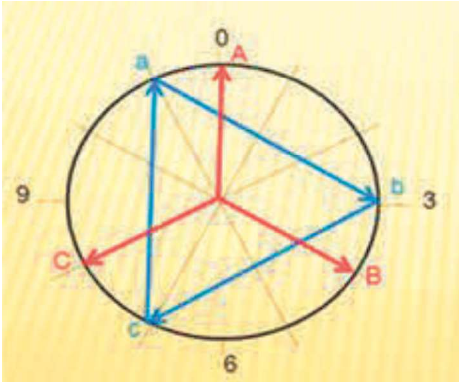


Fig.5.13 Three-phase transformer coupling time index

Example: Coupling /Yy_0

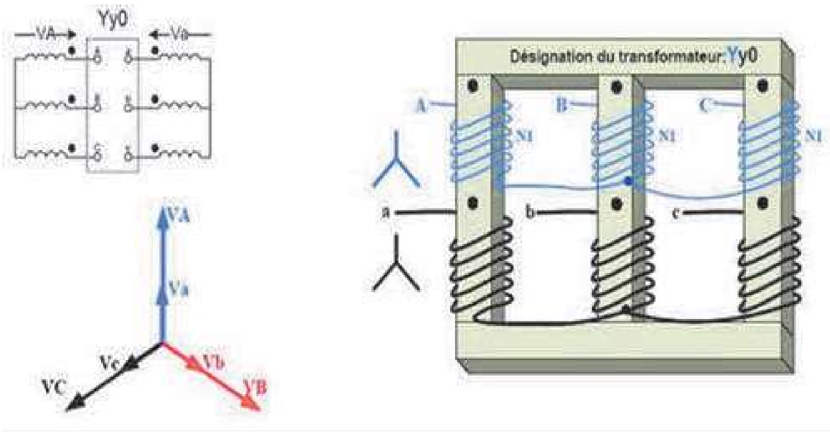


Fig.5.14 Connection diagram and vector diagram of the Yy_0 coupling

A 2nd example: Triangle-star -11h (D-y_{11})

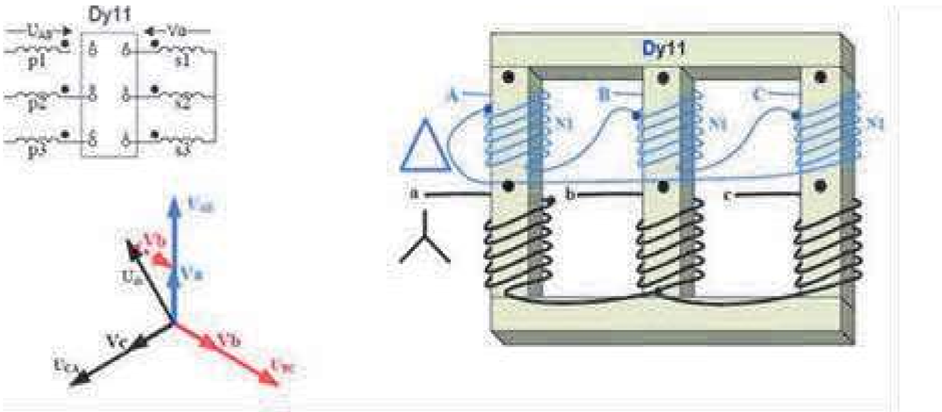
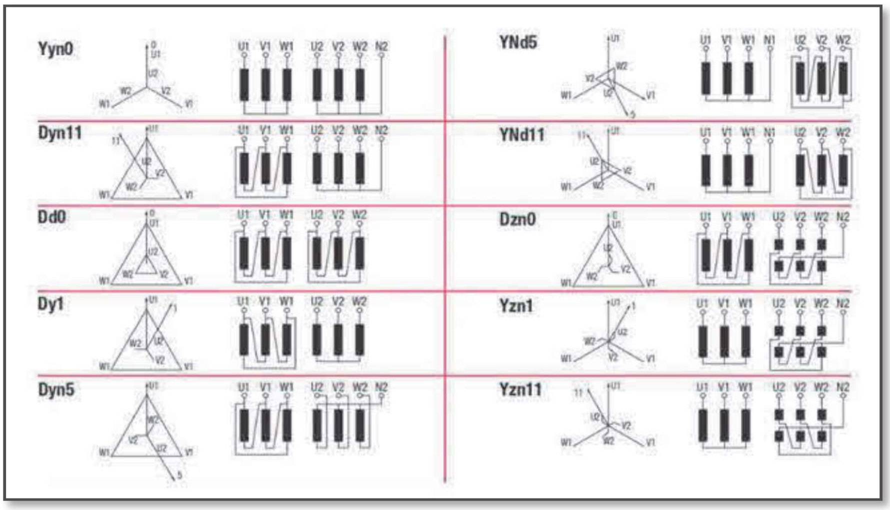


Fig.5.15 Connection diagram and vector diagram of *Dy11* coupling

Tab 5.1 Table summarizing hourly index according to coupling



5.12 Connecting transformers in parallel

To connect three-phase transformers in parallel, it is necessary that:

- The respective voltages must be equal
- The coupling groups must be the same
- The short-circuit voltages must be equal

TD N°5 : Exercise in Electrical Transformers

Exercice 5.1 :

The transformation ratio of a perfect transformer is equal to 0.127.

Calculate:

1- The effective value of the secondary voltage when $U_1 = 220\ V$.

2- The secondary winding has 30 turns, what is the number of turns in the primary.

3- Under load, the primary absorbs an effective current of 0.5 A. Calculate the effective value of the current in the secondary.

Exercice 5.2 :

Measuring the effective values of the primary and secondary voltages of a perfect transformer gave: $U_1 = 230\ V$, $U_{20} = 24$ V

Calculate:

1- the transformation ratio and the number of turns in the secondary if $N_1 = 1030$.

2- The secondary delivers 2.5 A in an inductive load with a power factor equal to 0.8.

3- Calculate the effective current I_1 and the different powers of the primary.

Exercice 5.3 :

A single-phase control and signaling transformer has the following characteristics:

230 V/ 24 V 50 Hz, 630 VA

1- The total losses at nominal load are 54.8 W. Calculate the nominal efficiency of the transformer for Cos φ_1 = 1 and Cos φ_2 = 0.3.

2- Calculate the nominal current at the secondary I_{2N}.

3- The no-load losses (iron losses) are 32.4 W. Deduce the Joule losses at nominal load. Deduce R_S, the resistance of the windings brought back to the secondary.

Exercice 5.4 :

A single-phase transformer has the following information on its nameplate:

$S=2200VA$, $\eta= 0.95$, Primary $V_{1n} = 220\ V$, secondary $V_{2n} = 127\ V$.

1- Calculate the nominal primary current I_{1n} and the nominal secondary current I_{2n}

2- The efficiency is specified for a load absorbing the nominal current under nominal secondary voltage and having a power factor Cosφ= 0.8.

Calculate the value of the losses in the transformer under these conditions.

Exercice 5.5 :

The magnetic flux density in the core of a 4.4-kVA, 4400/440-V, 50-Hz, step-down transformer is 0.8 T (rms). If the induced emf per turn is 10 V, determine:

a- the primary and secondary turns,

b- the cross-sectional area of the core,

c- the full-load current in each winding.

Exercice 5.6 :

A 200 turn coil is immersed in a 60 Hz flux with an effective value of 4 mWb. Obtain an expression for the instantaneous value of the induced EMF. If a voltmeter is connected between its two ends, what will be the reading on the voltmeter?

Exercice 5.7 :

The number of turns in the primary and the secondary of an ideal transformer are 200 and 500, respectively. The transformer is rated at 10 kVA, 250 V, and 60 Hz on the primary side. The cross-sectional area of the core is 40 cm^2. If the transformer is operating at full load with a power factor of 0.8 lagging, determine

a- the effective flux density in the core,

b- the voltage rating of the secondary,

c- the primary and secondary winding currents,

d- the load impedance on the secondary side and as viewed from the primary side

Chapter 6:

Introduction of Rotating Electrical Machines

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Introduction of Rotating Electrical Machines

6.1 Introduction

Rotating electrical machines are fundamental components in modern energy systems, enabling the conversion of energy between electrical and mechanical forms. They are used extensively in industrial processes, transportation systems, household appliances, and power generation. These machines are essential in both motor and generator applications: motors convert electrical energy into mechanical motion, while generators perform the reverse operation.

The study of rotating electrical machines forms a cornerstone of electrical engineering education, as these machines embody key electromagnetic principles and serve as a bridge between theoretical knowledge and real-world applications. Understanding their structure, operation, and classification is vital for designing, analyzing, and controlling electromechanical systems.

Rotating electrical machines are electromechanical devices that convert electrical energy into mechanical energy (motors) or vice versa (generators). They have a rotating part called a rotor, which interacts with a fixed part, the stator, to produce movement or generate electricity using electromagnetic principles. When it converts electrical energy into mechanical energy, it is called an electric motor, and when it converts mechanical energy into electrical energy, it is called a generator. Each electrical machine can switch from being a motor to a generator by changing its energy source.

This chapter aims to provide an introductory overview of rotating electrical machines. It will first classify the different types of machines and describe their main components and working principles. A comparative discussion between AC and DC machines will be presented, followed by an outline of typical applications and recent developments in the field. This foundational knowledge will prepare students for more advanced topics in electrical machines, control systems, and power electronics.

6.2 Classifications of Rotating Electrical Machines

Rotating electrical machines are generally classified into three main types: Direct current machines, alternating current machines (including synchronous and induction machines), and special-purpose machines. These machines can operate either as motors, converting electrical energy into mechanical energy, or as generators, converting mechanical energy into electrical energy.

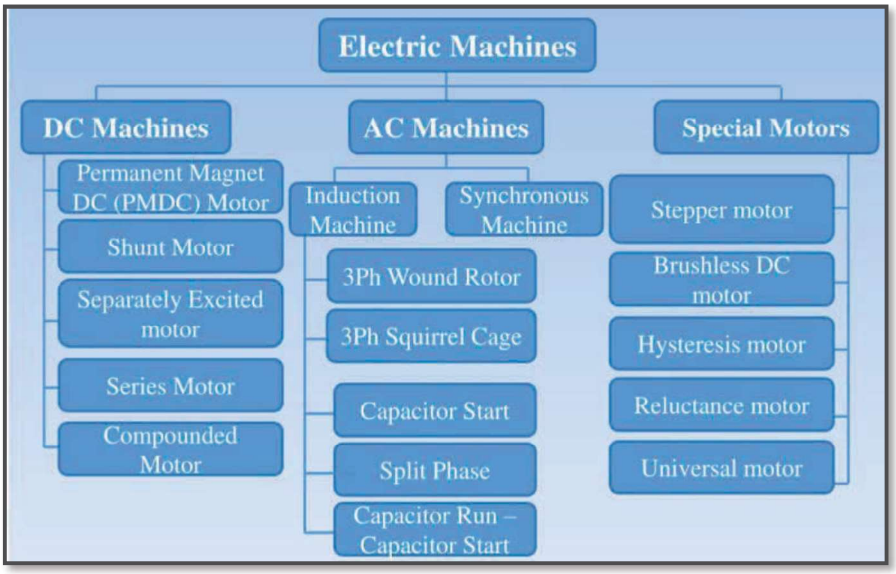

Fig.6.1 Classifications of Rotating Electrical Machines

6.3 Historical Development of Rotating Electrical Machines

The evolution of rotating electrical machines is deeply intertwined with the broader history of electromagnetism and industrialization. These machines are the result of centuries of discoveries, innovations, and engineering advancements.

6.3.1 Early Discoveries (18th–19th Century)

The foundation of electrical machines was laid in the late 18th and early 19th centuries with the discovery of the relationship between electricity and magnetism. In 1820, Hans Christian Ørsted demonstrated that an electric current produces a magnetic field, and shortly after, André-Marie Ampère formulated the laws governing electromagnetic forces. These principles paved the way for the concept of electromechanical conversion.

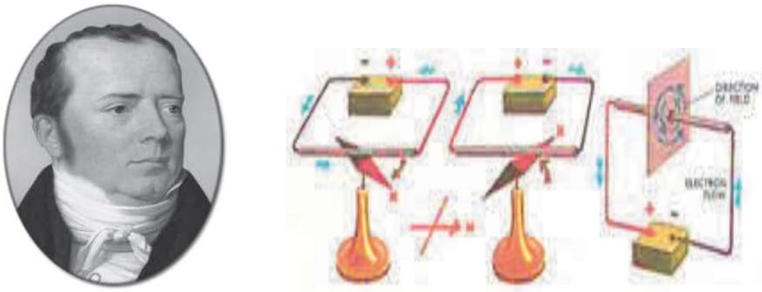

Fig 6.2 Hans Christian Ørsted and relationship between electricity and magnetism

In 1831, Michael Faraday discovered electromagnetic induction, showing that a changing magnetic field could induce an electric current. This principle remains the cornerstone of all electric generators and many motor types.

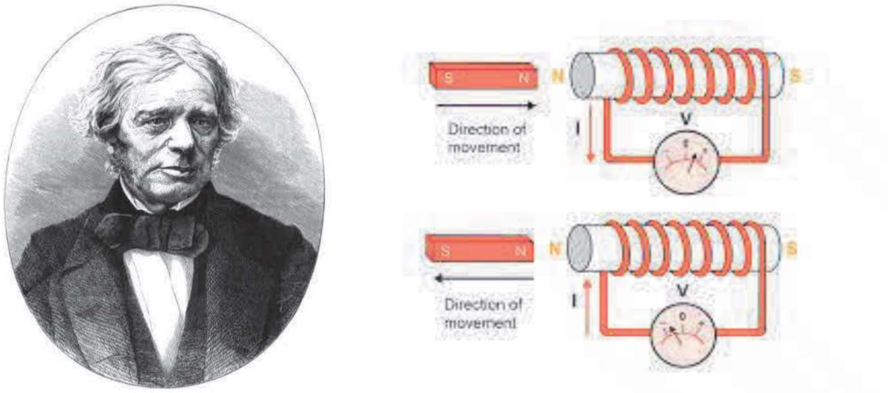

Fig 6.3 Michael Faraday discovered electromagnetic induction

6.3.2 First Rotating Machines

The first practical rotating machine was the commutator-type DC generator (dynamo), developed by Hippolyte Pixii in 1832, followed by improvements by Zénobe Gramme, who invented the Gramme ring in the 1870s, making continuous direct current generation feasible.

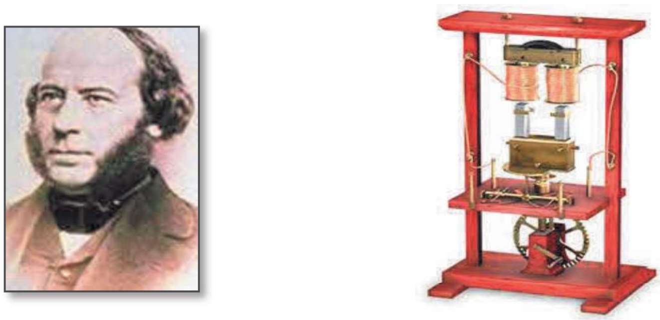

Fig 6.4 Commutator-type DC generator (dynamo), developed by Hippolyte Pixii

Fig 6.5 Zénobe Gramme, who invented the Gramme ring

Meanwhile, Thomas Edison and Werner von Siemens contributed to the development of DC motors and power systems in the late 19th century.

Fig 6.6 Meanwhile, Thomas Edison and Werner von Siemens

6.3.3 The Rise of Alternating Current (*AC*) Machines

The late 19th century saw the "War of Currents" between direct and alternating current systems. Nikola Tesla made significant contributions by developing the induction motor and advocating for AC power transmission. His work, along with that of George Westinghouse, laid the foundation for modern AC machines.

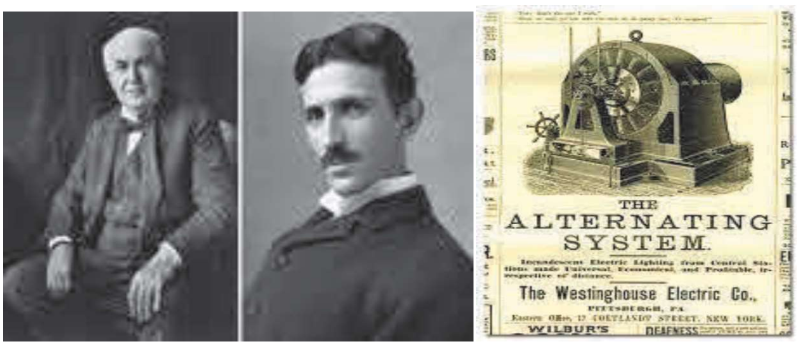

Fig 6.7 Nikola Tesla and George Westinghouse modern foundation for AC machines

The synchronous machine, vital for large-scale power generation, also emerged during this period. It became the standard machine in hydroelectric and thermal power plants. Throughout the 20^{th} century, rotating machines became more efficient, compact, and reliable. Advances in materials (e.g., better magnetic steels, insulation) and the development of power electronics greatly improved control and performance. The introduction of variable-speed drives, vector control, and digital control systems transformed the way motors are used in industry, enabling precision, energy savings, and automation.

6.3.4 Modern Era and Technological Integration

Today, rotating electrical machines are an integral part of smart grids, renewable energy systems, and electric vehicles. High-efficiency machines such as permanent magnet synchronous motors (*PMSMs*) and brushless DC motors (BLDCs) are now common in robotics, aerospace, and clean energy applications. Research continues into high-speed machines, superconducting machines, and compact drives to meet the demands of modern industry and environmental sustainability. Rotating electrical machines can be further categorized based on the type of current they use and their operational characteristics:

6.4 Types of Rotating Electrical Machines

There are two main types of electric motors: direct current (*DC*) motors and alternating current (*AC*) motors. Electric motors are used in industrial and residential applications.

6.4.1 Direct Current (DC) Machines

DC machines operate using a direct current supply. They consist of a stator, which provides a stationary magnetic field, and a rotor (also known as the armature), where the electromotive force is induced. *DC* machines are known for their excellent speed control characteristics and are widely used in applications requiring variable speed, such as electric traction, robotics, and precision tools.

- *DC* machines are subdivided into:
- *DC* motors (energy conversion: electrical → mechanical)
- *DC* generators (energy conversion: mechanical → electrical)

6.4.2 Alternating Current (*AC*) Machines

AC machines use alternating current and dominate most industrial and commercial applications due to their simplicity, robustness, and efficiency.

The electric motor uses electromagnetic force to generate movement. It converts electricity into mechanical energy through magnetization. The stator (static part) turns the rotor (moving part) using the force of the current. The motor can create movement by receiving electricity, but it can also create electricity when it is set in motion.

6.5 Direct current machine (DC)

The direct current machine is an energy converter, totally reversible, it can operate either as a motor, converting electrical energy into mechanical energy, or as a generator, converting mechanical energy into electrical energy. In both cases a magnetic field is necessary for the different conversions. This machine is therefore an electromechanical converter.

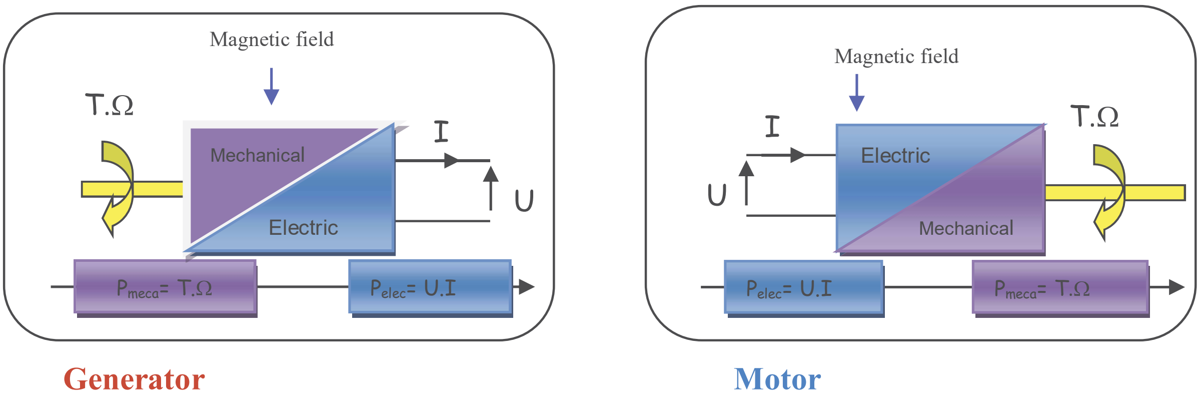

Fig.6.8 Generator and Motor *DC*

Mechanical energy is characterized by a moment torque T associated with an angular speed Ω, the product of these two quantities defines the mechanical power.

$$P_{mec} = T.\Omega \qquad (6.1)$$

P_{meca} : Mechanical power in watts [W]

T : Moment of mechanical torque in newton meters [Nm]

Ω : Angular velocity in radians per second [$rad.s^{-1}$]

Electrical energy is evaluated by a direct current I and a direct voltage U, the electrical power will be the product of these two quantities:

$$P_{elec} = U.I \qquad (6.2)$$

P_{elec} : Electrical power in watts $[W]$

U : Voltage in volts $[V]$

I : Current intensity in amperes $[A]$

Tab: 6.1 Type of motors

Energy absorbed	Running	Energy supplied
Electrical	Motor	Mecanical
Mecanical	Generator	Electrical

6.6 DC Machine Construction

The stator of the dc motor has poles, which are excited by dc current to produce magnetic fields. In the neutral zone, in the middle between the poles, commutating poles are placed to reduce sparking of the commutator. The commutating poles are supplied by *dc* current. Compensating windings are mounted on the main poles. These short-circuited windings damp rotor oscillations.

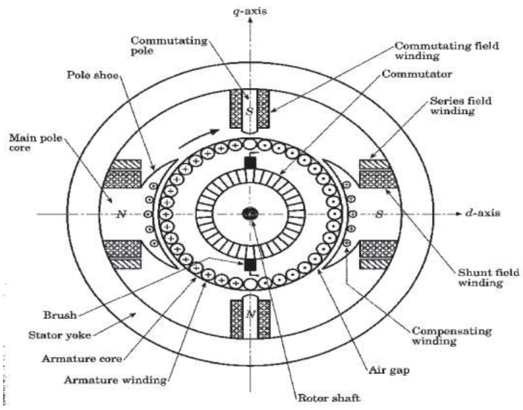

Fig:6.9 General arrangement of a *DC* machine

The poles are mounted on an iron core that provides a closed magnetic circuit. The motor housing supports the iron core, the brushes and the bearings. The rotor has a ring-shaped laminated iron core with slots. Coils with several turns are placed in the slots. The distance between the two legs of the coil is about 180 electric degrees. The coils are connected in series through the commutator segments. The ends of each coil are connected to a commutator segment. The commutator consists of insulated copper segments mounted on an insulated tube. Two brushes are pressed to the commutator to permit current flow. The brushes are placed in the neutral zone, where the magnetic field is close to zero, to reduce arcing. The rotor has a ring-shaped laminated iron core with slots. The commutator consists of insulated copper segments mounted on an insulated tube. Two brushes are pressed to the commutator to permit current flow. The brushes are placed in the neutral zone, where the magnetic field is close to zero, to reduce arcing.

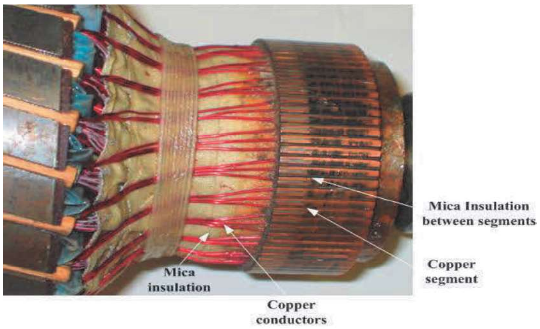

Fig.6.10 DC Machine construction

The commutator switches the current from one rotor coil to the adjacent coil, the switching requires the interruption of the coil current. The sudden interruption of an inductive current generates high voltages .The high voltage produces flashover and arcing between the commutator segment and the brush.

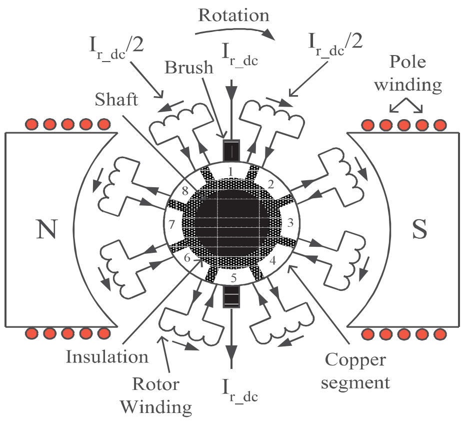

Fig.6.11 Commutator with the rotor coils connections

The active conductors, of number N, cut the lines of the magnetic field, they are therefore the seat of induced electromotive forces, the electromotive force *F.e.m* resulting from all of these *N* turns:

$E = N.n.\emptyset$ (6.3)

E : The *F.e.m* in volts [*V*]

N : Rotation frequency in revolutions per second [$tr.s^{-1}$]

$\emptyset$: The flow in Webers [Wb]

N: The number of active drivers.

This relationship is essential for the machine, because it is the link between the flux , a magnetic quantity, the voltage E, an electrical quantity, and the rotation frequency n, a mechanical quantity. Knowing that the $\Omega = 2.\pi.n$, another relation, linking the three types of quantities, is frequently used, it takes into account the angular speed Ω expressed in radians per second:

$$E = K.\emptyset.\Omega \qquad (6.4)$$

K : Constant

6.7 DC Generator

The N-S poles produce a dc magnetic field and the rotor coil turns in this field. A turbine or other machine drives the rotor. The conductors in the slots cut the magnetic flux lines, which induce voltage in the rotor coils. The coil has two sides: one is placed in slot a, the other in slot b.

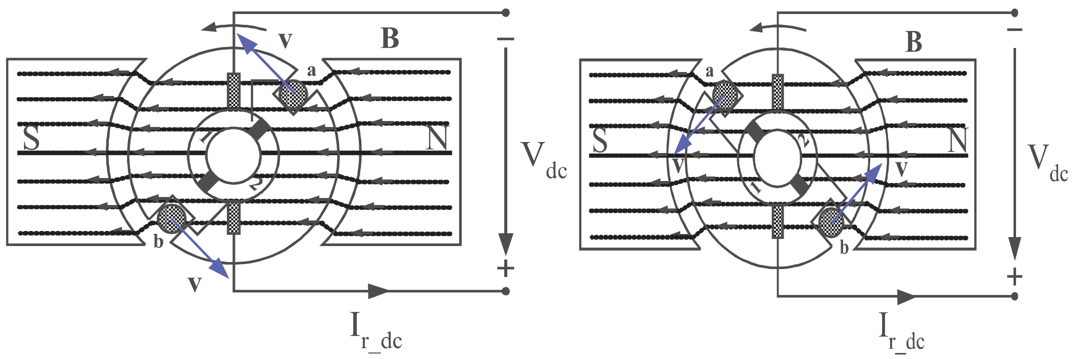

Fig.6.12
(a) Rotor current flow from segment 1 to 2 **(b)** Rotor current flow from segment 2 to 1

In figure 6.12**,** the conductors in slot a are cutting the field lines entering into the rotor from the north pole. The conductors in slot b are cutting the field lines exiting from the rotor to the south pole. The cutting of the field lines generates voltage in the conductors. The voltages generated in the two sides of the coil are added. The induced voltage is connected to the generator terminals through the commutator and brushes.

In Figure 6.12.a, the induced voltage in b is positive, and in a is negative. The positive terminal is connected to commutator segment 2 and to the conductors in slot b.

The negative terminal is connected to segment 1 and to the conductors in slot a. When the coil passes the neutral zone: Conductors in slot a are then moving toward the south pole and cut flux lines exiting from the rotor .Conductors in slot b cut the flux lines entering the in slot b. This changes the polarity of the induced voltage in the coil. The voltage induced in a is now positive, and in b is negative. The simultaneously the commutator reverses its terminals, which assures that the output voltage (V_{dc}) polarity is unchanged.

In Figure 6.12.b ;the positive terminal is connected to commutator segment 1 and to the conductors in slot a and the negative terminal is connected to segment 2 and to the conductors in slot b.

6.7.1 Operation without load and at constant rotation frequency

The rotor of the machine is driven by an external source at the rotation frequency n. We will say that the generator is running empty when it does not deliver any current.

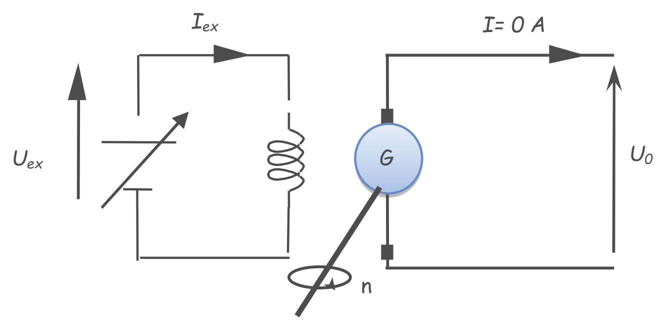

Fig.6.13 Operation of a no-load generator

Relation $E=N.n.\Phi$ is therefore characterized by two constants, the number of conductors N, and the rotation frequency n with which the generator is driven. The *f.e.m E* is in this case proportional to the flow ϕ, it is therefore to the nearest coefficient the image of the magnetization curve of the machine. The index "*o*" characterizes empty operation.

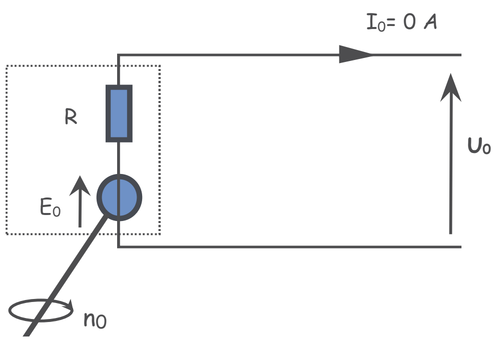

Fig.6.14 Voltage of U_0

The voltage U_0 measured directly on the armature of the generator is exactly equal to the $F.e.m$. E_0 of the machine because the current intensity is zero, there is therefore no voltage drop due to the resistance of the armature.

6.7.2 Operation with resistive load

The generator is driven by an auxiliary motor, it delivers a current of intensity I into a load rheostat.

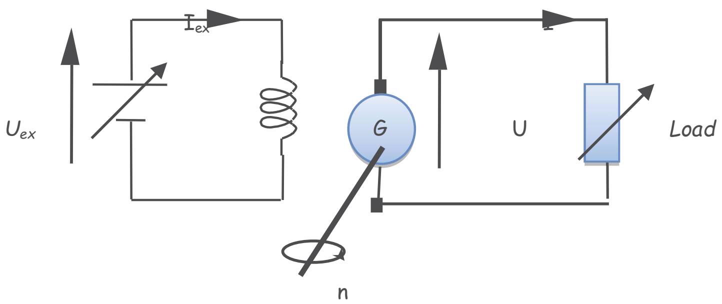

Fig.6.15 Operation with resistive load

The generator armature can be replaced by its equivalent model:

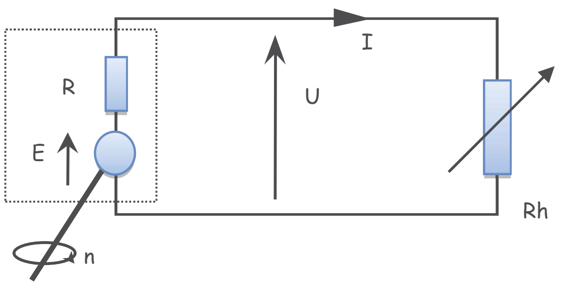

Fig.6.16 Equivalent Model of Generator Armature

Ohm's law of the armature is easily deduced from its equivalent model:

$$U = E - RI \qquad (6.5)$$

Depending on the values taken by the resistive load, the moment of the torque (*U; I*) of the voltage across the armature and the intensity of the current in the armature can only move on the straight line determined by two values particular:

I_{sc} maximum value of the current intensity in the short-circuited armature, $U = 0\ V$

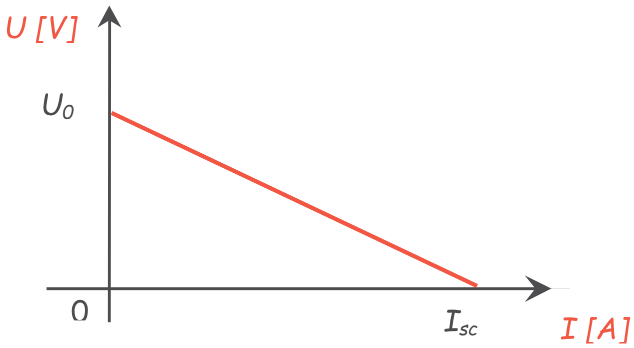

Fig.6.17 $U = f\,(I)$

We can plot the characteristic of the ohmic load *R* using Ohm's law, the moment of the couple (*U; I*) of the voltage across the load and the intensity of the current passing through it moves only on the straight line with directing coefficient equal to the value of *R*:

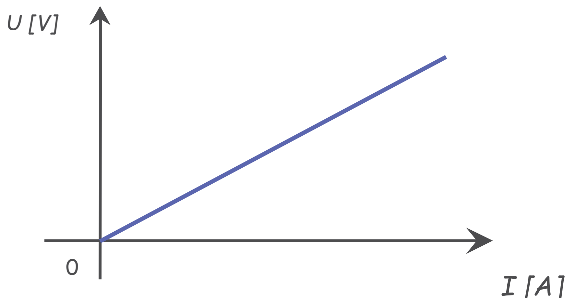

Fig.6.18 $U = f\,(I)$

The operating point of the Armature – Resistive load group can be determined graphically. It corresponds to the simultaneous operation of the power supply and the receiver. The two couples (current; voltage) resulting from the two characteristics must be equal since they are associated, as follows:

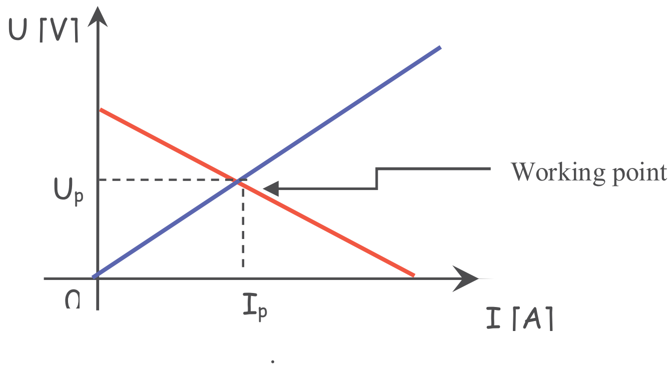

Fig.6.19 Graphical evaluation of the operating point

The operating point can also be calculated from the two equations:

$$U = E - RI \qquad (6.6)$$

$$U = R_h . I \qquad (6.7)$$

The point of intersection (U_{pf}; I_{pf}) of these two lines gives the quantities common to the two dipoles.

6.7.3 Power balance

The power balance breaks down all powers, from the absorbed power of mechanical origin to the useful power of an electrical nature. Between these two terms, the study will focus on all losses, both mechanical and electrical, and finally one power will be studied in particular, it corresponds to the transition from mechanical power to electrical power. The results can be summarized using the following diagram:

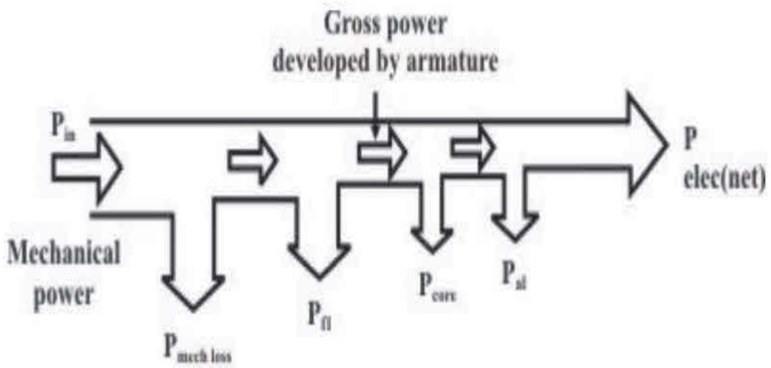

Fig:6.20 Balance of the powers of a generator

The generator receives power P_a , product of the moment of the mechanical torque T coming from an auxiliary system and the angular speed Ω .All the powers involved in this assessment can be calculated from the following relationships.

$P_a = T.\Omega$ (6.8)

$P_c = T_p.\Omega$ (6.9)

$P_{em} = T_{em}.\Omega$ (6.10)

$P_{em} = E.I$ (6.11)

$P_j = R.I^2$ (6.12)

$P_u = U.I$ (6.13)

The assessment highlights the fact that the absorbed power is necessarily the most important power, it continues to decrease as it progresses towards the useful power which is obviously the lowest, thus:

$P_{em} = P_a - P_c$ and $P_u = P_{em} - P_j$; So $P_u = P_a - P_c - P_j$ (6.14)

P_c represents the sum of mechanical losses and magnetic losses in the generator. T_p is the moment of the pair of losses corresponding to this lost power.

- Magnetic losses due to hysteresis and eddy currents occur in the rotor laminations.
- Mechanical losses due to friction are located at the level of the bearings.

The efficiency is the ratio between the useful electrical power and the mechanical power absorbed by the armature, hence:

$\eta = \frac{P_u}{P_a}$ (6.15)

The efficiency of the complete generator takes into account the power absorbed by the inductor, P_{ex} to the extent that it is electrically powered. This power is only used to magnetize the machine, all the active power absorbed by the excitation circuit is entirely consumed by the Joule effect therefore:

$P_{ex} = U_{ex}.I_{ex}$ (6.16)

$P_{ex} = r.{I_{ex}}^2$ (6.17)

$P_{ex} = r.{I_{ex}}^2$ (6.18)

$P_{ex} = \frac{{U_{ex}}^2}{r}$ (6.19)

The yield is therefore:

$$\eta = \frac{P_u}{P_a + P_{ex}} \qquad (6.20)$$

P_a : The power absorbed in watts [W];

T : The moment of the mechanical torque in newton meters [Nm]

P_C : Collective losses in watts [W]

T_p : The moment of the loss couple in newton-meters [Nm]

P_{em} : Electromagnetic power in watts [W]

T_{em} : The moment of the electromagnetic torque in newton meters [Nm]

P_u : Useful power in watts [W]; P_j : Joule effect losses in watts [W]

P_{ex} : The power absorbed by the inductor in watts [W]

U_{ex} : The supply voltage of the inductor in volts [V]

I_{ex} :The intensity of the current in the inductor in amperes [A]

r :The resistance of the inductor in ohms

6.8 DC Motor

In a dc motor, the stator poles are supplied by dc excitation current, which produces a dc magnetic field. The rotor is supplied by dc current through the brushes, commutator and coils. The interaction of the magnetic field and rotor current generates a force that drives the motor .Before reaching the neutral zone, the current enters in segment 1 and exits from segment 2, Therefore, current enters the coil end at slot a and exits from slot b during this stage. After passing the neutral zone, the current enters segment 2 and exits from segment 1, This reverses the current direction through the rotor coil, when the coil passes the neutral zone. The result of this current reversal is the maintenance of the rotation.

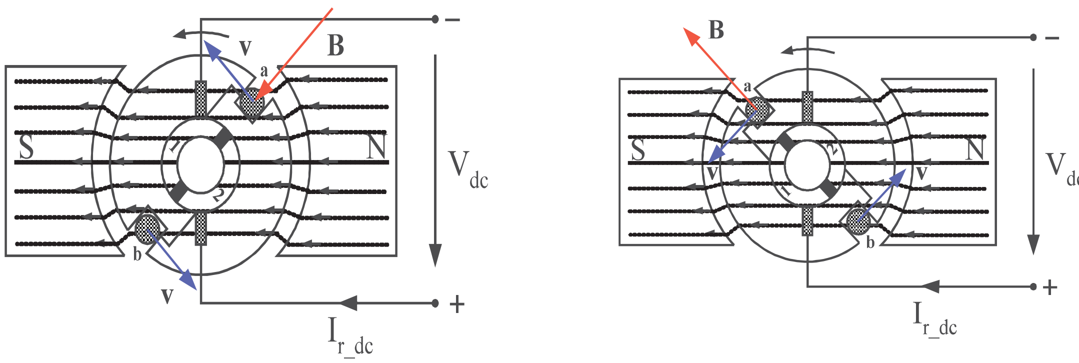

Fig. 6.21
(a) Rotor current flow from segment 1 to 2 (slot a to b)
(b) Rotor current flow from segment 2 to 1 (slot b to a)

6.8.1 Load operation

The motor armature is powered by a second *DC* voltage source, it drives a mechanical load at the rotation frequency n.

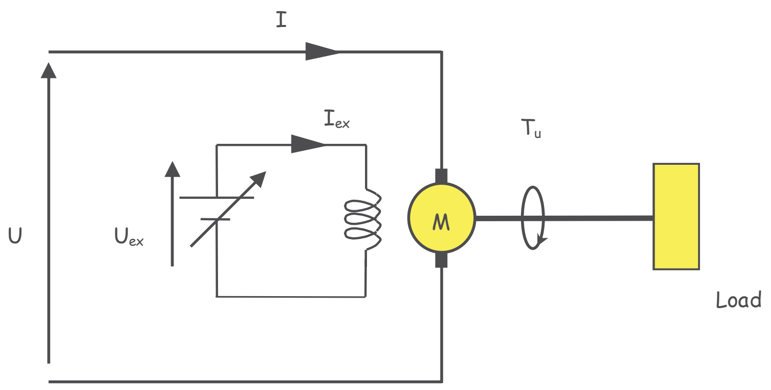

Fig.6.22 Fonctionnement d'un moteur en charge

The motor absorbs electrical power and returns mechanical power, a combination of the useful torque and the rotation frequency.

6.8.2 Ohm's law

The motor armature can be replaced by its equivalent model:

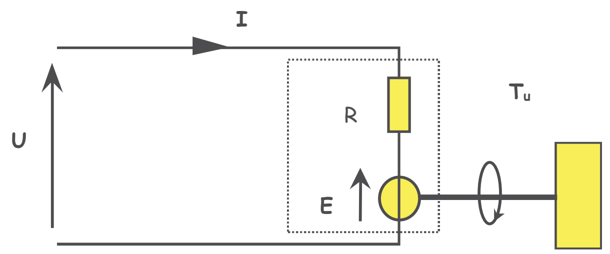

Fig.6.23 Equivalent model of the motor

Armature Ohm's law of the armature is easily deduced from its equivalent model:

$$U = E + RI \qquad (6.21)$$

The nameplate of a motor gives valuable information, it concerns the most appropriate operation, that is to say that which allows a very good efficiency, not necessarily the highest, but which ensures a very good longevity of the machine. The values mentioned for the armature are called the nominal values, they must not be exceeded by more than 1.25 times, they break down as follows:

6.8.3 Balance of powers

The power balance breaks down all powers, from the absorbed power of electrical origin to the useful power of a mechanical nature. Between these two terms, the study will focus on all losses, both mechanical and electrical, and finally one power will be studied in particular, it corresponds to the transition from electrical power to mechanical power. The results can be summarized using the following diagram:

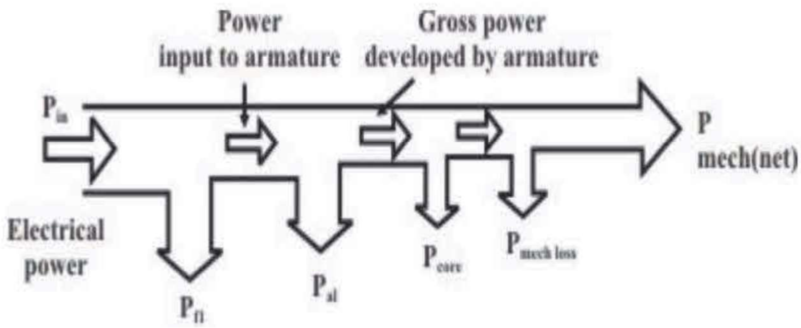

Fig:6.24 Assessment of the powers of an engine

All the powers involved in this assessment can be calculated from the following relationships. The motor receives a power Pa, product of the voltage applied to the terminals of the armature and the intensity of the current passing through it.

$P_a = U.I$ (6.22)

$P_j = R.I^2$ (6.23)

$P_{em} = E.I$ (6.24)

$P_{em} = T_{em}.\Omega$ (6.25)

$P_c = T_p.\Omega$ (6.26)

$P_a = T\Omega$ (6.27)

The assessment highlights the fact that the absorbed power is necessarily the most important power, it continues to decrease as it progresses towards the useful power which is obviously the lowest, thus:

$P_a = U.I$; $P_{em} = P_a - P_j$; and $P_u = P_{em} - P_c$ so $P_u = P_a - P_j - P_c$ (6.28)

• P_c represents the sum of mechanical losses and magnetic losses in the motor. T_p is the moment of the pair of losses corresponding to this lost power.

• Magnetic losses due to hysteresis and eddy currents occur in the rotor laminations.

• Mechanical losses due to friction are located at the level of the bearings.

The efficiency is the ratio between the useful mechanical power and the electrical power absorbed by the armature, hence: $\eta = \frac{P_u}{P_a}$

The efficiency of the complete motor takes into account the power absorbed by the inductor, P_{ex} , to the extent that it is electrically powered. This power is only used to magnetize the motor, all the active power absorbed by the excitation circuit is entirely consumed by the Joule effect therefore:

$$P_{ex} = U_{ex}.I_{ex} \quad (6.29)$$

$$P_{ex} = r.I_{ex}^2 \quad (6.30)$$

$$P_{ex} = \frac{U_{ex}^2}{r} \quad (6.31)$$

The yield is therefore:

$$\eta = \frac{P_u}{P_a+P_{ex}} \quad (6.32)$$

6.8.4 Load test

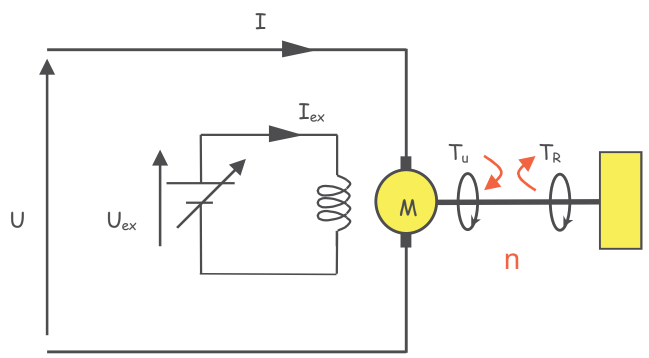

Fig.6.25 Equivalent model of the motor

The motor is now loaded, that is to say that the motor shaft drives a resistive load which opposes the movement of the rotor. At steady state, the moment of the useful torque delivered by the motor is equal to the moment of the resistant torque opposed to it by the mechanical load. In steady state :

$T_u = T_R$	T_u	The moment of the useful torque in newton meters [Nm]
	T_R	The moment of the resisting torque in newton meters [Nm]

6.8.5 Working point

The operating point is located at the intersection of the mechanical characteristic of the motor and the curve which characterizes the moment of the resisting torque of the load.

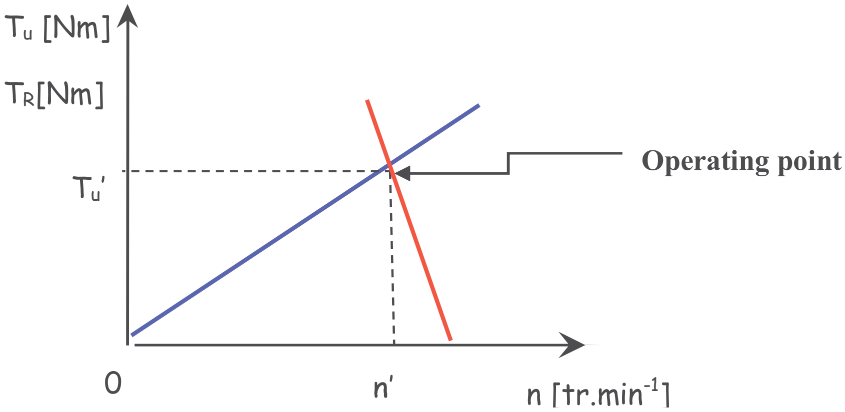

Fig. 6.26 Graphical evaluation of the operating point

The operating point graphically gives n', the rotation frequency of the motor as well as $T_{u'}$ the moment of the useful torque moment.

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Exercice 6.1 :

The nameplate of a DC generator with independent excitation indicates: Inductor 220V 6.8A; excitation 220V 0.26A mass 38kg. At nominal operation a torque of 11.2NM drives the generator at a speed of 1500 rpm:

1- Calculate the mechanical power consumed and the power consumed by the excitation.

2- Calculate the useful power and deduce the nominal efficiency.

Exercice 6.2 :

An independently excited generator delivers a constant emf of 210V for an inductor current of 2 A. The resistances of the armature and inductor windings are 0.6 Ω and 40 Ω respectively. The "constant" losses are 400 W. For a flow rate of 45 A, calculate:

1- The armature voltage U and the useful power P_u .

3- The armature and inductor Joule losses ;the absorbed power P_a and the efficiency η

Exercice 6.3 :

A DC motor with independent and constant excitation is supplied with 240 V. The armature resistance is equal to 0.5 Ω, the inductor circuit absorbs 250 W and the collective losses amount to 625 W. At nominal operation, the motor consumes 42 A and the rotation speed is 1200 rpm.

Calculate:

- The emf - the absorbed power, the electromagnetic power and the useful power; the useful torque and the efficiency

What is the rotation speed of the motor when the armature current is 30 A? What happens to the useful torque at this new speed (assuming that the collective losses are still equal to 625 W)? Calculate the efficiency.

Exercice 6.4 :

The nameplate of an independently excited motor has the following information: U= 240V; I = 35 A; P = 7 kW; n = 800 rpm. Calculate (at rated load):

1- The efficiency of the motor knowing that the inductor Joule losses are 150 watts.

2- The armature Joule losses knowing that the armature has a resistance of 0.5 Ω.

3- The electromagnetic power and the “constant” losses.

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